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Sub-extensions in the Default Knowledge Base Systems

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Abstract

The extension is the most important concept of default logic, but it is not constructible. In [6], we have provided two kinds of successively constructible extensions, sub-extensions and approximate extensions whose construction steps are all finite. In this paper, we consider a time-bounded default knowledge base system. Particularly, we pay attention to the tradeoff between constructing an infinite extension and restricting usable resources to finite ones. To reduce unnecessary computations, this system constructs the sub-extension, called the current belief set, for a user based on his/her interests (e.g. queries). We also show that this sub-extension is just a perfect belief set which the user can infer. Then, we discuss the concepts including consistency, existence and uniqueness of extensions, belief revision under the constraint of the time-bound, and give some more natural explanations for these concepts.

1 Introduction

Default logic is an important knowledge representation system for incomplete information. The extension is the most essential concept of default logic, but it is not constructible. To cope with this problem, we have provided two kinds of successively constructible extensions, sub-extensions and approximate extensions whose constructing steps are all finite [6]. However, two more problems to which we should address have been left unsolved. (a) An extension of a first order default theory may be infinite. (b) The resources which can be used to compute the extensions are finite. Therefore, we can only approximately compute some finite parts of extensions by using the time-bounded reasoning.

In this paper, we consider a time-bounded default knowledge base system. Particularly, we pay attention to the tradeoff between the above (a) and (b). In order to reduce the unnecessary computations and save the resources, this system will construct user's current belief set based on his/her interests (e.g. queries) instead of enumerating sub-extensions as shown in [6]. Then, we look over the notions of consistency, existence and uniqueness of extensions under the constraint of the time-bound.

In Section 3, we define a time-bounded default knowledge base system and the personal knowledge base for every user. In Section 4, we discuss the query form and the query processing method for this system. Then, we show a query of the form $< \alpha, \neg\alpha >$ is very useful to check the consistency of the personal knowledge base and to raise the reliability of the system. In Section 5, based on such a query processing method and the time-bound, we briefly reconsider several concepts of default logic, and offer some new explanations for them. We also show that the system can automatically determine the "perfect current belief set" for the user based on his/her queries. In Section 6, we discuss the belief revision problem in personal knowledge base. In Section 7, we describe some properties of the system.

2 Preliminaries

Let L be a first order language. Well-formed formulas (wff for short) are formed in the usual way. A wff is *closed* if it does not contain any free variable. Then a *default rule*(or *default*) is a rule of the form

$$\frac{\alpha : M\beta_1, \ldots, M\beta_m}{\omega},$$

where $\alpha, \beta_1, \ldots, \beta_m, \omega$ are wffs in L. We call α a *prerequisite*, $\beta_1, \ldots, \beta_m$ *justifications*, and ω a *consequence*. M is a meta-operator which can be read as "*it is consistent to assume*". The default is *closed* if $\alpha, \beta_1, \ldots, \beta_m, \omega$ are closed wffs.

Let D be a set of defaults, and W be a set of closed wffs. Then $\Delta = (D, W)$ is called a *default theory.* A default theory Δ is *closed* if every default in D is closed.

For a set D of default rules, we define $Cons(D)$, the set of all consequences of D, by

$$Cons(D) = \left\{ \omega \;\middle|\; \frac{\alpha : M\beta_1, \ldots, M\beta_m}{\omega} \in D \right\}.$$

For a set S of closed wffs and a closed wff ω, we use $S \vdash \omega$ to denote that ω is derived from S, and $S \not\vdash \omega$ to denote that ω is not derived from S. We also define the set $Th(S)$ by

$$Th(S) = \{\omega \mid \omega \text{ is a closed wff and } S \vdash \omega\}.$$

Definition 2.1 (Reiter [3]) Let $\Delta = (D, W)$ be a closed default theory, $S \subseteq L$ be a set of closed wffs, and $\Gamma(S)$ be the smallest set satisfying the following three conditions:

(1) $W \subseteq \Gamma(S)$,

(2) $Th(\Gamma(S)) = \Gamma(S)$,

(3) If $\dfrac{\alpha : M\beta_1, \ldots, M\beta_m}{\omega} \in D$, $\alpha \in \Gamma(S)$, $\neg\beta_1, \ldots, \neg\beta_m \notin S$ then $\omega \in \Gamma(S)$.

