Correct Definition of Finite Elasticity

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Correct Definition of Finite Elasticity

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Abstract: Finite elasticity is introduced by K. Wright as a sufficient condition for classes of languages to be inferable from positive data. However it is shown that there exists a counterexample, that is, a class of languages that has finite elasticity but cannot be inferred from positive data. In this paper, a correct definition of finite elasticity is presented and shown to be sufficient for inferability from positive data.

1. Preliminaries

First we briefly review basic notions and results on inductive inference according to literatures [1, 2, 3, 5]. The reader interested in more details should be referred to them.

Let Σ be a finite set of *symbols*. A *word* is a finite string of symbols. By Σ^* we denote the set of all words. A *formal language* (*language* for short) is a subset of Σ^*. A class of languages $C = L_1, L_2, \ldots$ is said to be *an indexed family of recursive languages* if there exists a computable function $f: I \times \Sigma^* \to \{0, 1\}$ such that $f(i, w) = 1$ if $w \in L_i$ and $f(i, w) = 0$ if $w \notin L_i$, where $I = \{ 1, 2, \ldots \}$ is the set of *indexes*. Throughout in this paper, we assume classes are indexed families of recursive languages. A *positive presentation* of a language L is an infinite sequence of all words in L. Duplicating occurrences of words are allowed to appear in any positive presentation. An *inductive inference machine* (*IIM* for short) is an effective procedure that requests input from time to time and produces output from time to time. A sequence is said to *converges to s* if all but finitely many elements in the sequence is equal to s.

DEFINITION [5] A class of languages $C = L_1, L_2, \ldots$ is said to be *inferable from positive data* if there exists an *IIM M* such that the outputs produced by M receiving any positive presentation of L_i are converges to an index j with $L_i = L_j$, for any i.

Gold [5] proved that any class of languages containing all finite languages and at least one infinite language is not inferable from positive data. From his result we can easily show that even the class of regular languages can not be inferred from positive data. In [5] only trivial classes are shown to be inferable from positive data. However Angluin gave a characterizing theorem for classes to be inferable from positive data and presented nontrivial classes [1, 2]. The class of pattern languages is one of the most important classes she presented. Shinohara showed the class of unions of two pattern languages is inferable from positive data [6]. Wright extended this to unions of three or more languages [8].

DEFINITION [1, 2] Let $C = L_1, L_2, \ldots$ be a class of languages. A finite subset $T_i \subseteq L_i$ is said to be a *finite tell-tale* if any language $L_j \subsetneq L_i$ fails to contain T_i.

THEOREM 1 [1, 2] A class of languages $C = L_1, L_2, \ldots$ is inferable from positive data if and only if there exists an effective procedure that enumerates all words in a finite tell-tale of L_i for any given i.

Wright [8] showed a property as a sufficient condition for languages to be inferable from positive data. The following is the definition of the property.

DEFINITION [8] A class C of languages has *infinite elasticity* if there exist two infinite sequences

$w_0, w_1, w_2, \ldots$ of words and $L_1, L_2, L_3, \ldots$ of languages in C

such that $w_k \in L_n$ if and only if $k < n$. A class has *finite elasticity* if it does not have infinite elasticity.

In the definition above, $w_i \neq w_j$ and $L_i \neq L_j$ whenever $i \neq j$. Because $w_k \notin L_k$ but $w_k \in L_{k+i}$ for any $i \geqq 1$. Note that any language L_i cannot contain infinitely many words $w_i, w_{i+1}, \ldots$.

2. A Counterexample

Wright showed a theorem that guarantees classes having finite elasticity to be inferable from positive data. However we can show a counterexample, that is, a class of languages which has finite elasticity but is not inferable from positive data.

Consider a class of languages $C = \{ L \mid L = \Sigma^* \text{ or } L = \Sigma^* - \{w\},\ w \in \Sigma^* \}$. Since the complement of any language in C is empty or singleton, it is clear that C does not have infinite elasticity. For any finite set S of words there exists a language $L \in C$ such that $S \subseteq L$ but $L \neq \Sigma^*$. Any language other than Σ^* is properly contained in Σ^*. Therefore any finite set S of words cannot be a finite tell-tale of Σ^*. Thus we can show that C has finite elasticity but C is not inferable from positive data from Theorem 1.

3. Correct Definition of Finite Elasticity

DEFINITION A class C of languages has *infinite elasticity* if there exist two infinite sequences

$$w_0, w_1, w_2, \ldots \text{ of words and } L_1, L_2, L_3, \ldots \text{ of languages in } C$$

such that $\{ w_0, w_1, \ldots, w_{n-1} \} \subseteq L_n$ but $w_n \notin L_n$ for any $n \geqq 1$. A class has *finite elasticity* if it does not have infinite elasticity.

If the property of finite elasticity is defined as the above, the class C presented in the previous section has infinite elasticity. Let $w_0, w_1, w_2, \ldots$ be any infinite sequence of mutually distinct words and let $L_k = \Sigma^* - \{w_k\}$ for $k = 1, 2, 3, \ldots$. Then, $\{w_0, w_1, \ldots, w_{k-1}\} \subseteq L_k$ but $w_k \notin L_k$. Therefore the elasticity of C is infinite.

THEOREM 2 If a class of languages has finite elasticity then it is inferable from positive data.

Proof. Let $C = L_1, L_2, \ldots$ be a class of languages having finite elasticity. Consider the following procedure A.

```
procedure A;
input:    an index i;
output:   an enumeration of a finite tell-tale of L_i;
method:
  begin
    let w_0, w_1, w_2, ... be a recursive enumeration of all words in L_i;
    output w_0;
    T := { w_0 };
    n := 1;
    do
      find (j, k) such that T ⊆ L_j and w_k ∉ L_j;
      if found then
      begin
        output w_k;
        T := T ∪ {w_k};
        n := n + 1
      end
    forever
  end;
```

First we show procedure A produces only finitely many outputs. Let (j_n, k_n) be the value of (j, k) when the n-th output is produced. Then $\{ w_0, w_{k_1}, \ldots, w_{k_{n-1}} \} \subseteq$

L_{j_n} but $w_{k_n} \notin L_{j_n}$. If A produces infinitely many outputs, then two infinite sequences

$$w_0, w_{k_1}, w_{k_2}, \ldots \text{ and } L_{j_1}, L_{j_2}, L_{j_3}, \ldots$$

show infinite elasticity of C. Therefore, by a contradiction, A produces only finitely many outputs.

Let t be the number of outputs produced by A for an index i. Then $T = \{ w_0, w_{k_1}, \ldots, w_{k_{t-1}} \}$ is a finite tell-tale of L_i. Because, if not so, then $T \subseteq L_j \subsetneq L_i$ for some j and $w_k \in L_i - L_j$ for some k, therefore a pair (j, k) should be found and another output should be produced. This contradicts to our assumption that $w_{k_{t-1}}$ is the last output.

By Theorem 1, C is inferable from positive data. □

4. Discussion

We have presented a counterexample for Wright's result [8], and modified the definition of finite elasticity to be sufficient for inferability from positive data. Wright also showed that finite elasticity is closed under unions. Using the notion of finite elasticity, Shinohara showed the class of languages defined by length-bounded elementary formal systems [4] with at most n axioms is inferable from positive data [7]. Fortunately such results are still valid when our new definition is adopted.

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