Elementary Formal Systems with the subword property characterize the class P

Ikeda, Daisuke
Department of Informatics Kyushu University

Arimura, Hiroki
Department of Informatics Kyushu University

出版情報：DOI Technical Report. 125, 1996-05-28. Department of Informatics, Kyushu University
バージョン：
権利関係：

DOI-TR-125

DOI Technical Report

Elementary Formal Systems with the subword property characterize the class P

Daisuke IKEDA and Hiroki ARIMURA

May 28,1996

Department of Informatics
Kyushu University 33
Fukuoka 812, Japan
E-mail: daisuke@i.kyushu-u.ac.jp Phone: +81-92-583-7635

Elementary Formal Systems with the subword property characterize the class P

Daisuke Ikeda
daisuke@i.kyushu-u.ac.jp

Hiroki Arimura
arim@i.kyushu-u.ac.jp

Department of Informatics
Graduate School of Information Science and Electrical Engineering
Kyushu University 33, Fukuoka 812, Japan

Abstract

An EFS is a kind of a grammar and generates a language. During an EFS's with the subword property generating an output, once a word appears, the word is a subword of the output. The property is not only natural and simple but ample to describe a computation of a polynomial time-bounded deterministic Turing machine. Our main result is that the class of elementary formal systems (EFSs for short) with the subword property is equal to the class P. We also give a membership problem for EFSs with the subword property and show that the class of languages generated by EFSs with the property is closed under some operations.

1 Introduction

Elementary formal systems (EFSs for short) introduced by Smullyan [1] has a rich structure to generate languages such as grammars. An EFS is a set of rules which transform patterns to patterns. A pattern consists of a variable and a constant symbol which correspond to a non-terminal and a terminal symbol, respectively, of a grammar.

Some classes defined by Turing machines and grammars has been studied by using EFSs. The following results are shown [3]: a language is recursively enumerable (resp. context-sensitive, context-free and regular) if and only if it is definable by a variable-bounded (resp. length-bounded, regular and linear) EFS. However there has been no study about computational complexity classes characterized by some EFSs. The purpose of this paper is to show computational complexities of EFSs with some properties, especially to show what property characterizes the class P.

Miyano et al. [4] showed that a language generated by an EFS with the *subword* property is accepted by a deterministic Turing machine in polynomial time. In each rule of an EFS with subword property, each pattern in body has to appear in the head, so that once a word appears during the EFS's generating a word w, w contains it. We show that any language in P is generated by an EFS with the subword property, the converse of their result. Thus P = H-EFS, which is the set of languages generated by EFSs with subword property.

Decision problems and closure properties for classes of grammars have been studied. We focus the membership problems and closure properties for H-EFS. We show that a membership problem for H-EFS is DEXPTIME-complete. We also show that H-EFS posses some closure properties.

In Section 2, the definitions of two-way multihead alternating finite automata and EFS are given. It is shown that the set of languages accepted by the automata is equal to the class P. We show the relation between P and tH-EFS using two-way multihead alternating finite automata, in Section 3. In Section 4, we give a membership problem and closure properties for EFSs.

2 Preliminaries

2.1 Two-way Multihead Alternating Finite Automata

A two-way multihead alternating finite automaton is intuitively a nondeterministic multihead finite automaton whose heads can move both left and right, and states are either existential or universal. Its definition is formally given as follows.

Definition 1. A *two-way alternating finite automaton with k heads* (2AFA(k) for short) is a 6-tuple $(K, \Sigma, \delta, q_0, F, U)$, where

- K is the nonempty finite set of states,
- Σ is the input alphabet which does not contain the endmarkers ¢ and \$,
- δ is a mapping from $K \times (\Sigma \cup \{¢, \$\})^k$ into a subset of $K \times \{-1, 0, +1\}^k$ with the restriction that for each $1 \leq j \leq k$, $d_j \geq 0$ if $a_j = ¢$, and $d_j \leq 0$ if $a_j = \$$,
- $q_0 \in K$ is the initial state, and $F \subseteq K$ is the set of accepting states, and
- $U \subseteq K$ is the set of *universal* states and $K - U$ is the set of *existential* states.

