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Stable Model Semantics of Circumscription

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Abstract

This paper is concerned with circumscription of logic programs and its stable model semantics. We introduce two conditions, minimal condition and uniqueness condition, which determine predicate symbols to be minimized. The minimal condition is imposed on parallel predicate circumscription, while the uniqueness condition is imposed on parallel formula circumscription. We show that each of them also determines a unique model of circumscription, and show that it coincides with a unique stable model as declarative semantics for a program.

1 Introduction

Circumscription, introduced by McCarthy [4] and developed by him [5] and Lifschiz [2, 3], is an important tool for formalizing the nonmonotonic aspects of commonsense knowledge and reasoning. There are many other approaches to the commonsense knowledge and reasoning such as *nonmonotonic modal logic* by McDermott and Doyle [7, 6], *default logic* by Reiter [11], and *autoepistemic logic* by Moore [8]. These approaches, called *consistency-based*, are formalized by using the modal operator to represent logical consistency. Hence, they are a variant of the modal logic, and beyond the classical logic. In contrast, circumscription is based on the classical logic.

Circumscription is a nonmonotonic reasoning based on the truth in all the minimal models of the classical logic, and in this sense it is called a *minimal model approach*. There are also many kinds of circumscription, *predicate circumscription* by McCarthy [4], *formula circumscription* by McCarthy [5] and Lifschitz [2], *prioritized circumscription* by Lifschitz [2], and *pointwise circumscription* by Lifschitz [3]. Predicate circumscription is a minimization which does not include predicate variables in the process of minimization, while formula circumscription is a minimization which includes such variables. Prioritized circumscription is a minimization with a priority on predicate symbols in the process of minimization.

Recently, nonmonotonic logics have been characterized by the declarative semantics of logic programs with negation, the perfect model semantics [10], the stable model semantics [1] and the well-founded model semantics [12]. In particular, circumscription is characterized by the perfect model semantics [10]. Parallel circumscription is a special prioritized circumscription, and hence the perfect model semantics is suitable for parallel or prioritized circumscription.

However, parallel circumscription is not merely a special case of prioritized circumscription, but it has another important model called a *stable model*. In this paper we discuss such a

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model for parallel predicate circumscription and formula circumscription of logic programs. We introduce two conditions, a *minimal condition* and a *uniqueness condition*, under which there uniquely exists the model of parallel circumscription. A positive disjunctive database satisfies the minimal condition if and only if, for any clause, only one predicate symbol in the head of the clause is included in the tuple of minimized predicate symbols, and the other predicate symbols in the head are not included in the tuple. A positive disjunctive database satisfies the uniqueness condition if and only if, for any clause, only one predicate symbol in the head of the clause is included in the tuple of varied predicate symbols, and the other predicate symbols in the head are included in the tuple of minimized predicate symbols. We prove that the least Herbrand model of a positive disjunctive database which satisfies the minimal condition coincides with a model of predicate circumscription, while one which satisfies the uniqueness condition coincides with a model of formula circumscription. Furthermore, these models coincide with a stable model of the given database.

2 Preliminaries

By $p, q, \cdots$, we denote predicate constants and by $P, Q, \cdots$, we denote predicate variables. We use $p(\overline{x})$ instead of $p(x_1, \cdots, x_n)$, and $\forall\overline{x}$ instead of $\forall x_1 \cdots \forall x_n$.

We introduce an order $<$ on predicate symbols as follows. Let p, q be predicate symbols with the same arity. Then, $p \leq q$ stands for the formula $\forall\overline{x}(p(\overline{x}) \rightarrow q(\overline{x}))$. Let $\overline{p} = p_1, \cdots, p_n$ and $\overline{q} = q_1, \cdots, q_n$ be tuples of predicate symbols, where p_i and q_i have the same arity for any i. Then, $\overline{p} \leq \overline{q}$ stands for the formula

$$(p_1 \leq q_1) \wedge \cdots \wedge (p_n \leq q_n).$$

Furthermore, $\overline{p} < \overline{q}$ stands for the formula $(\overline{p} \leq \overline{q}) \wedge \neg(\overline{q} \leq \overline{p})$, and $\overline{p} = \overline{q}$ for the formula $(\overline{p} \leq \overline{q}) \wedge \neg(\overline{q} \leq \overline{p})$. In case $n = 1$, $p < q$ simply means that the extension of p is a proper subset of the extension of q.