A set $E \subseteq L$ satisfying $\Gamma(E) = E$ is an *extension* of Δ.

An intuitive characterization of the extensions is given by the following theorem:

Theorem 2.1 (Reiter [3]) *Let $\Delta = (D, W)$ be a closed default theory, $E \subseteq L$ be a set of closed wffs, and $E_i (i \geq 0)$ be sets of closed wffs defined as follows:*

$$\begin{aligned} E_0 &= W, \\ E_{i+1} &= Th(E_i) \bigcup \left\{ \omega \;\middle|\; \begin{array}{l} \frac{\alpha : M\beta_1, \ldots, M\beta_m}{\omega} \in D \\ E_i \vdash \alpha \\ E \not\vdash \neg\beta_1, \ldots, \neg\beta_m \end{array} \right\}. \end{aligned}$$

Then E is an extension of Δ if and only if

$$E = \bigcup_{i=0}^{\infty} E_i.$$

However, to compute the extensions, we need to solve the following problems.

(i) The definition of Reiter's extensions is self-recursive.

(ii) The computation of an extension may require an infinite time since the extension may be infinite.

For the problem (i), some researchers have been proposed some constructive notions of extensions [8, 2]. However, they have not discussed the problem (ii). In order to cope with these problems, we introduce the time-bounded reasoning[5, 7] into the computation of extensions. That is, we use "$\vdash_n$" instead of "$\vdash$".

Let α be a closed wff, $S, A_1, A_2, \cdots, A_k$ be sets of closed wffs. We define the following notations:

(1) $S \vdash_n \alpha$: α is derived from S in time n.

(2) $S \not\vdash_n \alpha$: α cannot be derived from S in time n.

(3) $S \not\vdash_n^F \alpha$: all derivations of α from S fail in time n.

(4) $S \not\vdash_n^I \alpha$: whether or not α is derived from S cannot be decided in time n.

(5) $Th_n(S) = \{\alpha \mid \alpha \text{ is a closed wff and } S \vdash_n \alpha\}$.

(6) $Th'_n(A_1 \cup A_2 \cup \cdots \cup A_k) = Th_n(A_1 \cup A_2 \cup \cdots \cup A_k) - Th_n(A_1 \cup \cdots \cup A_{k-1})$.
Then, clearly

$$S \not\vdash_n \alpha \quad \Leftrightarrow \quad (S \not\vdash_n^F \alpha \;\; or \;\; S \not\vdash_n^I \alpha),$$

As is described in [6], in order to solve the problem of self-recursive definition, we construct a sub-extension SE_j to replace the E_i, and use $SE_j \not\vdash_n \neg\beta$ to replace $E \not\vdash \neg\beta$ in the checking of $M\beta$. Furthermore, we also classify the knowledge in the extension to supply more accurate results for users.

Definition 2.2 Let $\Delta = (D, W)$ be a closed default theory, and the time-bound be n. Then the *i-th finite sub-extension* SE_i, a set of closed wffs, consists of three parts:

$$SE_i = K_i \cup B_i \cup G_i \qquad (i \geq 0)$$

which are constructed as follows:

$$\begin{aligned}
&K_0 = W, \\
&B_0 = G_0 = \phi, \\
&K_{i+1} = Th'_n(K_i), \\
&B_{i+1} = Th'_n(K_i \cup B_i) \cup Cons(D'_i), \\
&G_{i+1} = Th'_n(K_i \cup B_i \cup G_i) \cup Cons(D''_i),
\end{aligned}$$

where D'_i is an empty set or a singleton set $\{\delta\}$ such that

$$\begin{aligned}
&\delta = \frac{\alpha : M\beta_1, \ldots, M\beta_m}{\omega} \in D, \\
&\omega \notin K_{i+1}, \quad K_i, B_i \vdash_n \alpha, \quad \textit{and} \\
&K_i, B_i \not\vdash_n^F \neg\beta_1, \ldots, K_i, B_i \not\vdash_n^F \neg\beta_m,
\end{aligned}$$

and D''_i is an empty set or a singleton set $\{\delta\}$ such that

$$\begin{aligned}
&\delta = \frac{\alpha : M\beta_1, \ldots, M\beta_m}{\omega} \in D, \\
&\omega \notin K_{i+1} \cup B_{i+1}, \quad K_i, B_i, G_i \vdash_n \alpha, \quad \textit{and} \\
&K_i, B_i, G_i \not\vdash_n \neg\beta_i, \quad (j = 1, \ldots, m).
\end{aligned}$$

