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http://hdl.handle.net/2324/3158

出版情報：RIFIS Technical Report. 52, 1991-12-24. Research Institute of Fundamental Information Science, Kyushu University
バージョン：
権利関係：

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RIFIS-TR-CS-52

RIFIS Technical Report

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December 24, 1991

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Definite Inductive Inference as a Successful Identification Criterion

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Abstract. A majority of studies on inductive inference of formal languages and models of logic programings have mainly used Gold's identification in the limit as a correct inference criterion. However, in this criterion, we can not decide in general whether the inference terminates or not, and the results of the inference involve some risks. In this paper, we consider the definite inductive inference as a successful identification criterion. The definite inference machine produces a unique guess just once when it convinces the termination of the inference, and the result does not involve any risk at all. We discuss the necessary and sufficient conditions for definite inductive inferability from positive data and complete data respectively.

1. Introduction

Inductive inference is a process of hypothesizing a general rule from examples. In order to define an inductive inference problem precisely, we need to specify the following five items[3]:

- the class of rules being considered,
- the hypothesis space, that is, a set of descriptions such that each rule in the class has at least one description in the hypothesis space,
- for each rule, its set of examples, and the sequences of examples that constitute admissible presentations of the rule,
- the class of inference methods under consideration,
- the criterion for a successful inference.

As a correct inference criterion, or a criterion of a successful inference, for inductive inference of formal languages and models of logic programings, we have mainly used Gold's identification in the limit[5]. An inference machine *IM* identifies a rule R in the limit iff the sequence of outputs of *IM*, which is successively presented a sequence of examples of R, converges to an expression τ, i.e., all outputs of *IM* become a unique τ in a certain finite time, for a correct expression τ of R. Under this criterion, we have many productive results such as Angluin[1], Wright[13], Shinohara[12], Sato&Umayahara[10] and so on, concerning inductive inference from positive data, and many systems have been developed such as Shapiro's MIS[11], Muggleton&Buntine's CIGOL[8] and so on, concerning inductive inference from complete data.

Considering ordinary learning process of human beings, the criterion of identification in the limit seems to be natural. However, in this criterion, we can not decide in general whether a sequence of outputs of the inference machine converges or not at a certain time, and the results of the inference involve some risks. Clearly, it is important to have a definite answer, when we are to use the result of machine learning. There are some classes of which concepts can be learned definitely within a finite time.

In this paper, we consider the definite inductive inference as a successful identification criterion. An inference machine *DIM* is said to infer a rule R definitely iff *DIM*, which is successively presented a sequence of examples of R, produces a unique output at a certain time and the output is a correct expression of R. That is, in this criteria, the inference machine does not produce a guess until it convinces the guess is correct.

In Section 2, we give definitions of concepts necessary for our discussions. In Section 3 and 4, we discuss the necessary and sufficient conditions for inductive inferability from positive data and complete data respectively. Angluin[1] introduced the notion of a finite tell-tale of a language to discuss inferability of formal languages from positive data, and showed that a class is inferable from positive data iff there is a recursive procedure to enumerate all elements in the finite tell-tale of any given rule of the class. In this paper, we introduce the notions of a definite finite tell-tale and a pair of definite finite tell-tales of a rule, and show that a class is definitely inferable from positive data or complete data iff they are uniformly computable respectively.

2. Preliminaries

Let U be a recursively enumerable set to which we refer as a *universal set.* Then we call $R \subseteq U$ a *rule.* We do not consider the empty rule in this paper.

Definition 2.1. *A class of rules $C = R_1, R_2, \cdots$ is said to be an indexed family of recursive rules iff there exists a computable function $f : N \times U \to \{0,1\}$ such that*

$$f(i,w) = \begin{cases} 1, & \text{if } w \in R_i, \\ 0, & \text{if } w \notin R_i. \end{cases}$$

From now on, we assume the class of rules is an indexed family of recursive rules without any notice.

Definition 2.2. *A positive presentation of a rule R is an infinite sequence $w_1, w_2, \cdots$ of elements of U such that $\{w_1, w_2, \cdots\} = R$.*

A complete presentation of R is an infinite sequence $(w_1,t_1), (w_2,t_2), \cdots$ of elements of $U \times \{0,1\}$ such that $\{w_i \mid t_i = 1, i \geq 1\} = R$ and $\{w_j \mid t_j = 0, j \geq 1\} = U - R$.