In this paper, by $A(\overline{p})$ (*resp.* $A(\overline{p}; \overline{z})$) we mean the formula A including the tuple $\overline{p}$ (*resp.* $\overline{p}$ and $\overline{z}$) of predicate symbols.

Definition 1 Let $A(\overline{p}; \overline{z})$ be a formula, and $\overline{p}$ and $\overline{z}$ be disjoint tuples of predicate symbols. Then, *(parallel) formula circumscription* $CIRC(A(\overline{p}; \overline{z}); \overline{p}; \overline{z})$ *of* $\overline{p}$ *in* $A(\overline{p}; \overline{z})$ *with variables* $\overline{z}$ is defined by the following second-order formula :

$$CIRC(A(\overline{p}; \overline{z}); \overline{p}; \overline{z}) = A(\overline{p}; \overline{z}) \wedge \forall\overline{P}\overline{Z}(\neg A(\overline{P}; \overline{Z}) \vee \overline{P} \not< \overline{p}).$$

This formula expresses that $\overline{p}$ has a minimal possible extension under the condition $A(\overline{p}; \overline{z})$ when $\overline{z}$ is allowed to vary in the process of minimization. If $\overline{z}$ is empty, then *(parallel) predicate circumscription* $CIRC(A(\overline{p}); \overline{p})$ *of* $\overline{p}$ *in* $A(\overline{p})$ is defined by the following second-order formula.

$$CIRC(A(\overline{p}); \overline{p}) = A(\overline{p}) \wedge \forall\overline{P}(\neg A(\overline{P}) \vee \overline{P} \not< \overline{p}).$$

The model-theoretic meaning of circumscription can be expressed by the following notion :

Definition 2 Let $M_1 = (D, I_1)$ and $M_2 = (D, I_2)$ be the structures of first-order logic. Let $\overline{p} = p_1, \cdots, p_n$ and $\overline{q} = q_1, \cdots, q_n$ be tuples of predicate symbols.

1. $M_1 \leq^{\overline{p}} M_2$ if
   (a) for any $c \notin \overline{p}$, $I_1(c) = I_2(c)$, and
   (b) for any $c \in \overline{p}$, $I_1(c) \subseteq I_2(c)$.

2. $M_1 \leq^{\overline{p};\overline{z}} M_2$ if

   (a) for any $c \notin \overline{p} \cup \overline{z}$, $I_1(c) = I_2(c)$, and

   (b) for any $c \in \overline{p}$, $I_1(c) \subseteq I_2(c)$.

3. $M_1 <^{\overline{p}} M_2$ if $M_1 \leq^{\overline{p}} M_2$ and $\neg(M_2 \leq^{\overline{p}} M_1)$.

4. $M_1 <^{\overline{p};\overline{z}} M_2$ if $M_1 \leq^{\overline{p};\overline{z}} M_2$ and $\neg(M_2 \leq^{\overline{p};\overline{z}} M_1)$.

Let M be the structure of A. Then, M is *$\overline{p}$-minimal model (resp. $(\overline{p};\overline{z})$-minimal model) of A* if there exists no model N of A such that $N <^{\overline{p}} M$ (*resp.* $N <^{\overline{p};\overline{z}} M$).

The models of (parallel) circumscription is characterized as follows.

Theorem 1 (McCarthy [4], Lifschitz [2]) Let A be a formula, and $\overline{p}$ and $\overline{z}$ be tuples of predicate symbols.

1. M is the model of $CIRC(A(\overline{p});\overline{p})$ if and only if M is $\overline{p}$-minimal model of A.

2. M is the model of $CIRC(A(\overline{p};\overline{z});\overline{p};\overline{z})$ if and only if M is $(\overline{p};\overline{z})$-minimal model of A.

In this paper, we adopt the stable model semantics by Gelfond and Lifschitz [1] as the declarative semantics, because we do not deal with a priority on predicate symbols. A logic program in [1] is a *general program*, which is a set of clauses of the form

$$A \leftarrow B_1, \cdots, B_m, \neg C_1, \cdots, \neg C_l,$$

where A, B_i, C_j are atoms for any i $(1 \leq i \leq m)$ and j $(1 \leq j \leq l)$.

