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Furuya, Shinji
Department of Information Systems, Kyushu University

Miyano, Satoru
Department of Information Systems, Kyushu University

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Shinji Furuya
Satoru Miyano

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Research Institute of Fundamental Information Science
Kyushu University 33
Fukuoka 812, Japan

E-mail: miyano@rifis.sci.kyushu-u.ac.jp Phone: 092 (641)1101 Ex. 4471

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Shinji Furuya and Satoru Miyano
Department of Information Systems
Kyushu University 39

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Abstract

Analogy is described in predicate logic. This paper deals with the analogy without any function symbols except constants. We show that the problem of deciding whether a given atomic formula can be inferred by analogy is NP-hard even in such a simple case.

1 Introduction

Analogy is an inference method that acquires unknown facts or knowledge by finding similarities among given objects. Some theoretic formulations have been suggested to realize analogy on a computer [6][7][8]. But the computational complexity of analogy has not yet been studied very much. In this paper we deal with the analogy theory by Haraguchi and Arikawa [1][3][4][5] that is described in terms of predicate logic. We consider the case where no function symbols are allowed. We show that a problem of deciding whether a given atomic formula can be inferred by analogy is NP-hard even in such a simple case.

2 Analogy principle

A *definite clause* is a formula of the form

$$q_0(t_1^0, \ldots, t_{n_0}^0) \leftarrow q_1(t_1^1, \ldots, t_{n_1}^1), \ldots, q_r(t_1^r, \ldots, t_{n_r}^r) \quad (r \geq 0),$$

where t_j^i are terms and q_i are predicate symbols. In Haraguchi and Arikawa's analogy theory [1][3][4][5], an object of analogy is the minimal model M represented by a finite set S of definite clauses. In this paper we concentrate on the case where no function symbols are allowed except constant symbols. Therefore terms are constants or variables. We call each element in M a *fact*. An atomic formula containing no variables is simply called an *atom*.

Let S_i be a finite set of definite clauses and let $C(S_i)$ be the set of constants in S_i for $i = 1, 2$. A *partial identity* between S_1 and S_2 is a subset φ of $C(S_1) \times C(S_2)$ such that for each $a \in C(S_1)$ (resp., $a' \in C(S_2)$) there is at most one $a' \in C(S_2)$ (resp., $a \in C(S_1)$) with $\langle a, a' \rangle \in \varphi$. Hence φ gives a one-to-one correspondence between some subsets of $C(S_1)$ and $C(S_2)$.

Let $t_j \in C(S_1), t'_j \in C(S_2)$ for $j = 1, 2, \ldots, n$ and let α, α' be atoms in S_1, S_2, respectively. For a partial identity φ, we say that α and α' are *identified* by φ, denoted by $\alpha\varphi\alpha'$, if they are written as

$$\begin{aligned} \alpha &= p(t_1, t_2, \ldots, t_n), \\ \alpha' &= p(t'_1, t'_2, \ldots, t'_n), \end{aligned}$$

and $\langle t_j, t'_j \rangle \in \varphi$ for $i = 1, 2, \ldots, n$.

Haraguchi and Arikawa's analogy is explained with these terminologies as follows. We assume that there exist facts $\beta_1, \beta_2, \ldots, \beta_n$ in S_1 such that $\alpha \leftarrow \beta_1, \beta_2, \ldots, \beta_n$ holds in S_1. Then if there exist facts $\beta'_1, \beta'_2, \ldots, \beta'_n$ in S_2 with $\beta_i\varphi\beta'_i$ for $i = 1, 2, \ldots, n$, then we infer α' in S_2 by identifying it with α.

An atom α' inferred in this way is not always a fact in S_2. But the partial identity φ gives a reason of possibility that α' holds in S_2. Then we can continue to infer by analogy, assuming such α' to be a fact in S_2. Conversely, we also infer atoms in S_1 from S_2 by analogy in the same way. Let $M_i(*)$ be the set of atoms in S_i which can be inferred in this way. Formally, $M_i(*)$ is defined inductively as follows.

Definition. Let S_i be a finite set of definite clauses and let M_i be the minimal model of S_i for $i = 1, 2$. For a partial identity φ, we define $M_i(*)$ as follows, where we set i (resp., i') to 1 (resp., 2) or 2 (resp., 1).

