

## A Reasoning System using Inductive Inference of Analogical Union

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<https://doi.org/10.5109/3111>

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出版情報 : Bulletin of informatics and cybernetics. 23 (3/4), pp.121-128, 1988-09-30. Research Association of Statistical Sciences  
バージョン :  
権利関係 :

## **A REASONING SYSTEM USING INDUCTIVE INFERENCE OF ANALOGICAL UNION**

By

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### **Abstract**

Analogical reasoning derives a new fact based on the analogous facts previously known. Inductive inference is a process of gaining a general rule from examples. We propose a new reasoning system using inductive inference and analogical reasoning, which is applicable to intellectual information processing and we characterize its power. Given an enumeration of paired examples, this system inductively infers a program representing the paring and constructs an analogical union. It reasons by analogy new facts in the limit, i.e., reasons them correctly from some time on, by using a deducibility of them from the analogical union.

### **1. Introduction**

Analogical reasoning is one which derives a new fact based on the analogous facts previously known. Of many researches on analogical reasoning, Haraguchi and Arikawa's theory [1, 2, 3] is effective. According to this theory, analogical reasoning finds an analogy or a similarity between some objects and predicts a new fact by transferring facts and knowledges in one object to another object. The reasoning system *ARTS* based on this theory reasons by analogy, not using an analogy given a priori but calculating an analogy when given an unknown fact which may be true. In order to reduce the computational complexity, the analogy the current *ARTS* system admits is confined. Given the descriptions of two worlds and an unknown fact, it determines whether the fact holds or not based on an analogy and answers it.

Although an automatic analogical reasoning described above is very useful, whether one identifies or not an object in one world with an object in another world heavily depends on time, place and his subjectivity. Thus the system which infers an analogy, given examples of pairs of similar and unsimilar objects by a user, and reasons based on the analogy is also useful. Since the system reasons given neither an analogy a priori nor no analogy, it is realistic. It infers an analogy by using inductive inference which can gain a general rule from examples.

In order to make the framework of the system mentioned above, in Section 2 we define *U*-paring by extending the notion of paring [2] and show that the relation between analogical reasoning and deduction also holds under the extension. In Section 3,

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we propose a new reasoning system which receives examples of corresponding terms and inductively infers the program representing  $U$ -paring and, using the above relation, correctly reasons in the limit, i.e., correctly reasons from some time on.

## 2. Analogical Reasoning using $U$ -paring

First we give the notion of logic programs [4]. Let  $a, b_i (0 \leq i \leq n)$  be an atom. A *rule* or a *definite clause* is a formula of the form

$$a \leftarrow b_1, \dots, b_n.$$

A *logic program* (*program*, for short) is a finite set of rules. Let  $S$  be a program.  $U(S)$ ,  $B(S)$  and  $M(S)$  denote the Herbrand universe of  $S$ , the Herbrand base of  $S$  and the least Herbrand model for  $S$ , respectively.

Second we summarize the points of the Haraguchi and Arikawa's theory. Let  $S_1$  and  $S_2$  be programs. A *paring*  $\phi$  is a finite subset of  $U(S_1) \times U(S_2)$ . The relation  $\phi^+$  of terms is defined to be the smallest one satisfying the following conditions:

- (1)  $\phi \subseteq \phi^+$ ,
- (2) For any function symbol  $f$  appearing in both  $S_1$  and  $S_2$ ,

$$\langle t_i, t'_i \rangle \in \phi^+ (1 \leq i \leq n) \Rightarrow \langle f(t_1, \dots, t_n), f(t'_1, \dots, t'_n) \rangle \in \phi^+.$$

Terms  $t$  and  $t'$  such that  $\langle t, t' \rangle \in \phi^+$  are conceived to be corresponding ones. Haraguchi and Arikawa [2] show that the Proposition 1 and Theorem 1 hold when we substitute  $\phi^+$  defined above for the  $U$ -paring  $\phi$  in Definitions from 2 to 5. Furthermore they [1] have implemented a reasoning system *ARTS* which detects a paring  $\phi$  such that  $\phi^+$  is a one to one relation, and simultaneously reasons a new fact by analogy based on  $\phi$ .

Here we consider a new reasoning system stated in Section 1. Given an enumeration of the corresponding elements in  $\phi \subseteq U(S_1) \times U(S_2)$ , it inductively infers the expression of  $\phi$ , i.e., the program for the predicate representing the correspondence. In order to make the framework of this system, we extend the definitions appearing in [2] by substituting  $U$ -paring for  $\phi^+$ .

DEFINITION 1. A  $U$ -paring is a subset of  $U(S_1) \times U(S_2)$ .

DEFINITION 2. Let  $t_j \in U(S_1)$  and  $t'_j \in U(S_2)$  for  $j (1 \leq j \leq n)$ . Let  $\phi$  be a  $U$ -paring,  $a \in M(S_1)$  and  $a' \in M(S_2)$ . Then  $a$  and  $a'$  is *identified by*  $\phi$ , denoted by  $a\phi a'$ , if for some predicate symbol  $p$

$$a = p(t_1, \dots, t_n), \quad a' = p(t'_1, \dots, t'_n) \quad \text{and} \quad \langle t_j, t'_j \rangle \in \phi (1 \leq j \leq n).$$

DEFINITION 3. Let  $\phi$  be a  $U$ -paring and  $I_i \subseteq B(S_i)$ . Let  $R = a \leftarrow b_1, \dots, b_n$  and  $R' = a' \leftarrow b'_1, \dots, b'_n$  be rules containing no variables. Then  $R'$  is a  $(\phi, I_1, I_2)$ -analogue of  $R$  if

$$b_j \in I_1, \quad b'_j \in I_2 (1 \leq j \leq n), \quad a\phi a', \quad b_j\phi b'_j (1 \leq j \leq n).$$