K_i, B_i, and G_i are called the *knowledge part*, the *belief part*, and the *guess part* of SE_i, respectively.

Definition 2.3 Let $\Delta = (D, W)$ be a closed default theory, and $SE_i = K_i \cup B_i \cup G_i$ $(i = 1, 2, \cdots)$ be any sequence of finite sub-extensions constructed by Definitions 2.2. Then, $AE = K \cup B \cup G$, an *approximate extension* of the theory Δ, is a set of closed wffs, where

$$K = \bigcup_{i=0}^{\infty} K_i, \qquad B = \bigcup_{i=0}^{\infty} B_i, \qquad G = \bigcup_{i=0}^{\infty} G_i.$$

Theorem 2.2 *Let E be any extension of closed default theory Δ. Then, there exists an approximate extension AE of Δ such that $AE = E$.*

3 Personal Knowledge Bases

In order to reduce the unnecessary computation and save the resources of systems, we need to decide in what beliefs a user is interested. In knowledge base systems, a user's interest will just be represented by his/her queries. Therefore, it is more natural to approximately construct a finite part of some extension by user's queries, rather than successively enumerating them. For discussing the computation and explanation of the extensions in an inference system, we consider a multi-user knowledge base based on the default theory $\Delta = (D, W)$, as shown in Figure 1.

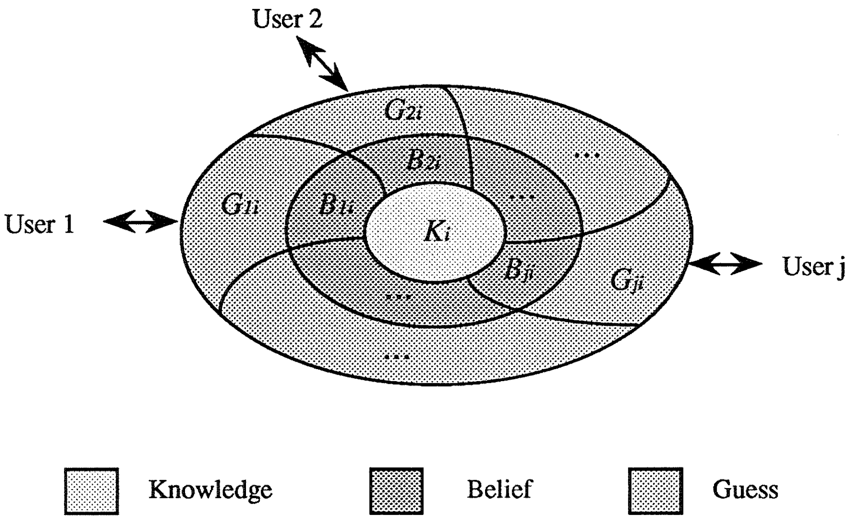

Figure 1: The structure of multi-user knowledge base

Definition 3.1 Let $\Delta = (D, W)$ be a default theory. A *default knowledge base* DKB is a set of personal knowledge bases such as

$$DKB = \{PKB^{(1)}, PKB^{(2)}, \cdots, PKB^{(m)}\},$$

where $PKB^{(i)}$ is the finite set of wffs called a personal knowledge base of the user i.

Then, we define PKB as a variant of the sub-extensions (Definition 2.2) by queries from a user.