Let $M = (K, \Sigma, \delta, q_0, F, U)$ be a 2AFA(k). A *configuration* of M on input $w \in \Sigma^*$ is a $(k+1)$-tuple $(q, h_1, \ldots, h_k) \in K \times \{0, \ldots, n+1\}^k$, which means that the state of M is q and the j-th head is scanning the h_j-th symbol of the input tape for each $1 \leq j \leq k$. We define by convention that the 0-th and $(n+1)$-st symbols of $¢w\$$ are ¢ and \$, respectively. A transition relation of M is a binary relation on the configurations of M on input w given by

$$(p, h_1, \ldots, h_k) \vdash_M (q, h_1 + d_1, \ldots, h_k + d_k)$$

if $(q, d_1, \ldots, d_k) \in \delta(p, a_1, \ldots, a_k)$ and for each $1 \leq j \leq k$, a_j is the h_j-th symbol of $¢w\$$. If $C \vdash D$ for some configurations C and D, then we say that D is an *immediate descendant* of C. We denote by $\vdash_M^*$ the reflexive transitive closure of $\vdash_M$. The *initial configuration* is $(q_0, 1, \ldots, 1)$ and an *accepting configuration* is any configuration $(q, h_1, \ldots, h_k)$ with $q \in F$. A configuration $(q, h_1, \ldots, h_k)$ is *universal* (*existential*) if q is universal (existential).

Definition 2. An *accepting computation tree* of M on the input w is a finite tree T whose nodes are labeled with configurations of M, where

- The root is labeled with the initial configuration of M.
- Let u be an internal node labeled with a configuration C, and $D_1, \ldots, D_n$ be all the immediate descendants of C. If C is universal then u has the children $u_1, \ldots, u_n$ labeled with $D_1, \ldots, D_n$, respectively. If C is existential then u has exactly one child labeled with u_i for some $1 \leq i \leq n$.
- The leaves are labeled with accepting configurations.

We say that M *accepts* w if there is an accepting computation tree of M on input w. Then, the *language accepted by* M is the set of strings accepted by M. We denote by 2AFA(k) the class of languages accepted by a 2AFA(k) . We define 2AFA $= \cup_{k \geq 1}$2AFA(k).

Theorem 1. (Chandra el al.[5], King [2]) P = ASPACE(log n) = 2AFA.

2.2 Elementary Formal Systems

Let Σ be a finite alphabet, X a set of variables, and Π a set of predicate symbols. We assume that Σ, X, and Π are mutually disjoint. Let Σ^* be the set of all words, Σ^+ the set of all nonempty words.

First we define a non-erasing EFS. A *pattern* is an element of $(\Sigma \cup X)^+$. An *atom* is an expression of the form $p(\tau_1, \ldots, \tau_n)$, where p is a predicate symbol in Π with arity n and $\tau_1, \ldots, \tau_n$ are patterns. A *definite clause* is a clause of the form

$$A \leftarrow B_1, \ldots, B_m,$$

where $m \geq 0$ and $A, B_1, \ldots, B_m$ are atoms. The atom A is called the *head* and the part $B_1, \ldots, B_m$ the *body* of the definite clause. We say that a definite clause

$$q(\pi_1, \ldots, \pi_n) \leftarrow q_1(\tau_1, \ldots, \tau_{t_1}), q_2(\tau_{t_1+1}, \ldots, \tau_{t_2}), \ldots, q_l(\tau_{t_{l-1}+1}, \ldots, \tau_{t_l})$$

is *hereditary* if, for each $1 \leq j \leq t_l$, a pattern τ_j is a subword of some π_i.

An *elementary formal system* (EFS for short) is a triplet $S = (\Sigma, \Pi, \Gamma)$, where Γ is a finite set of definite clauses. An EFS (Σ, Π, Γ) is *hereditary* if all definite clauses in Γ is hereditary.

A *substitution* θ is a homomorphism from patterns to themselves such that $\theta(a) = a$ for each symbol $a \in \Sigma$. For a pattern π and a substitution θ, we denote by $\pi\theta$ the pattern obtained from π by applying θ. For an atom $A = p(\pi_1, \ldots, \pi_n)$ and a definite clause $C = A \leftarrow B_1, \ldots, B_m$, we define $A\theta = p(\pi_1\theta, \ldots, \pi_n\theta)$ and $C\theta = A\theta \leftarrow B_1\theta, \ldots, B_m\theta$. A substitution θ is said to be *erasing* if θ maps some variables to the empty string, and *non-erasing* otherwise.