Definition 3 Let P be a general program. For any set M of atoms from P, let P_M be the program obtained from P by deleting

1. all rules that have a negative literal $\neg B$ in their bodies with $B \in M$,

2. all negative literals in the bodies of remaining rules.

If the least Herbrand model of P_M, denoted by $model(P_M)$, coincides with M, then M is a *stable model* of P. The *stable model semantics* is defined for a logic program P, if P has exactly one stable model, and it declares the model to be a canonical model of P.

The stable model semantics can be characterized as follows.

Theorem 2 (Gelfond and Lifschitz [1]) Any stable model of P is a minimal Herbrand model of P.

3 Circumscription and Logic Programs

Przymusinski [9] showed the relationship between the least Herbrand model semantics and the model of predicate circumscription for a definite program.

Theorem 3 (Przymusinski [9]) Let P be a definite program and Π be all predicate symbols appearing in P. Then, M is the least Herbrand model of P if and only if M is the model of $CIRC(P;\Pi)$.

Przymusinski also introduced the semantics of logic programs with negation. There are many kinds of declarative semantics, the *perfect model semantics* by Przymusinski [10, 9], the *stable model semantics* by Gelfond and Lifschitz [1], and the *well-founded model semantics* by Van Gelder *et.al.* [12]. Przymusinski proved a relationship between the perfect model semantics and the model of prioritized circumscription.

Prioritized circumscription is a kind of circumscription which has a priority on predicate symbols in the process of the minimization. *Parallel circumscription* is circumscription without a priority. Predicate and formula circumscription which we defined in Section 2 is parallel. In general, parallel circumscription is a special case of prioritized circumscription. Przymusinski also proved a relationship between the perfect model semantics and the model of prioritized circumscription.

Theorem 4 (Przymusinski [9]) Let P be a stratified program and $P_1, \cdots, P_n$ be strata of P. Then, M is a perfect model of P if and only if M is a model of $CIRC(P; P_1 > \cdots > P_n)$.

He showed the above two theorems for parallel and prioritized *predicate* circumscription. On the other hand, formula circumscription is more suitable for nonmonotonic logic than predicate circumscription. For example, let

$$P = \left\{ \begin{array}{l} fly(X), abnormal(X) \leftarrow bird(X) \\ bird(tweety) \leftarrow \end{array} \right\}$$

and

$$\begin{array}{l} M_1 = \{fly(tweety), bird(tweety)\}, \\ M_2 = \{abnormal(tweety), bird(tweety)\}. \end{array}$$

Then, the models of $CIRC(P; abnormal)$, i.e., $abnormal$-minimal models of P, are M_1 and M_2. However, we should infer intuitively that normal birds fly, i.e., $abnormal$-minimal model of P is only M_1. Formula circumscription is based on this point. The model of $CIRC(P; abnormal; fly)$ ($CIRC(P; abnormal; fly, bird)$), i.e., $(abnormal; fly)$ $((abnormal; fly, bird))$-minimal model, is M_1. In this paper, we study parallel formula circumscription. In particular, we make clear the relationship between the *stable model semantics* and parallel predicate and formula circumscription in the following sections.

4 Minimal Condition

In this section, we study the relationship between the stable model semantics and the model of predicate circumscription. At first, we define the programs to discuss the model of circumscription. A *clause* is a formula of a form $A_1, A_2, \cdots, A_n \leftarrow B_1, B_2, \cdots, B_m$, where A_i, B_j are atoms for any i $(1 \leq i \leq n)$ and j $(1 \leq j \leq m)$. We call A_i a *head* and B_j a *body*. A *program* is a set of clauses. Let $pred(A)$ be the predicate symbol of an atom A.

Let P be a program and let $C = A_1, \cdots, A_n \leftarrow B_1, \cdots, B_m$ be a clause in P. Then, we call a definite clause $A_i \leftarrow B_1, \cdots, B_m$ a *projection* D_C of C, and a set $\{D_C \mid C \in P\}$ a *projection* of P. Note that there may exist many projections for one program.