Definition 3.2 Let $\Delta = (D, W)$ be a default theory, and $(Q_1, Q_2, \cdots)$ be a sequence of queries from a user. Then the sequence of *personal knowledge bases* is successively constructed as follows:

Initial base:

$$K_0 = W,$$
$$B_0 = \phi,$$
$$G_0 = \phi,$$

To the query Q_i, $PKB_i = K_i \cup B_i \cup G_i - \Psi(Q_i)$ are constructed by:

$$K_i = \begin{cases} K_{i-1} \cup \{Q_i\} & if\ \ Q_i \in Th'_n(K_{i-1}), \\ K_{i-1} & otherwise, \end{cases}$$
$$B_i = \begin{cases} B_{i-1} \cup \{Q_i\} & if\ \ Q_i \in Th'_n(K_{i-1} \cup B_{i-1})\ or\ Q_i \in Cons(D'_i), \\ B_{i-1} & otherwise, \end{cases}$$
$$G_i = \begin{cases} G_{i-1} \cup \{Q_i\} & if\ \ Q_i \in Th'_n(K_{i-1} \cup B_{i-1} \cup G_{i-1})\ or\ Q_i \in Cons(D''_i), \\ G_{i-1} & otherwise, \end{cases}$$

where

$$D'_i = \left\{ \delta = \frac{\alpha : M\beta_1, \ldots, M\beta_m}{\omega} \;\middle|\; \begin{array}{l} \delta \in D, \quad \omega \notin K_i, \\ K_{i-1}, B_{i-1} \vdash_n \alpha, \\ K_{i-1}, B_{i-1} \not\vdash^F_n \neg\beta_j \ (j = 1, \ldots, m) \end{array} \right\},$$
$$D''_i = \left\{ \delta = \frac{\alpha : M\beta_1, \ldots, M\beta_m}{\omega} \;\middle|\; \begin{array}{l} \delta \in D, \quad \omega \notin K_i \cup B_i, \\ K_{i-1}, B_{i-1}, G_{i-1} \vdash_n \alpha, \\ K_{i-1}, B_{i-1}, G_{i-1} \not\vdash_n \neg\beta_j \ (j = 1, \ldots, m) \end{array} \right\},$$

and $\Psi(Q_i)$ is a set of beliefs to be revised when Q_i is added to PKB_{i-1}. (Refer to Section 6 for details)

PKB_i is also called the personal knowledge base constructed by the query sequence $Q_1, \cdots, Q_i$. K_i, B_i, and G_i are called *knowledge part, belief part*, and *guess part* of PKB_i, respectively.

We have shown that $K_i^{(j)}$ (the knowledge) are essentially different from $B_i^{(j)}$ (the belief) and $G_i^{(j)}$ (the guess) in the paper [6], and the individual users may have individual belief sets. The knowledge of $K_i^{(j)}$ may be shared since it is true for all users. But $B_i^{(j)}$ ($G_i^{(j)}$) is just the set of beliefs (guesses) of *a user*. They can not be shared with the other users. $PKB_i^{(j)} = K_i^{(j)} \cup B_i^{(j)} \cup G_i^{(j)}$ just represents the current belief set of *user j*.

For simplicity's sake, we assume these users are independent of each other in the rest of this paper. Thus, it is sufficient to consider one pair of a user and his/her personal knowledge base which uses the default theory as its knowledge representation. In the following sections, we denote the personal knowledge base by $PKB = K \cup B \cup G$ if there are no confusions.

4 Query Forms and Query Processing

From Definition 3.2 and the discussions above, we can see that queries are very important and critical. Now, we discuss the forms of queries and query processing.

Suppose a knowledge base system uses the NAF principle as its deduction rule, and a user asks a query α, which is a literal, to the system. Then, the system returns "yes" if it can prove α, and returns "no" if it cannot prove α as shown in Table 1.

$KB \vdash \alpha$	Answers to query α	Answers to query $\neg\alpha$
×	no	yes
○	yes	no

○ : *provable*, × : *unprovable.*

Table 1: The answers to query

However, for a knowledge base which explicitly represents the negation, since it does not use NAF principle as its a deduction rule, we have

$$KB \not\vdash \alpha \quad \not\Rightarrow \quad KB \vdash \neg\alpha, \quad and \quad KB \not\vdash \neg\alpha \quad \not\Rightarrow \quad KB \vdash \alpha.$$

That is, the proof of a query α ($\neg\alpha$) is not related to those of $\neg\alpha$ (α). A user always expects to get an answer, e.g.,"yes", "no" or some other information, for his/her query.