In this paper, we use a somewhat different inference machine from that in the criterion of identification in the limit. That is, an inference machine is an effective procedure that requests inputs from time to time and stops with a unique output. We call such an inference machine a *definite inference machine.* The unique output produced by the machine is called a *guess.*

Definition 2.3. *A class C of rules is said to be definitely inferable from positive (complete) data iff there exists a definite inference machine DIM which satisfies the following: For any rule R of C and for any positive (complete) presentation σ of R, DIM presented σ's data successively produces a unique output, say j, after some finite time and $R_j = R$ holds.*

In this criterion, an inference machine produces a unique guess when the inference process terminates.

3. Definite inductive inference from positive data

In this section, we discuss the necessary and sufficient conditions for inductive inferability from positive data.

From now on, let $C = R_1, R_2, \cdots$ be an indexed family of recursive rules.

In the criterion of identification in the limit, the following definition and theorem are well-known.

Definition 3.1. (Angluin[1]) *A set S_i is said to be a finite tell-tale of R_i iff*
(1) S_i is a finite subset of R_i, and
(2) there is no index j such that $S_i \subseteq R_j \subsetneq R_i$.

Theorem 3.1. (Angluin[1]) *A class C is inferable from positive data in the limit if and only if there exists an effective procedure that enumerates all elements in a finite tell-tale of a rule R_i for any index i.*

Now, we introduce our definition and theorem that form a remarkable contrast to the above definition and theorem.

Definition 3.2. *A set S_i is said to be a definite finite tell-tale of R_i iff*
(1) S_i is a finite subset of R_i, and
(2') $R_j \neq R_i$ implies $S_i \not\subseteq R_j$ for any index j.

This definition means that if R includes a definite finite tell-tale S_i of R_i, then $R = R_i$. This has a more specific meaning than that of a finite tell-tale.

Theorem 3.2. *A class C is definitely inferable from positive data if and only if a definite finite tell-tale of a rule R_i is uniformly computable for any index i, i.e., there exists an effective procedure that on input i outputs all elements of a definite finite tell-tale of R_i and then halts.*

Proof: (i) The 'only if' part. Suppose that C is definitely inferable from positive data. Then there exists a definite inference machine *DIM* which satisfies Definition 2.3. The definite finite tell-tale of R_i is uniformly computable by the following procedure:

```
procedure Q(i);
begin
    let σ be a positive presentation of R_i;
    for k := 1 to ∞ do begin
        feed the next datum in σ to DIM;
        if DIM produces a guess then output the first k data of σ and stop
    end
end.
```

Note that we can take a positive presentation of R_i effectively, because the universal set U is effectively enumerable and whether $w \in R_i$ or not is decidable for any $w \in U$.

Since *DIM* infers C definitely, this procedure is guaranteed to terminate. Now, we show by contradiction that the output of this procedure, say S, is a definite finite tell-tale of R_i. Suppose that S is not a definite finite tell-tale of R_i. Clearly, S is a finite subset of R_i. Therefore, there exists a j such that $R_i \neq R_j$ and $S \subseteq R_j$. Let k_0 be the value of k when this procedure terminates, and let δ be an arbitrary positive presentation of R_j. Then $\sigma[k_0] \cdot \delta$ is a positive presentation of R_j, where $\sigma[k_0]$ is σ's initial segment of length k_0. Since *DIM* infers R_i definitely from σ, it follows that *DIM* can not infer R_j from $\sigma[k_0] \cdot \delta$.

This contradicts the assumption.

(ii) The 'if' part. Suppose that a definite finite tell-tale of R_i is uniformly computable for any index i, and we denote the result of computation by $S(i)$. We can infer C definitely from positive data by the following procedure:

```
procedure DIM;
begin
    T := φ;
    for j := 1 to ∞ do begin
        read the next sample and add it to T;
        for i := 1 to j do
            if S(i) ⊆ T then output i and stop
    end
end.
```

Note that whether $S(i) \subseteq T$ or not is decidable, because $S(i)$ and T are explicitly given finite sets. Suppose we are going to feed a positive presentation σ of R_h, and we denote a set of σ's first k data by $\sigma(k)$.

(1) When this procedure terminates, the output is a correct guess. In fact, let g be the output of this procedure. Since $S(g) \subseteq R_h$, it follows that $R_g = R_h$ by Definition 3.2.