$M_i(*) = \bigcup_k M_i(k)$,
$M_i(0) = M_i$,
$R_i(k) = \{\alpha \leftarrow \beta_1, \beta_2, \ldots, \beta_n \mid \beta_j \in M_i(k), \beta'_j \in M_{i'}(k)$ $(j = 1, 2, \ldots, n)$ and $\alpha' \leftarrow \beta'_1, \beta'_2, \ldots, \beta'_n$ holds in $S_{i'}$ and $\alpha\varphi\alpha', \beta_j\varphi\beta'_j\}$,

$$M_i(k+1) = \{\alpha \mid R_i(k) \cup M_i(k) \cup S_i \vdash \alpha\}.$$

Example. Consider the following sets S_1 and S_2 of definite clauses, where upper-case letters are variables and lower-case letters are constants or predicate symbols.

$$\begin{aligned} S_1 = & \{ \ p(a,b),\ q(b,c), \\ & \quad r(Y,X) \leftarrow q(X,Y), \\ & \quad s(X,Z) \leftarrow p(X,Y), r(Z,Y) \ \} \\ S_2 = & \{ \ p(a',b'),\ q(b',c') \ \} \end{aligned}$$

Then take the following partial identity φ:

$$\varphi = \{ \ \langle a,a' \rangle,\ \langle b,b' \rangle,\ \langle c,c' \rangle \ \}.$$

For S_1, S_2 and φ, the inference by analogy goes as follows. First, we obtain $M_2(0) = \{p(a',b'), q(b',c')\}$. Next we get $r(c,b) \leftarrow q(b,c)$ from $r(Y,X) \leftarrow q(X,Y) \in S_1$. Then we get $r(c',b') \leftarrow q(b',c') \in R_2(0)$ by $q(b,c)\varphi q(b',c')$ and $r(c',b') \in M_2(1)$. Moreover, we get $s(a,c) \leftarrow p(a,b), r(c,b)$ from $s(X,Z) \leftarrow p(X,Y), r(Z,Y) \in S_1$. Then we get $s(a',c') \leftarrow p(a',b'), r(c',b') \in R_2(1)$ by $p(a,b)\varphi p(a',b')$, $r(c,b)\varphi r(c',b')$ and $s(a',b') \in M_2(2)$. No more atoms can be inferred by analogy. Hence $M_2(2) = M_2(*)$.

3 Computational complexity

Analogy consists of two phases. One is to search a partial identity φ and the other is to infer using φ. We consider the following decision problem on analogy.

ANALOGY
Instance: Two finite sets of definite clauses S_1, S_2 and an atom $p(t_1, t_2, \ldots, t_n)$.
Problem: Decide whether there exists a partial identity φ between S_1 and S_2 such that $p(t_1, t_2, \ldots, t_n)$ is in $M_2(*)$.

We obtain the following theorem about the complexity of searching a partial identity.

Theorem. ANALOGY is NP-hard.

Proof. We give a reduction from 3-SAT (3-satisfiability problem) [2] to ANALOGY. For a Boolean formula $F = C_1C_2\cdots C_m$ in three conjunctive normal form (3-CNF), S_1 and S_2 are constructed as follows, where $x_1,\ldots,x_n$ are the variables in F. For $i = 1,2,\ldots,n$,

$$h_i(a_i),\ h_i(\bar{a}_i) \in S_1 \text{ and } h_i(a'_i) \in S_2,$$

where a_i and $\bar{a}_i$ are constant symbols in S_1, a'_i is a constant symbol in S_2 and h_i is a predicate symbol.

Next, for each clause C_j, we use predicate symbols p_j of zero argument and q_j of one argument. Let α be a literal in C_j.

If $\alpha_j = x_u$, then

$$q_j(a_u) \in S_1,\quad q_j(a'_u) \in S_2 \text{ and } p_j \leftarrow h_u(X), q_j(X) \in S_1.$$

If $\alpha_j = \bar{x}_u$, then

$$q_j(\bar{a}_u) \in S_1,\quad q_j(a'_u) \in S_2 \text{ and } p_j \leftarrow h_u(X), q_j(X) \in S_1.$$

Moreover,

$$p \leftarrow p_1, p_2, \ldots, p_m \in S_1,$$

where p is a predicate symbol of zero argument. Then we show that F is satisfiable if and only if there exists a partial identity φ such that p is in $M_2(*)$.