We think that a more natural query form should be a pair of $< \alpha, \neg\alpha >$ in such a system. As shown in Table 2, the system can provide more complete and accurate answers to users, and checking of some consistency of knowledge base at the same time by such a query form.

$KB \vdash \alpha$	$KB \vdash \neg\alpha$	Answers to Query α	Answers to Query $\neg\alpha$
×	×	unknown	unknown
×	○	unknown	yes
○	×	yes	unknown
○	○	contradictory	contradictory

Table 2: The answers to query in classical first order logic

The default knowledge base is similar to such one. But since a default theory may have many extensions, the default knowledge base system may provide many different answers to the same query. These answers may belong to different extensions and may be inconsistent with each other. Therefore, $KB \vdash \alpha$ and $KB \vdash \neg\alpha$ do not always mean contradiction. "?" in Table 3 represents that the answer is more complicated.

$KB \vdash \alpha$	$KB \vdash \neg\alpha$	Answers to Query α	Answers to Query $\neg\alpha$
×	×	unknown	unknown
×	○	unknown	yes
○	×	yes	unknown
○	○	?	?

Table 3: The answers to query in default logic

No.	$PKB \vdash_n \alpha$	$PKB \vdash_n \neg\alpha$	Answers to Query $< \alpha, \neg\alpha >$	Comments
1	×	×	unknown	PKB can not answer this query with time-bound n
2	×	**G**	$\mathbf{G}\neg\alpha$	$\neg\alpha$ is a guess of PKB
3	×	**B**	$\mathbf{B}\neg\alpha$	$\neg\alpha$ is a belief of PKB
4	×	**K**	$\mathbf{K}\neg\alpha$	$\neg\alpha$ is a knowledge of PKB
5	**G**	×	$\mathbf{G}\alpha$	α is a guess of PKB
6	**G**	**G**	Selective	The answer to this query will be selected by other conditions
7	**G**	**B**	$\mathbf{B}\neg\alpha$	$\neg\alpha$ is a belief of PKB
8	**G**	**K**	$\mathbf{K}\neg\alpha$	$\neg\alpha$ is a knowledge of PKB
9	**B**	×	$\mathbf{B}\alpha$	α is a belief of PKB
10	**B**	**G**	$\mathbf{B}\alpha$	α is a belief of PKB
11	**B**	**B**	Selective	The answer to this query will be selected by other conditions
12	**B**	**K**	$\mathbf{K}\neg\alpha$	$\neg\alpha$ is a knowledge of PKB
13	**K**	×	$\mathbf{K}\alpha$	α is a knowledge of PKB
14	**K**	**G**	$\mathbf{K}\alpha$	α is a knowledge of PKB
15	**K**	**B**	$\mathbf{K}\alpha$	α is a knowledge of PKB
16	**K**	**K**	contradictory	shows W is inconsistent

Table 4: The answers to query in default logic with time-bound

Now, we discuss this problem in our personal knowledge base. Since there are three kinds of different correctness results in the personal knowledge base, it may provide the answers in $\{\times$, $\mathbf{K}\alpha$, $\mathbf{B}\alpha$, and $\mathbf{G}\alpha\}$ to the query α, where $\mathbf{G}\alpha$, $\mathbf{B}\alpha$, and $\mathbf{K}\alpha$ mean $\alpha \in Th'_n(K \cup B \cup G)$, $\alpha \in Th'_n(K \cup B)$, and $\alpha \in Th'_n(K)$, respectively.

As is described in the paper [6], a default theory $\Delta = (D, W)$ consists of a complete knowledge part W (such as a first order theory) and an incomplete knowledge part D (such as a commonsense knowledge). Their reliabilities are different. In general, we think that the reliability of W is higher than D's. Furthermore, Definition 3.2 shows that

$$\mathbf{K}\alpha \succ \mathbf{B}\alpha \succ \mathbf{G}\alpha,$$

where $A \succ B$ means the reliability of A is higher than B. In general, the system should select a more reliable result as its answer. According to these observations, we can give and explain the query processing method as in Table 4. With such classification and selection, the system will reply queries in more detail and accurately. Lines 6, 11, and 16 in Table 4 will be discussed in the next sections.