Definition 4 Let P be a program, $\overline{\Pi}$ be the set of all predicate symbols of P and $\overline{q} \subseteq \overline{\Pi}$. Then, $(P, \overline{q})$ satisfies a *minimal condition* if for any clause $A_1, \cdots, A_n \leftarrow B_1, \cdots, B_m \in P$, there exists an i $(1 \leq i \leq n)$ such that $pred(A_i) \notin \overline{q}$ and $pred(A_j) \in \overline{q}$ for any $j \neq i$

The idea behind the minimal condition is that the p-minimal models of $p(a) \vee q(b)$ are $\{p(a)\}$ and $\{q(b)\}$, and in general $p_1, \cdots, p_{n-1}$-minimal models of $p_1(a_1) \vee \cdots \vee p_n(a_n)$ are $\{p_i(a_i)\}$ for any i $(1 \leq i \leq n)$.

Theorem 5 Let P be a program, $\overline{\Pi}$ be the set of all predicate symbols of P and $\overline{q} \subseteq \overline{\Pi}$. If $(P, \overline{q})$ satisfies the minimal condition, then the least Herbrand model of any projection of P is the model of $CIRC(P; \overline{q})$.

Proof. For any $D_C \in proj(P)$, if $pred(A_i) \notin \overline{q}$ then $M_{proj(P)} \cap B_P^{\overline{q}} = \phi$. Hence $M_{proj(P)}$ is $\overline{q}$-minimal model of P.

We use induction on n for $T_P \uparrow n$ to prove that $\overline{q}$-minimality is independent of the selection of heads.

In case $n = 1$, suppose, for $C = A_1, \cdots, A_n \leftarrow \in P$, that $pred(A_i) \notin \overline{q}$ and $pred(A_j) \in \overline{q}$. Let $P_1 = \{A_j \leftarrow\}$, $P_2 = \{A_i \leftarrow\}$, and $P_3 = P_1 \cup P_2$. Then, $T_{P_2} \uparrow 1$ is $\overline{q}$-minimal, and $A_i \notin M_{P_1}$, $A_i \in M_{P_2}$, $A_i \in M_{P_3}$, $pred(A_i) \notin \overline{q}$. Hence, $T_{P_1} \uparrow 1 \not\leq^{\overline{q}} T_{P_2} \uparrow 1$ and $T_{P_1} \uparrow 1 \not\leq^{\overline{q}} T_{P_3} \uparrow 1$, Therefore, $T_{P_1} \uparrow 1$ is also $\overline{q}$-minimal.

Assume that the claim holds for n. For $n + 1$, suppose $C = A_1, \cdots, A_n \leftarrow B_1, \cdots B_m$ and $\{B_1, \cdots, B_m\} \subseteq T_{proj(P)} \uparrow n$. Suppose $pred(A_i) \notin \overline{q}$ and $pred(A_j) \in \overline{q}$. Let $P_1 = \{A_j \leftarrow\} \cup T_{proj(P)} \uparrow n$, $P_2 = \{A_i \leftarrow\} \cup T_{proj(P)} \uparrow n$ and $P_3 = P_1 \cup P_2$. Then, $T_{P_2} \uparrow (n+1)$ is $\overline{q}$-minimal, and $A_i \notin M_{P_1}$, $A_i \in M_{P_2}$, $A_i \in M_{P_3}$, $pred(A_i) \notin \overline{q}$. Hence, $T_{P_1} \uparrow (n+1) \not\leq^{q} T_{P_2} \uparrow (n+1)$ and $T_{P_1} \uparrow (n+1) \not\leq^{q} T_{P_3} \uparrow (n+1)$. Therefore, $T_{P_1} \uparrow (n+1)$ is also $\overline{q}$-minimal.

Thus $\overline{q}$-minimality is independent on the selection of heads, and hence the least Herbrand model of all projection of P is $\overline{q}$-minimal model, i.e., the model of $CIRC(P; \overline{q})$. □

We introduce the following notion to see a relationship between the stable model and the model of predicate circumscription.

Definition 5 Let $proj(P)$ be the projection of P. Then, we define a *program generated by* $proj(P)$ to be the general program $\{A_i \leftarrow B_1, \cdots, B_m, \neg A_1, \cdots, \neg A_{i-1}, \neg A_{i+1}, \cdots, A_n \mid A_i \leftarrow B_1, \cdots, B_m \in proj(P)\}$.