First, if F is satisfiable, then let $\hat{x_1},\ldots,\hat{x_n}$ be a truth assignment to the variables $x_1,\ldots,x_n$ that satisfies each clause C_j of F. We define the partial identity φ by

$$\langle a_i, a'_i\rangle \in \varphi \quad \text{if } \hat{x}_i = 1$$
$$\langle \bar{a}_i, a'_i\rangle \in \varphi \quad \text{if } \hat{x}_i = 0$$

for each $i = 1,\ldots,n$. Then we can infer each p_j as follows. If C_j contains a literal $\hat{x}_i$ with $\hat{x}_i = 1$, then φ contains $\langle a_i, a'_i\rangle$. We get $p_j \leftarrow h_i(a_i), q_j(a_i) \in S_1$ from $p_j \leftarrow h_i(X), q_j(X) \in S_1$. Then we get $p_j \leftarrow h_i(a'_i), q_j(a'_i) \in R_2(0)$ by $h_i(a_i)\varphi h_i(a'_i)$ and $q_j(a_i)\varphi q_j(a'_i)$. Therefore $p_j \in M_2(1)$. If C_j is satisfiable by a literal $\hat{x}_i$ with $\hat{x}_i = 0$, we can show in a similar way that p_j is inferred by analogy. Hence p is in $M_2(2)$.

Conversely, assume that there exists a partial identity φ such that p is in $M_2(*)$. For each $i = 1,\ldots,n$, we define a truth assignment $\hat{x}_1,\ldots,\hat{x}_n$ as follows: If φ contains $\langle a_i, a_i' \rangle$, then $\hat{x}_i = 1$. If φ contains $\langle \bar{a}_i, a_i' \rangle$, then $\hat{x}_i = 0$. Otherwise $\hat{x}_i$ is arbitrary. If p is $M_2(*)$, each p_j must be inferred by analogy using φ since it is not in S_2. Then, there exists i such that $p_j \leftarrow h_i(a_i'), q_j(a_i')$ is in $R_2(0)$ and $p_j \leftarrow h_i(a_i), q_j(a_i)$ or $p_j \leftarrow h_i(\bar{a}_i), q_j(\bar{a}_i)$ holds in S_1 since both h_i and q_j are not in the left side of definite clauses. Therefore φ must contain either $\langle a_i, a_i' \rangle$ or $\langle \bar{a}_i, a_i' \rangle$. If $\langle a_i, a_i' \rangle \in \varphi$, then we can satisfy C_j by $\hat{x}_i = 1$. If $\langle \bar{a}_i, a_i' \rangle \in \varphi$, then we can satisfy C_j by $\hat{x}_i = 0$. It is not hard to see that this reduction is computable in polynomial time or log space. Hence ANALOGY is NP-hard. □

Remark. If the argument of each predicate symbol is bounded by a fixed constant and if the number of atomic formulas containing variables is also bounded by a fixed constant in each definite clause, then we can see that ANALOGY is solvable in NP. The definite clauses constructed in our reduction satisfies these conditions. Moreover, S_2 consists of only facts.

4 Conclusion

Our analysis shows that searching a partial identity φ such that a given atom can be inferred using φ is at least as hard as finding a truth assignment that satisfies a given 3-CNF formula. Therefore the computational complexity of analogy is fairly large even in the case where no function symbols are allowed. But the analogical reasoning is used not only to decide whether a given atom is inferred but also to obtain new knowledge. In this respect, analogy can be a useful technique in Artificial Intelligence in spite of its large complexity

References

[1] Arikawa, S. and Haraguchi, M.: *Knowledge Acquisition and Learning, Osuga, S. and Saeki, Y.,Eds*, Chapter 8, 221-251, Ohm (1987) (in Japanese).

[2] Garey, M.R. and Johnson, D.S.: *Computers and Intractability: A Guide to the Theory of NP-Completeness*, W.H. Freeman and Company (1979).

[3] Haraguchi, M. and Arikawa, S.: *A formulation of analogical reasoning and its realization*, *Journal of Japanese Society for Artificial Intelligence* **1** (1), 132-139 (1986) (in Japanese).

[4] Haraguchi, M. and Arikawa, S.: *A foundation of reasoning by analogy–analogical union of logic programs*, *LNCS* **264**, Springer-Verlag, 58-69 (1987).

[5] Haraguchi, M. and Arikawa, S.: *Reasoning by analogy as a partial identity between models*, *LNCS* **265**, Springer-Verlag, 61-87 (1987).

[6] Kling, R. E.: *A paradigm for reasoning by analogy*, *Artif. Intell.* **2**, 147-178 (1971).

[7] Plaisted, D. A.: *Theorem proving with abstraction*, *Artif. Intell.* **16**, 47-108 (1981).

[8] Winston, P. E.: *Learning and reasoning by analogy*, *CACM* **23**, 689-703 (1980).