5 Approximate Extensions and Perfect Belief Sets

In this section, we discuss the consistency and extension of the default knowledge base system based on the following assumption:

Assumption 1 *The time used to infer a query is finite.*

Assumption 2 *A user does not need to know all conclusions from the knowledge base.*

Since a default knowledge base system is a computational machine and a user may only be interested in some parts of knowledge base, such assumptions should be natural and reasonable.

5.1 Local Consistency

Due to the constraint of time-bound, the consistency of our system can not be guaranteed. Hence a weaker notion of consistency which depends on time-bound is needed.

Definition 5.1 A personal knowledge base PKB is *locally consistent* if there does not exist an atom α such that $\alpha \in PKB$ and $\neg\alpha \in PKB$.

As shown in Table 4, there may be many kinds of local inconsistency in a default knowledge base. We can divide them into two: (a) The first order theory W in the default theory $\Delta = (D, W)$ is inconsistent as in Line 16; (b) The beliefs in different extensions may be inconsistent as in Lines 6, 7, 8, 10, 11, 12, 14, and 15.

Reiter [3] defined that a closed default theory has *an inconsistent extension* iff one of its extensions is the set of all closed wffs of L. He also proved the following facts:

Corollary 5.1 (Reiter [3]) *A closed default theory* (D, W) *has an inconsistent extension iff* W *is inconsistent.*

Corollary 5.2 (Reiter [3]) *If a closed default theory has an inconsistent extension then it is only the extension.*

These corollaries show that W is the most important and basic part of the default knowledge base. The inconsistency of W results in that the system believes any wff. To find and solve the inconsistency in a knowledge base is an important subject of AI research, and there are many investigations such as the TMS systems (Doyle [1]) and the contradiction backtracing algorithm (Shapiro [4]). Since this subject beyond the scope of this paper, we will not discuss the local inconsistency (a). We give another assumption.

Assumption 3 W *is consistent.*

Before discussing the local inconsistency (b), we briefly look over some notions which is related to the extensions.

5.2 Extensions

Extensions play an important role in default logic. An extension is a set of beliefs which are in some sense "justified" or "reasonable" in light of what is known about a world.

There are significant problems for extensions such as:

(1) existence problem of extensions,

(2) uniqueness problem of extensions,

(3) how to select the answer from extensions.

Unfortunately, all of these problems can not be solved in general.

However, based on Assumptions 1, 2, and 3 above, we can supply some new explanations for these notions in default knowledge base systems.

The inconsistency among defaults in D is a main reason that affects the existence and uniqueness of extensions. Since the defaults are very delicate, it is very difficult to guarantee existence and uniqueness of extensions in a large base. But, since a knowledge base is an abstraction of some domain, a user may just be interested in some part of this base (Assumption 2). That is, a user merely uses a sub-theory, a part of default theory, as his/her theory. Thus, to such a user, it is more important that there is a unique extension for a sub-theory rather than for the whole theory. Unfortunately, it is also impossible to know whether such a sub-theory has a unique consistent extension or not within a given time-bound. The system and the user just can select his/her perfect current belief set at that time. In the next subsection, we will show this.

5.3 Perfect Belief Set

Although a user may have many methods to decide his/her sub-theory, we only consider the method which depends on the interactions between the system and the user. Now, we discuss the inconsistency as shown in Table 4, except Lines 6, 11, and 16.

Since the individual extensions of a default theory are both ground and complete, it is quite natural to require any default inference system to restrict its conclusions to some extension. If conclusions are drawn from different extensions, they may be incompatible. That is, the beliefs which belong to different extensions may be inconsistent with each other.

Example 5.1 Consider the following default theory $\Delta_1 = (D, W)$ with

$$D = \left\{ \frac{student(X) : M\neg married(X)}{\neg married(X)}, \quad \frac{adult(X) : Mmarried(X)}{married(X)} \right\},$$

$$W = \left\{ adult(X) \leftarrow student(X)., \quad student(bob). \right\}.$$

There are two extensions $E^{(1)}$ and $E^{(2)}$ for Δ_1 as shown in Table 5.