Theorem 6 The least Herbrand model of the projection $proj(P)$ of P is the stable model of the program generated by $proj(P)$. Hence, the model of $CIRC(P; \overline{q})$ is the stable model of the program generated by $proj(P)$.

5 Uniqueness Condition

In this section, we show a relationship between the stable model and the model of formula circumscription

Definition 6 Let P be a program, $\overline{\Pi}$ be the set of all predicate symbols of P, and $\overline{q} \cup \overline{z} = \overline{\Pi}$. Then, $(P; \overline{q}; \overline{z})$ satisfies a *uniqueness condition* if for any clause $A_1, \cdots, A_n \leftarrow B_1, \cdots, B_m \in P$, there exists an i such that

1. $pred(A_i) \notin \overline{q}$ and $pred(A_j) \in \overline{q}$ for any $j \neq i$,
2. $pred(A_i) \in \overline{z}$.

The idea behind the uniqueness condition is that the $(p; q)$-minimal model of $p(a) \vee q(b)$ is $\{q(b)\}$, and in general $(p_1, \cdots, p_{n-1}; p_n)$-minimal model of $p_1(a_1) \vee \cdots \vee p_n(a_n)$ is $\{p_n(a_n)\}$.

Theorem 7 Let P be a program, $\overline{\Pi}$ be the set of all predicate symbols of P, and $\overline{q} \cup \overline{z} = \overline{\Pi}$. If $(P; \overline{q}; \overline{z})$ satisfies the uniqueness condition, there exists a unique model of $CIRC(P; \overline{q}; \overline{z})$.

Proof. At first, we define the *transformation* of P in $\overline{q}$ as follows :

$$trans(P,\overline{q}) = \{A_i \leftarrow B_1,\cdots,B_m \in D_C \mid C = A_1,\cdots,A_n \leftarrow B_1\cdots,B_m \in P,\ pred(A_i) \notin \overline{q}\}.$$

We show $M_{trans(P,\overline{q})} \models P$, where $M_{trans(P,\overline{q})}$ is the least Herbrand model of $trans(P,\overline{q})$. By $M_{trans(P,\overline{q})} \models trans(P,\overline{q})$, for any $A_i \leftarrow B_1,\cdots,B_m \in trans(P,\overline{q})$, we have $M_{trans(P,\overline{q})} \models A_i \leftarrow B_1,\cdots,B_m$. Since $A_i \leftarrow B_1,\cdots,B_m$ is equivalent to $A_i \vee \neg B_1 \vee \cdots \vee \neg B_m$, $M_{trans(P,\overline{q})} \models A_i \vee \neg B_1 \vee \cdots \vee \neg B_m$. So $M_{trans(P,\overline{q})} \models A_1 \vee \cdots \vee A_i \vee \cdots \vee A_n \vee \neg B_1 \vee \cdots \vee \neg B_m$. For any $C \in P$, $M_{trans(P,\overline{q})} \models C$. Hence, $M_{trans(P,\overline{q})} \models P$.

Let $B_P^{\overline{q}}$ to be the set $\{q(\overline{t}) \in B_P | q \in \overline{q}\}$. Note that $M \cap B_P^{\overline{\Pi}-\overline{q}\cup\overline{z}} = \phi$ for any model M of P. Then, $M_{trans(P,\overline{q})} \cap B_P^{\overline{q}} = \phi$, and hence $M_{trans(P,\overline{q})}$ is the $(\overline{q};\overline{z})$-minimal model of P.