A definite clause C is *provable from* an EFS $S = (\Sigma, \Pi, \Gamma)$, denoted by $\Gamma \vdash C$, if C is obtained from Γ by finitely many applications of non-erasing substitutions and modus ponens. That is, we define the relation $\Gamma \vdash C$ inductively as follows:

(1) if $\Gamma \ni C$ then $\Gamma \vdash C$,

(2) if $\Gamma \vdash C$ then $\Gamma \vdash C\theta$ for any *non-erasing* substitution θ, and

(3) if $\Gamma \vdash A \leftarrow B_1, \ldots, B_m, B_{m+1}$ and $\Gamma \vdash B_{m+1} \leftarrow$, then $\Gamma \vdash A \leftarrow B_1, \ldots, B_m$.

Note that the empty pattern ε and erasing substitutions are not allowed in the definitions of EFS and the provability $\vdash$. If we allow ε and erasing substitutions, then we obtain *erasing EFS* and *erasing provability* $\vdash_\varepsilon$ instead. To emphasize the difference, we sometimes write non-erasing EFS and non-erasing provability for ordinary ones.

For $p \in \Pi$ of arity one, we define $L(\Gamma, p) = \{w \in \Sigma^+ \mid \Gamma \vdash p(w) \leftarrow\}$ and $L_\varepsilon(\Gamma, p) = \{w \in \Sigma^* \mid \Gamma \vdash_\varepsilon p(w) \leftarrow\}$. A language $L \subseteq \Sigma^+ (L \subseteq \Sigma^*)$ is non-erasingly (erasingly) *definable by EFS* if there exist a non-erasing (erasing) EFS $S = (\Sigma, \Pi, \Gamma)$ and some $p \in \Gamma$ such that $\Gamma \vdash p(w) \leftarrow (\Gamma \vdash_\varepsilon p(w) \leftarrow)$. The class of languages definable by non-erasing (erasing) hereditary EFS is denoted by H-EFS (H-EFS$_\varepsilon$).

A *proof tree* for an atom A from a non-erasing (erasing) EFS $S = (\Sigma, \Pi, \Gamma)$ is a finite tree such that

(1) if $\Gamma \ni A \leftarrow$, then a tree consisting of a single node labeled with A is a proof tree for A from S, and

(2) if there exists a clause $B \leftarrow B_1, \ldots, B_n$ such that $A = B\theta$ for some non-erasing (erasing) substitution θ, and if there are proof trees $T_1, \ldots, T_n$ for atoms $B_1\theta, \ldots, B_n\theta$ from S, respectively, then a tree whose root node is labeled with A and has children labeled with the atoms $B_1\theta, \ldots, B_n\theta$ is a proof tree for A from S.

The following lemma is immediate from the definition.

Lemma 1. For any atom A and any non-erasing (erasing) EFS S, a definite clause $A \leftarrow$ is provable from S if and only if there exists a proof tree for A from S.

3 H-EFS is equal to P

To show H-EFS = P, we pay attention to the result showed by Miyano et al. [4]. They studied the hereditary EFS from the viewpoint of PAC learnability and showed that the class H-EFS(m, k, t, r) is polynomial time learnable for any $m, k, t, r \geq 0$, where definite clauses are at most m and each of them satisfies the following: (1) the munber of variables occurrences in the head is at most k: (2) the number of atoms in the body is at most t: (3) the arity of each predicate symbol is at most r. In this proof, they construct a deterministic Turing machine that, given $w \in \Sigma^+$ and H-EFS S, decides whether $w \in L(S, p)$ in polynomial time.

Theorem 2. (Miyano el al. [4]) P = 2AFA $\supseteq$ H-EFS.

To simplify the proof of the converse of the above theorem, we give the following lemma. It makes a 2AFA(k) standarized. It is easy to prove the lemma by adding new states.

Lemma 2. For every 2AFA(k), there exists a 2AFA(k) M such that when p is the state of M, (1) if p is universal, M does not move its heads but change its state: (2) if p is existential, M moves at most one head at a time.

Theorem 3. P $\subseteq$ H-EFS$_\varepsilon$.

Proof: Let $L \in$ P, then there exists a 2AFA(k) $M = (K, \Sigma, \delta, q_0, F, U)$ such that M accepts L and satisfies Lemma 2.