(2) This procedure always terminates after some finite time. In fact, let

$$a = \min\{k \mid S(h) \subseteq \sigma(k)\} \qquad \text{and} \qquad b = \max\{a, h\}.$$

Note that $h \leq b$ holds. Suppose this procedure does not terminate. Then it reaches the case of $j = b$ and $i = h$. In this case, $S(i) \subseteq T(= \sigma(b))$ holds, which contradicts the assumption. ∎

We can show that procedure *DIM* terminates with a guess c when it reaches the case $j = b$ and $i = c$, where

$$\begin{aligned} a_n &= \min\{k \mid S(m_n) \subseteq \sigma(k)\}, \qquad (n \geq 1) \\ b &= \min\{\max\{m_1, a_1\}, \max\{m_2, a_2\}, \cdots\}, \\ c &= \min\{m_k \mid \max\{m_k, a_k\} = b,\ k \geq 1\} \end{aligned}$$

and $m_1, m_2, \cdots$ are all the m's with $R_m = R_h$. Note that $R_i = R_j$ does not imply $S(i) = S(j)$.

Corollary 3.3. *If a class C has two rules R_i, R_j with $R_i \subsetneq R_j$, then the class C is not definitely inferable from positive data.*

Here, we present an example of a class of rules definitely inferable from positive data.

Example 3.1. *Let p_i be the i-th prime and put $R_i = \{n \mid n$ is a multiple of $p_i\}$ $(i \geq 1)$. Since p_i is a primitive recursive function of i, the class $C = R_1, R_2, \cdots$ is an indexed family of recursive rules. This class C is definitely inferable from positive data. In fact, we can take a set $\{p_i\}$ as the definite finite tell-tale of R_i.*

4. Definite inductive inference from complete data

In this section, we discuss the necessary and sufficient conditions for inductive inferability from complete data.

The following Definition 4.1 and Theorem 4.1 form a remarkable contrast to Definition 3.2 and Theorem 3.2 concerning positive data.

Definition 4.1. *A rule R is said to be consistent with a pair of sets $\langle T, F\rangle$ iff $T \subseteq R$ and $F \subseteq U - R$. A pair of sets $\langle T_i, F_i\rangle$ is said to be a pair of definite finite tell-tales of R_i iff*
(1) T_i is a finite subset of R_i,
(2) F_i is a finite subset of $U - R_i$ and
(3) if R_j is consistent with the pair $\langle T_i, F_i\rangle$, then $R_i = R_j$.

Note that if S_i is a definite finite tell-tale of R_i, then a pair $\langle S_i, \phi\rangle$ is a pair of definite finite tell-tales of R_i.

The following theorem can be shown in a similar way to the proof of Theorem 3.2.

Theorem 4.1. *A class C is definitely inferable from complete data if and only if the pair of definite finite tell-tales of a rule R_i is uniformly computable for any index i.*

We present a sufficient condition for a class to be definitely inferable. This condition has more specified meaning than that of the condition of "finite thickness", which Angluin[1] introduced as a sufficient condition for a class to be inferable from positive data.

Theorem 4.2. *A class C is definitely inferable from complete data if*
(1) the set $\{i \mid w \in R_i\}$ is finite and uniformly computable for any $w \in U$, and
(2) whether $R_i = R_j$ or not is computable for any indices i, j.

Proof: Suppose that (1) and (2) hold. The definite finite tell-tale of R_i is uniformly computable by the following procedure, where a sequence $w_1, w_2, \cdots$ is an effective enumeration of the universal set U:

```
procedure Q(i);
begin
    w := w_j, where j is the least number such that w_j ∈ R_i;
    T := {w};
    F := φ;
    compute the set {j | w ∈ R_j} and set it to S;
    for each j ∈ S do
        if R_i ≠ R_j then begin
            m := 1;
            while (w_m ∈ R_i ∧ w_m ∈ R_j) or (w_m ∉ R_i ∧ w_m ∉ R_j) do m := m + 1;
            if w_m ∈ R_i ∧ w_m ∉ R_j then T := T ∪ {w_m} else F := F ∪ {w_m}
        end;
    output a pair ⟨T, F⟩ and stop
end.
```

Since the while loop above is executed only when $R_i \neq R_j$, this while statement always terminates. Therefore, the procedure $Q(i)$ always terminates. It is clear that the output of $Q(i)$ is a pair of definite finite tell-tales of R_i. ∎

We present an example of a class of rules definitely inferable from complete data.