$E^{(1)}$	$E^{(2)}$
$adult(bob) \leftarrow student(bob).$, $student(bob), adult(bob)$, $\neg married(bob)$	$adult(bob) \leftarrow student(bob).$, $student(bob), adult(bob)$, $married(bob)$

Table 5: The extensions of Δ_1

$\neg married(bob)$ and $married(bob)$ are two conclusions in Δ_1. However, they are drawn from different extensions, and concluding both leads to inconsistency.

For a query $married(bob)$ ($\neg married(bob)$), the system can prove the beliefs $married(bob)$ and $\neg married(bob)$ simultaneously. Obviously, it does not represent that the system is inconsistent. In this case, the system needs to select exactly one of them as its answer. Depending on the observation that the simpler knowledge is more reliable, the system will select $\mathbf{B}\neg married(bob)$ as the user's current belief, since the complexity of $\mathbf{B}married(bob)$ is greater than that of $\mathbf{B}\neg married(bob)$.

Example 5.2 Consider a default theory $\Delta_2 = (D, W)$ with

$$\begin{aligned} D &= \left\{ \delta_1 = \frac{: MA}{\neg B},\ \delta_2 = \frac{: MB}{\neg C},\ \delta_3 = \frac{: MC}{\neg A} \right\}, \\ W &= \phi. \end{aligned}$$

Although there are no extensions for Δ_2, A user can successively constructs his/her current belief set and sub-theory. Assume that the user is only concerned with A and C. If he/she firstly asks the query A, the answer $\mathbf{B}\neg A$ is obtained by using the default δ_3. Then, if he/she asks the another query C, the answer $\mathbf{B}\neg C$ is obtained by using the default δ_2. Obviously, $\mathbf{B}\neg A$ becomes a false belief because of the addition of $\mathbf{B}\neg C$. To keep the consistency of current belief set of the user, belief revision is needed. Furthermore, the belief revision operation deletes the false belief $\mathbf{B}\neg A$. That is, this user uses the sub-theory $(\{\delta_2, \delta_3\}, W)$ as the theory of himself/herself, and $\{\mathbf{B}\neg C\}$ as his/her belief set.

Thus, The existence of proofs $PKB \vdash \alpha$ and $PKB \vdash \neg\alpha$ does not mean that the theory is inconsistent, rather it means that there are several different candidates for the next "current belief set". In general, the system will select result whose reliability is higher as its answer and the next "current belief set".

To Lines 6 and 11 which have the same reliability, the system selects the answer by the run-time of their derivations. According to Table 4 and the discussions above, the default knowledge base system can automatically determine the next "current belief set" for the user based on his/her query. We also call the PKB constructed by such a selection method as a *perfect belief set* of the user at that time.

6 Belief Revision

As described in Examples 5.1 and 5.2, to keep that the current belief set is locally consistent, the belief revision is needed.

In order to represent the scope of belief revision, we define some notations. Firstly we simply denote a negative predicate symbol by an new individual predicate symbol. For example, we take the predicate symbol of "$\neg married(x)$" as "$\neg married$".

Definition 6.1 Let PKB be a knowledge base which use $\Delta = (D, W)$ as its theory. A binary relation Rel on $PS(PKB)$ is defined as follows:

$$Rel_+ = \left\{ (pred(A), pred(B_i)) \;\middle|\; \begin{array}{c} A \leftarrow B_1, \cdots, B_k \in W, \\ or \\ \dfrac{B_i : MC_1, \cdots, MC_m}{A} \in D \end{array} \right\},$$

$$Rel_- = \left\{ (pred(A), pred(C_i)) \;\middle|\; \dfrac{B : MC_1, \cdots, MC_m}{A} \in D \right\},$$

where $pred(L)$ denotes the predicate symbol of the literal L, and $PS(PKB)$ is the set of predicate symbols in PKB.