Let $w \in \Sigma^*$ an input for M, $C = (p, h_1, \ldots, h_k)$ be a configuration of M on input w and $C' = (q, h_1, \ldots, h_{s-1}, h_s + d, \ldots, h_k)$ be an immediate descendant of C. The idea of this proof is that, if $d = +1$, the position of the s-th head is expressed by a triplet $(x_s, ay_s, x_s a y_s)$, where a is the h_s-th symbol on the input tape and $x_s a y_s = w$, and one at C' is also expressed by another triplet $(x_s a, y_s, x_s a y_s)$. The position of all the other heads at C is expressed by a triplet $(x_j, y_j, x_j y_j)$ for all $1 \leq j \neq s \leq k$ and it is also expressed by the same triplet $(x_j, y_j, x_j y_j)$ at C' since only the s-th head can move. Therefore, we can describe the above transition of M, $C \vdash_M C'$, by the definite clause

$$p(t_1, \ldots, t_k) \leftarrow q(t_1, \ldots, t_{s-1}, t'_s, t_{s+1}, \ldots, t_k, x_s a y_s),$$

where $t_s = (x_s, ay_s), t'_s = (x_s a, y_s), t_j = (x_j, y_j)$ for each $j \neq s$, and p, q are the predicate symbols with arity $2k+1$ since the last patterns in all t_s, t'_s and $t_j (1 \leq j \neq s \leq k)$ are the same. If the s-th head is scanning leftmost symbol of w, we use the empty word ε instead of words over Σ because erasing substitutions are allowed.

Although the input tape contains $|w| + 2$ symbols including the endmarkers, the above triplet $(x_s a, y_s, x_s a y_s)$ can express only $|w|$ positions for each head. So we introduce the *boundary flag* $b = b_1 \dots b_k \in \{¢, \$, 1\}^k$ for a configuration of a 2AFA(k) and use predicate symbols with the boundary flag. A predicate symbol p_b with $b = b_1 \dots b_k$ means that if $b_j = ¢$, the j-th head is at the left endmarker, if $b_j = \$$, at the right endmarker, and if $b_j = 1$, at any symbol of an input for each $1 \leq j \leq k$. If $b_j \neq 1$, we use a triplet $(\varepsilon, ay_s, ay_s)$ to indicate the s-th head position.

We construct an erasing hereditary EFS $S = (\Sigma, \Pi_M, \Gamma_M)$ by which L is definable. The alphabets of S and M are the same. The set of predicate symbols Π_M is defined as

$$\Pi_M = \{p_b \mid p \in K, b \in \{¢, \$, 1\}^k\} \cup \{p_0\},$$

where p_0 is the symbol such that $p_0 \notin K$. The predicate symbol p_0 is arity one and the other predicate symbols are arity $2k+1$.

The set of definite clauses of S is defined as $\Gamma_M = \cup_{i=0}^{5}\Gamma_i$. A definite clause in Γ_0 corresponds to the start and the end of a computation of M. The initial configuration is $(q_0, 1, 1, \dots, 1)$, so that Γ_0 consists of the following definite clauses:

- $p_0(x) \leftarrow q_{0b}(\varepsilon, x, \dots, \varepsilon, x, x)$ with $b = 1 \dots 1$,
- $q_b(t_1, \dots, t_k, x_1 y_1) \leftarrow$ for all $q \in F$ and $b \in \{¢, \$, 1\}^k$,

where $t_i = x_i, y_i \in X$ for each $1 \leq i \leq k$. In the rest of the proof, we assume $x_i, y_i \in X$ and denote a pair x_i, y_i by t_i.

A definite clause in Γ_1, Γ_2 and Γ_3 corresponds to a transition with heads of M moving. In this case, all we have to do is to consider existential states.