Assume that there exists an M such that $M \models P$ and $M \cap B_P^{\overline{q}} = \phi$. By $M \models P$, there exists the projection $proj(P)$ of P such that $M \models proj(P)$. If $M \not\models proj(P)$ for any projection $proj(P)$ of P, then $M \not\models A_i$ for any head A_i of P, which contradicts the assumption. Suppose that $M \models proj(P)$ and $proj(P) \neq trans(P,\overline{q})$. Then, there exists a definite clause $A_j \leftarrow B_1,\cdots,B_m \in proj(P)$ such that $pred(A_j) \in \overline{q}$. Hence, the instantiation $inst(A_j)$ of an atom A_j is an element of $B_P^{\overline{q}}$. On the other hand, by $M \models proj(P)$, $M \models A_j$, i.e., $inst(A_j) \in M$. Then, $M \cap B_P^{\overline{q}} \neq \phi$, which contradicts the $\overline{q}$-minimality of M. Hence, $M \models trans(P,\overline{q})$. For such an M, $M_{trans(P,\overline{q})} \subseteq M$. Therefore, the unique $(\overline{q};\overline{z})$-minimal model of P exists and it is $M_{trans(P,\overline{q})}$. $\Box$

Example 1 Let

$$P = \left\{ \begin{array}{l} fly(X), abnormal(X) \leftarrow bird(X) \\ bird(tweety) \leftarrow \end{array} \right\}.$$

Then, there are two minimal models of P:

$$\begin{array}{l} M_1 = \{fly(tweety), bird(tweety)\}, \\ M_2 = \{abnormal(tweety), bird(tweety)\}. \end{array}$$

The $(abnormal; fly, bird)$-minimal model of P, i.e., the unique model of $CIRC(P; abnormal; fly, bird)$ is M_1. We can infer that *normal birds fly*, while the $(fly; abnormal, bird)$-minimal model of P, i.e., the least model of $CIRC(P; fly; abnormal, bird)$ is M_2. We can also infer that *birds not to fly are abnormal.*

Furthermore, let

$$\begin{array}{l} P_1 = P \cup \left\{ \begin{array}{l} abnormal(X) \leftarrow ostrich(X), bird(X) \\ ostrich(tweety) \leftarrow \end{array} \right\}, \\ P_2 = P \cup \left\{ \begin{array}{l} fly(X) \leftarrow swallow(X), bird(X) \\ swallow(tweety) \leftarrow \end{array} \right\}. \end{array}$$

Then, the $(fly; abnormal, bird, ostrich)$-minimal model of P_1 is

$$\{bird(tweety), ostrich(tweety), abnormal(tweety)\},$$

while the $(abnormal; fly, bird, swallow)$-minimal model of P_2 is

$$\{bird(tweety), swallow(tweety), fly(tweety)\}.$$

There is only one selection of predicate symbols that $P_i (i = 1,2)$ satisfies the uniqueness condition. Hence, the uniqueness condition can avoid the nonmonotonicity. When we select predicate symbols so that a program has the unique model, the uniqueness condition leads us to the conclusion we have expected.

As the logic program to be circumscribed is a *positive disjunctive database* and stable model semantics is developed for a general program, we need to transform the program which satisfies the uniqueness condition into general programs.

Definition 7 Let P be a program, $\overline{\Pi}$ be the set of all predicate symbols of P, and $\overline{q} \cup \overline{z} = \overline{\Pi}$, for which $(P; \overline{q}; \overline{z})$ satisfies the uniqueness condition. Then, $P^{\overline{q};\overline{z}}$ consists of the clauses of the form

$$A_i \leftarrow B_1, \cdots, B_m, \neg A_1, \cdots, \neg A_{i-1}, \neg A_{i+1}, \cdots, \neg A_n,$$

where $pred(A_i) \notin \overline{q}$ and $pred(A_i) \in \overline{z}$ for any clause $A_1, \cdots, A_n \leftarrow B_1, \cdots, B_m \in P$.

Theorem 8 Let P be a program, $\overline{\Pi}$ be the set of all predicate symbols of P, and $\overline{q} \cup \overline{z} = \overline{\Pi}$, for which $(P; \overline{q}; \overline{z})$ satisfies the uniqueness condition. Then, there exists a unique stable model of $P^{\overline{q};\overline{z}}$ which coincides with the least model of $CIRC(P; \overline{q}; \overline{z})$.