Definition 6.2 Let p be a predicate symbol in knowledge base PKB.

$$IS_+(p) = \{q \mid (q,p) \in Rel_+^+\},$$
$$IS_-(p) = \{q \mid (q,p) \in Rel_-^+\}$$

are the sets of the predicate symbols called *an positive influence set of the predicate symbol* p, and *an negative influence set of the predicate symbol* p, where Rel_+^+ and Rel_-^+ are the transitive closures of Rel_+ and Rel_-, respectively.

Definition 6.3 Let PKB be a current belief set of some user as defined in Definition 3.2, and Q be a literal. Then, $PKB' = K' \cup B' \cup G'$ is constructed from PKB in the following way:

(a) when $\mathbf{K}Q$ is added to K:

$$\begin{aligned} B' &= B - IS_-(pred(Q)) \\ G' &= G - IS_-(pred(Q)) \end{aligned}$$

(b) when $\mathbf{B}Q$ is added to B:

$$\begin{aligned} B' &= B - IS_-(pred(Q)) \\ G' &= G - IS_-(pred(Q)) \end{aligned}$$

(c) when $\mathbf{B}Q$ is deleted from B:

$$\begin{aligned} B' &= B - IS_+(pred(Q)) \\ G' &= G - IS_+(pred(Q)) \end{aligned}$$

(d) when $\mathbf{G}Q$ is added to G:

$$G' = G - IS_-(pred(Q))$$

(e) when $\mathbf{G}Q$ deleted from G:

$$G' = G - IS_+(pred(Q))$$

7 Properties of PKB

With Definition 3.2 and Definition 6.3, we give a new method to approximately construct the sub-extensions by user's queries. Therefore, we can see that the personal knowledge base is a variant of the sub-extension and the approximate extension in the knowledge base systems. We know that W is the most basic part of the default knowledge base. The inconsistency of W will lead to that the whole knowledge base becomes meaningless. If W is consistent, then by the definition of PKB we have the following results.

Proposition 7.1 *Let PKB_1, PKB_2, $\cdots$, PKB_k be a sequence of personal knowledge bases defined by Definition 3.2 and Definition 6.3. Then,*

$$K_1 \subseteq K_2 \subseteq \cdots \subseteq K_k \subseteq \cdots$$

Theorem 7.2 (local soundness) *Let PKB_i be a personal knowledge base, and α be a ground literal. Then,*

$$K_i \vdash_n \alpha \Rightarrow \alpha \in Th(W).$$

Theorem 7.3 (local validness) *Let PKB_i be a personal knowledge base, and α be a ground literal. Then,*

$$\alpha \in Th(W) \;\Leftrightarrow\; \textit{there is a series of queries } \alpha_1, \cdots, \alpha_k (= \alpha) \textit{ such that } K_i \vdash_2 \alpha.$$

Proposition 7.4 *Let PKB_0, PKB_1, $\cdots$, PKB_k be a sequence of personal knowledge bases defined by Definition 3.2 and Definition 6.3. If PKB_0 is locally consistent, then so are PKB_i $(i > 0)$.*

8 Conclusions

In the previous paper [6], to provide a more effective method for computing extensions, we have introduced the time-bounded reasoning into Reiter's default theory, and given two kinds of successively constructible extensions, the sub-extensions and approximate extensions, whose construction steps terminate in a finite time.

To show the approximate extensions is practically useful, and to discuss the notions (e.g., the infiniteness, existence and uniqueness of extensions) under the constraint of the resource limitation (time-bound), we have defined a multi-user time-bounded default knowledge base system and a personal knowledge base which is a variant of the sub-extension, and given a new query form and described its necessity and efficiency for such a system. We have shown that a user just uses a sub-theory of the whole default theory as his/her theory, and offered some new explanations for the above notions based on this system. We have seen that the system can automatically select a perfect belief set for the user.

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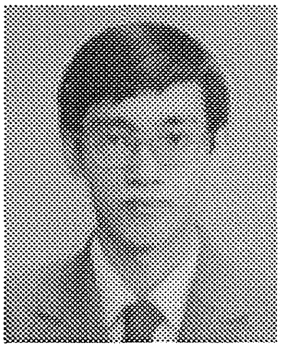

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