A definite clause in Γ_1 represents the s-th head at C is not at an endmarker and the head at C' is not at it. Thus the set Γ_1 consists of the following: for all $(q, d_1, \dots, d_k) \in \delta(p, a_1, \dots, a_k)$ with $d_j = 0 (1 \leq j \neq s \leq k)$ and $b = b_1 \dots b_k, b' = b'_1 \dots b'_k$ with $b_s = b'_s = 1$,

- $p_b(t_1, \dots, t_{s-1}, x_s, a_s y_s, \dots, t_k, x_s a_s y_s) \leftarrow q_{b'}(t_1, \dots, t_{s-1}, x_s a_s, y_s, \dots, t_k, x_s a_s y_s)$ if $d = +1$
- $p_b(t_1, \dots, t_{s-1}, x_s a_s, y_s, \dots, t_k, x_s a_s y_s) \leftarrow q_{b'}(t_1, \dots, t_{s-1}, x_s, a_s y_s, \dots, t_k, x_s a_s y_s)$ if $d = -1$

Definite clauses in Γ_2 and Γ_3 represent transitions between two configurations at one of which the s-th head is at an endmarker. A definite clause in Γ_2 corresponds to a transition from a configuration with $b_s \neq 1$, and one in Γ_3 corresponds to a transition into a configuration with $b_s \neq 1$. Therefore, Γ_2 consists of the following: for all $(q, d_1, \dots, d_k) \in \delta(p, a_1, \dots, a_k)$ such that p is existential and $d_j = 0 (1 \leq j \neq s \leq k)$, and for all $b = b_1 \dots b_k, b' = b'_1 \dots b'_k$,

- $p_b(t_1, \dots, t_{s-1}, \varepsilon, a_s x_s, \dots, t_k, a_s x_s) \leftarrow q_{b'}(t_1, \dots, t_{s-1}, \varepsilon, a_s x_s, \dots, t_k, a_s x_s)$ if $d = -1$,
- $p_b(t_1, \dots, t_{s-1}, x_s, a_s, \dots, t_k, x_s a_s) \leftarrow q_{b'}(t_1, \dots, t_{s-1}, x_s a_s, \varepsilon, \dots, t_k, x_s a_s)$ if $d = +1$,

where $b_s = 1, b'_s = ¢$ in the first dinite clauses and $b_s = 1, b'_s = \$$ in the second. Γ_3 consists of the same definite clauses except that the boundary flags b and b' are exchanged.

A definite clause in Γ_4 and Γ_5 represents a transition without the heads of M moving, when a configuration of M is universal and existential, respectively. So Γ_4 consists of the following definite clauses: for all $b \in \{¢, \$, 1\}^k$,

- $p_b(t_1, \ldots, t_k, x_1y_1) \leftarrow q_b^1(t_1, \ldots, t_k, x_1y_1), \ldots, q_b^m(t_1, \ldots, t_k, x_1y_1)$,

where $p \in K$ is universal and $q_1, \ldots, q_m$ are the states of the immediate descendants. Note that $x_iy_i = x_jy_j$ for all $1 \leq i, j \leq k$.

Even if the configuration of M is existential, M does not have to move heads of it. Thus Γ_5 consists of the following definite clauses: for all $(q, 0, \ldots, 0) \in \delta(p, a_1, \ldots, a_k)$ and for all $b \in \{¢, \$, 1\}^k$

- $p_b(t_1, \ldots, t_k, x_1y_1) \leftarrow q_b(t_1, \ldots, t_k, x_1y_1)$,

where p is existential.

The last patterns in each atoms of the definite clauses in Γ_M except for p_0 assures for hereditariness. Thus the EFS $(\Sigma, \Pi_M, \Gamma_M)$ is hereditary.

Let T be a proof tree for the clause $p_0(w) \leftarrow$ from the EFS $(\Sigma, \Pi_M, \Gamma_M)$. An atom which labels a node of T represents a configuration of an accepting computation tree. Therefore, M accepts $w \in \Sigma^*$ if and only if the clause $p_0(w) \leftarrow$ is provable from the erasing hereditary EFS $(\Sigma, \Pi_M, \Gamma_M)$. Thus $L(M) = L_\varepsilon(\Gamma_M, p_0)$. □

In the above theorem, we construct an erasing hereditary EFS. We can remove the erasing substitutions from the above proof.

Theorem 4. For any erasing hereditary EFS $S = (\Sigma, \Pi, \Gamma)$, there exists a non-erasing hereditary EFS $S' = (\Sigma, \Pi, \Gamma')$ such that $L_\varepsilon(\Gamma, p) - \{\varepsilon\} = L(\Gamma', p)$.

Proof: We define Γ' as

$$\Gamma' = \{C\theta \mid C \in \Gamma, \theta \subseteq \{x_1 := \varepsilon, \ldots, x_n := \varepsilon\}\},$$

where $x_1, \ldots, x_n$ are the variable symbols in C. Thus, proof trees for S and S' are the same. □

Finally we get the main theorem.