Proof. Suppose that M is the unique model of $CIRC(P; \overline{q}; \overline{z})$ but not the stable model of $P^{\overline{q};\overline{z}}$. Since M is the least model of $CIRC(P; \overline{q}; \overline{z})$, it is the least Herbrand model of $trans(P, \overline{q})$. Also suppose that for some clause $A_i \leftarrow B_1, \cdots, B_m, \neg A_1, \cdots, \neg A_{i-1}, \neg A_{i+1}, \cdots, \neg A_n \in P^{\overline{q};\overline{z}}$, there exists a j $(1 \leq j \leq n)$ such that $A_j \in M$ $(i \neq j)$. Then, if $A_j \in M$ and a $pred(A_j) \in \overline{q}$ then there exists a model M' of P such that $M' = M - \{A_j\}$, which contradicts the $(\overline{q}; \overline{z})$-minimality of M. Hence, there exists no j $(1 \leq j \leq n)$ such that $A_j \in M$. By the consistency of $P^{\overline{q}}$, $model({P^{\overline{q};\overline{z}}}_M) = M$. Hence, M is the stable model of $P^{\overline{q};\overline{z}}$.

Now suppose that $M \neq M'$ and M' is the stable model of $P^{\overline{q};\overline{z}}$. Then, for some clause $C = A_i \leftarrow B_1, \cdots, B_m, \neg A_1, \cdots, \neg A_{i-1}, \neg A_{i+1}, \cdots, \neg A_n \in P^{\overline{q}}$, there exists a j $(1 \leq j \leq n)$ such that $A_j \in M'$ $(i \neq j)$. Then, ${P^{\overline{q};\overline{z}}}_{M'} \subseteq {P^{\overline{q};\overline{z}}}_M - \{C\}$, ${P^{\overline{q};\overline{z}}}_{M'} \subset {P^{\overline{q};\overline{z}}}_M$. By the property of T_p-operator and least Herbrand model, $model({P^{\overline{q};\overline{z}}}_{M'}) \subseteq model({P^{\overline{q};\overline{z}}}_M)$. Hence, $M' \subseteq M$, which contradicts that M is stable, because the stable model of $P^{\overline{q};\overline{z}}$ is the minimal Herbrand model of $P^{\overline{q};\overline{z}}$. Thus, M is the unique stable model of $P^{\overline{q};\overline{z}}$. □

Example 2 Let P be the program of Example 1, and put

$$P_1 = P^{abnormal;bird,fly} = \left\{ \begin{array}{l} fly(X) \leftarrow bird(X), \neg abnormal(X) \\ bird(tweety) \leftarrow \end{array} \right\},$$

and

$$P_2 = P^{fly;bird,abnormal} = \left\{ \begin{array}{l} abnormal(X) \leftarrow bird(X), \neg fly(X) \\ bird(tweety) \leftarrow \end{array} \right\}.$$

1. The instantiation of P_1 is

$$\left\{ \begin{array}{l} fly(tweety) \leftarrow bird(tweety), \neg abnormal(tweety) \\ bird(tweety) \leftarrow \end{array} \right\}.$$

   Let $M_1 = \{fly(tweety), bird(tweety)\}$. Then, $model({P_1}_{M_1}) = M_1$, and M_1 is the unique stable model of P_1, and hence M_1 is the model of $CIRC(P; abnormal; bird, fly)$.

2. The instantiation of P_2 is

$$\left\{ \begin{array}{l} abnormal(tweety) \leftarrow bird(tweety), \neg fly(tweety) \\ bird(tweety) \leftarrow \end{array} \right\}.$$

Let $M_2 = \{abnormal(tweety), bird(tweety)\}$. Then, $model(P_{2M_2}) = M_2$, and M_2 is the unique stable model of P_2, and hence M_2 is the model of $CIRC(P; fly; bird, abnormal)$.

The above theorem does not hold without the uniqueness condition. In fact, if P is

$$\left\{ \begin{array}{l} q(X) \leftarrow p(X,Y), \neg q(Y) \\ p(1,2) \leftarrow \end{array} \right\},$$

then $M = \{p(1,2), q(1)\}$ is the unique stable model of P, but the least model of circumscription does not exists.

6 Conclusion

In this paper, we have discussed circumscription of logic programs, defined the sufficient conditions under which the model exists uniquely, and showed that these conditions are well-defined.

In general, the problem whether or not a logic program has the model of circumscription is not decidable, while the problem is decidable for function-free logic programs whose domains are finite. Furthermore, the semantics of logic programs in this paper is not beyond the Herbrand model semantics. We have shown that the problem is still decidable for programs with function symbols which satisfy the conditions given in this paper.

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