Example 4.1. *We consider the class of pattern languages. Here, we define a pattern and a pattern language briefly. (For more details, see Angluin[2] or Mukouchi[9].)*

Fix a finite alphabet with at least two constant symbols. A pattern is a nonnull finite string of constant and variable symbols. The pattern language $L(\pi)$ generated by a pattern π is the set of all strings obtained by substituting nonnull strings of constant symbols for the variables of π. Since two patterns that are identical except for renaming of variables generate the same pattern language, we do not distinguish one from the other. We can enumerate all patterns recursively and whether $w \in L(\pi)$ or not for any w and π is effectively decidable. Therefore, we can consider the class of pattern languages as an indexed family of recursive rules, where the pattern itself is considered to be an index.

(i) The class of pattern languages satisfies the condition (1) of the above theorem. In fact, fix an arbitrary constant string w. If $w \in L(\pi)$, then π is not longer than w. The set of all patterns shorter than a fixed length is finite and uniformly computable, and whether $w \in L(\pi)$ or not for any w and π is decidable. Therefore, the set $\{\pi \mid w \in L(\pi)\}$ is finite and uniformly computable.

(ii) Angluin[2] showed that $L(\pi) = L(\tau)$ if and only if $\pi = \tau$.

Therefore, we see that the class of pattern languages is definitely inferable from complete data by (i) and (ii).

Note that using theorems in Angluin[2], we can show that $\langle T, F\rangle$ is a pair of definite finite tell-tales of $L(\pi)$, where T is the set of all elements of $L(\pi)$ with the same length as π, and F is the set of all constant strings each of which is not longer than π and does not belong to T. Furthermore, we see that the class of pattern languages is not definitely inferable from positive data by Corollary 3.3.

5. Discussion

In this paper, we have discussed the definite inferability of an indexed family of recursive rules from positive data and complete data respectively. We also presented concrete classes that are definitely inferable.

This criterion of the definite inductive inference is shown to be equivalent to that of finite identification (cf. Freivald&Wiehagen[4]) as in the following way.

First, we define finite identification briefly. Let $\sigma = d_1, d_2, \cdots$ be a positive presentation of a rule, and we denote a Gödel numbering of $(d_1, d_2, \ldots, d_k)$ by $\sigma[\![k]\!]$. A sequence of natural numbers $j_1, j_2, \cdots$ is said to be *finitely convergent to a number* j iff it is convergent to j and $j_n = j_{n+1}$ implies $j_m = j$ $(m \geq n)$ for any n. Then a class $C = R_1, R_2, \cdots$ of rules is said to be *finitely identifiable* iff there exists a partial recursive function f which satisfies the following: For any rule R of C and for any positive (complete) presentation σ of R,

(1) $f(\sigma[\![n]\!])$ is defined for any n, and

(2) A sequence $f(\sigma[\![1]\!]), f(\sigma[\![2]\!]), \cdots$ is finitely convergent to a number j such that $R_j = R$.

Now, suppose C be finitely identifiable. Then clearly *DIM* which definitely infers C can be constructed using f.

To the contrary, suppose C be definitely inferable. Considering a recursive function computed by the following algorithm, we see that C is finitely identifiable:

procedure $f(m)$;
begin
    let $d_1, d_2, \cdots, d_n$ be the numbers obtained by decoding m;
    present $d_1, d_2, \cdots, d_n$ to *DIM*;
    if *DIM* produces a guess **then** return the guess **else** return n
end.

For the inductive inference using the criterion of finite identification for a class of functions, refer to, for example, Freivald&Wiehagen[4], Klette&Wiehagen[7] and Jantke&Beick[6].

Finally, we consider the definite inductive inference in relation to the results shown in Angluin[1]. As easily seen, if a set S is a definite finite tell-tale of a rule R, then S is also a finite tell-tale of R, but the converse is not always true. That is, the definite finite tell-tale has stronger meaning than the finite tell-tale. Furthermore, if a class C is definitely inferable from positive data, then C is also inferable from positive data in the limit. It seems that whether finitely many "mind changes" are allowed or not makes the difference between recursive enumerability of a finite tell-tale and uniform computability of a definite finite tell-tale.

Acknowledgement

The author wishes to thank Setsuo Arikawa for many suggestions, that made me start this research, and productive discussions.

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