Theorem 5. P = H-EFS.

4 A membership problem and closure properties.

The membership problem for a class $\mathcal{L}$ of languages (MEMB($\mathcal{L}$)) is, given any string w and any grammar G for a language in $\mathcal{L}$, to determine whether $w \in L(G)$.

Let DEXPTIME be the class of languages that is accepted by deterministic Turing machines in time $O(2^{p(n)})$ for some polynomial p. An alternating Turing machine (ATM for short) is a nondeterministic Turing machine with universal states in addition to existential states. Configurations and accepting computation trees of ATMs are defined similarly as those of 2AFA(k). Let ASPACE($s(n)$) denotes the class of languages accepted by an ATM with space $s(n)$.

Theorem 6. The membership problem for H-EFS is DEXPTIME-complete.

Proof: Since ASPACE(poly) = DEXPTIME, it is sufficient to show that the problem is log-space complete for ASPACE(poly). First we describe an ATM that, given an H-EFS S and an atom A, decides whether $S \vdash A$. M starts with the atom A on the first work tape and the H-EFS $S = (\Sigma, \Pi, \Gamma)$ on the input tape. M nondeterministically guesses an definite clause $C = B \leftarrow B_1, \ldots, B_m$ in Γ and a substitution θ such that $A = B\theta$. Then, M universally branches for all $1 \leq i \leq m$ to recursively check whether $S \vdash B_i\theta$ holds. If we start with $p(w)$ on the work tape, any atom on the first work tape contains only substrings of w as its arguments since S is hereditary. Thus, M uses $O(rn)$ space to decide $S \vdash A$, where r is the maximum arity of $q \in \Pi$. This prove that MEMB(H-EFS) is in ASPACE(poly).

Let $L \subseteq \{0,1\}^*$ be a language in ASPACE(poly). Then for some polynomial $s(n)$, there is an ATM $M = (K, \Sigma, \Delta, \delta, q_0, B, F, U)$ such that (1) M has only one work tape and no input tape: (2) $\Delta = \Sigma = \{0,1\}^*$: (3) Given an input w of length n, M starts with the initial state p_0 and the work tape $\cent\, wB^{s(n)-(n+1)}$ of length $s(n)$ padded with the blank symbol B: (4) M changes only its state in any transition from universal configuration: (5) M accepts L using at most $s(n)$ space.

Given an input string w and a one-tape ATM M, we define an atom A and an H-EFS $S = (\Sigma, \Pi, \Gamma)$ as follows. Let $\Sigma = \{0,1\}$ and $\Pi = K$, where every predicate symbols are arity $s(n) + 4$. The idea is to represent a configuration $J = (p, a_1 \ldots a_{i-1} @ a_i \ldots a_{s(n)})$ of M by an atom $A_J = p(a_1, \ldots, a_{i-1}, @, a_i, \ldots, a_{s(n)}, 0, 1, B)$, where the symbol @ stands for the position of the head. The last three arguments $0, 1, B$ are dummy ones to ensure the hereditariness of definite clauses below. We assume an appropriate encoding of $0, 1, B, @, \cent$ over $\{0,1\}$.

Let $p \in K - U$ be an existential state. Then for each transition $((p, a), (q, b, d)) \in \delta \subseteq (K \times \Sigma) \times (K \times \Sigma \times \{+1, 0, -1\})$ and for each $1 \leq i \leq s(n)$, we add the following definite clauses to Γ:

- $p(x_1, \ldots, x_{i-1}, @, a, x_{i+1}, \ldots, x_{s(n)}, 0, 1, B) \leftarrow q(x_1, \ldots, x_{i-1}, b, @, x_{i+1}, \ldots, x_{s(n)}, 0, 1, B)$ if $d = +1$
- $p(x_1, \ldots, x_{i-1}, @, a, x_{i+1}, \ldots, x_{s(n)}, 0, 1, B) \leftarrow q(x_1, \ldots, x_{i-1}, @, b, x_{i+1}, \ldots, x_{s(n)}, 0, 1, B)$ if $d = 0$
- $p(x_1, \ldots, x_{i-1}, a, @, x_{i+1}, \ldots, x_{s(n)}, 0, 1, B) \leftarrow q(x_1, \ldots, x_{i-1}, @, b, x_{i+1}, \ldots, x_{s(n)}, 0, 1, B)$ if $d = +1$

For a universal state $p \in U$, every transition is of the form $((p, a), (q_i, a, 0)) \in \delta$ for some states $q_1, \ldots, q_m$ by the above assumption. Thus, for each $1 \leq i \leq s(n)$, we add the following definite clause to Γ:

- $$p(x_1, \ldots, x_{i-1}, @, a, x_{i+1}, \ldots, x_{s(n)}, 0, 1, B) \leftarrow q_1(x_1, \ldots, x_{i-1}, @, a, x_{i+1}, \ldots, x_{s(n)}, 0, 1, B)$$
$$\vdots$$
$$q_m(x_1, \ldots, x_{i-1}, @, a, x_{i+1}, \ldots, x_{s(n)}, 0, 1, B)$$

Finally, we add the definite clause below to Γ for the special unary predicate q:

- $q(x_1 \ldots x_{s(n)+1} 01B) \leftarrow q_0(x_1, \ldots, x_{s(n)+1}, 0, 1, B)$

In these definite clauses, it is easy to see that every arguments in the body appears in the head. Thus, they are hereditary. It is not difficult to see that there is an accepting computation tree for the initial configuration $I = (q_0, \not{c}\, w B^{s(n)-n})$ corresponding to w by M if and only if there is a proof tree for the atom $q(\not{c}\, w B^{s(n)-n} 01B)$ from S. The transformation is obviously computable in logarithmic space. Hence MEMB(H-EFS) is log-space hard for ASPACE(poly). This completes the result. □

We give some closure properties for H-EFS. The following theorem obviously holds since the class P is closed under the following operations. But an importance of the theorem is to show how to describe such operations using hereditary EFSs, while it is not easy to do using usual grammars.

Theorem 7. The class H-EFS is closed under the operations of union, intersection and concatenation.

Proof: We sketch the outline. Let $L_1, L_2 \in$ H-EFS and $L_i = L(\Gamma_i, p_i)$, where Γ_i is the set of definite clauses and p_i is the predicate symbol with arity one for each $i = 1, 2$.

We define the set of hereditary definite clauses $\Gamma_{\mathrm{u}}, \Gamma_{\mathrm{i}}, \Gamma_{\mathrm{c}}$ as

$$\Gamma_{\mathrm{u}} = \Gamma_1 \cup \Gamma_2 \cup \{p_{\mathrm{u}}(x) \leftarrow p_1(x)\} \cup \{p_{\mathrm{u}}(x) \leftarrow p_2(x)\},$$
$$\Gamma_{\mathrm{i}} = \Gamma_1 \cup \Gamma_2 \cup \{p_{\mathrm{i}}(x) \leftarrow p_1(x), p_2(x)\},$$
$$\Gamma_{\mathrm{c}} = \Gamma_1 \cup \Gamma_2 \cup \{p_{\mathrm{c}}(xy) \leftarrow p_1(x), p_2(y)\}.$$

Then $L(\Gamma_{\mathrm{u}}, p_{\mathrm{u}}) = L_1 \cup L_2, L(\Gamma_{\mathrm{i}}, p_{\mathrm{i}}) = L_1 \cap L_2$, and $L(\Gamma_{\mathrm{c}}, p_{\mathrm{c}}) = L_1 \cdot L_2$.

□

The closure properties under the operations in Theorem 7 are effective since we give effective procedure how to construct a hereditary EFS from a 2AFA(k) and it it easy to construct a polynomial time-bounded deterministic Turing machine which accepts the complement of a language.

References

[1] R. M. Smullyan, Theory of Formal Systems, (Princeton Univ. Press, Princeton, 1961).

[2] K. N. King, Alternating Multihead Finite Automata, *Theoretical Computer Science* **61** (1988) 149-174.

[3] S. Arikawa, T. Shinohara and A Yamamoto, Learning elementary formal systems, *Theoretical Computer Science* **95** (1992) 97-113.

[4] S. Miyano, A. Shinohara and T. Shinohara, Which Classes of Elementary Formal Systems Are Polynomial-Time Learnable?, Technical Report RIFIS-TR-CS-37, Research Institute of Fundamental Information Science, Kyushu University, 1991.

[5] A. K. Chandra, D. C. Kozen and L. J. Stockmeyer, Alternation, *J. Assoc. Comput.* (1981) 114-133.