DEFINITION 4.  $M_i(*)$  is defined as follows:

$$M_i(*) = \bigcup_n M_i(n),$$

$$M_i(0) = M(S_i),$$

$$M_i(n+1) = \{a \in B(S_i) \mid R_i(n) \cup M_i(n) \cup S_i \vdash a\},$$

$$R_i(n) = \{R' = a' \leftarrow b'_1, \dots, b'_n \mid \text{there exists a ground instance } R \text{ of a rule of } S_j (j \neq i) \text{ such that } R' \text{ is a } \langle \phi, M_j(n), M_i(n) \rangle\text{-analogue of } R\}.$$

We call  $a \in M_i(*)$  a *fact reasoned by analogy* based on a  $U$ -paring  $\phi$ . The following proposition asserts that this definition is admissible.

PROPOSITION 1.  $M_i(*)$  is an Herbrand model for  $S_i$  ( $i=1, 2$ ).

PROOF. A proof similar to that of Proposition 2.2 [2] holds.  $\square$

We define an analogical union based on a paring extended above. Let  $\sim$  be a predicate symbol which never appears in  $S_1$  nor  $S_2$ . A program  $S$  such that  $M(S) = \{s \sim t \mid \langle s, t \rangle \in \phi\}$  is a *program* for  $U$ -paring  $\phi$ . Suppose  $a = p(t_1, \dots, t_n)$  be an atom. We denote  $p_i(t_1, \dots, t_n)$  by  $\langle a \rangle_i$ , where  $p_i$  is a new predicate symbol with a suffix  $i$ . We define  $copy(S_i)$  as follows:

$$copy(S_i) = \{\langle a \rangle_i \leftarrow \langle b_1 \rangle_i, \dots, \langle b_n \rangle_i \mid a \leftarrow b_1, \dots, b_n \in S_i\}.$$

Let  $W_1, \dots, W_n, V_1, \dots, V_n$  be variables never appearing in  $S_1$  nor  $S_2$ . The set  $tran(S_i)$  which consists of rules transforming the rules in  $S_j (j \neq i)$  to those in  $S_i$  based on  $\phi$  is defined as follows:

$$\begin{aligned} tran(S_1) = & \{p_1(W_1, \dots, W_n) \leftarrow \dots \\ & W_1 \sim t_1, \dots, W_n \sim t_n, V_1 \sim s_1, \dots, V_k \sim s_k, \\ & q_2(s_1, \dots, s_k), q_1(V_1, \dots, V_k), \dots \\ & \mid p(t_1, \dots, t_n) \leftarrow \dots, q(s_1, \dots, s_k), \dots \\ & \text{is a rule in } S_2 \text{ with a none-empty body}\}, \end{aligned}$$

$$\begin{aligned} tran(S_2) = & \{p_2(W_1, \dots, W_n) \leftarrow \dots \\ & t_1 \sim W_1, \dots, t_n \sim W_n, s_1 \sim V_1, \dots, s_k \sim V_k, \\ & q_1(s_1, \dots, s_k), q_2(V_1, \dots, V_k), \dots \\ & \mid p(t_1, \dots, t_n) \leftarrow \dots, q(s_1, \dots, s_k), \dots \\ & \text{is a rule in } S_1 \text{ with a none-empty body}\}. \end{aligned}$$

DEFINITION 5. Let  $S_1$  and  $S_2$  be programs,  $\phi$  be a  $U$ -paring and  $P$  be a program for  $\phi$ . Then the *analogical union* of  $S_1$  and  $S_2$  based on a program  $P$  for a  $U$ -paring  $\phi$ , denoted by  $S_1PS_2$ , is

$$copy(S_1) \cup copy(S_2) \cup tran(S_1) \cup tran(S_2) \cup P.$$

We have the following results analogous to the one of [2] under the extension of the notion of paring. Thus an analogical reasoning based on  $U$ -paring  $\phi$  is characterized as a deduction from the analogical union, if a program for  $\phi$  is known in a sense.

THEOREM 1. Let  $a(t_1, \dots, t_n)$  be an atom and  $P$  be a program for a  $U$ -paring  $\phi$ . Then the following two conditions are equivalent:

- (1)  $a(t_1, \dots, t_n) \in M_i(*)$ .
- (2)  $S_1PS_2 \vdash a_i(t_1, \dots, t_n)$ .

In order to prove this theorem we need a proposition.

PROPOSITION 2. [4] Let  $P$  be a program and  $I$  be an Herbrand interpretation of  $P$ .

Let mapping  $T(P): 2^{B(P)} \rightarrow 2^{B(P)}$  is defined as follows:

$T(P)(I) = \{A \in B(P) \mid A \leftarrow A_1, \dots, A_n \text{ is a ground instance of a rule in } P \text{ and } \{A_1, \dots, A_n\} \subseteq I\}$ . Then the following three conditions are equivalent:

- (1)  $I$  is a model for  $P$ .
- (2)  $T(P)(I) \subseteq I$ .
- (3)  $T(P)(\{C\})(I) \subseteq I$  for any rule  $C$  in  $P$ .

PROOF OF THEOREM 1. First we show that (2) implies (1). It suffices to prove that  $M(S_1PS_2) \subseteq M'_1(*) \cup M'_2(*) \cup M(P)$ , where  $M'_i(n) = \{a_i(t_1, \dots, t_n) \mid a(t_1, \dots, t_n) \in M_i(n)\}$  and  $n$  is a natural number or  $*$ . Since  $M(S_1PS_2)$  is the least Herbrand model for  $S_1PS_2$ , it suffices to prove that  $M'_1(*) \cup M'_2(*) \cup M(P)$  is a model for  $S_1PS_2$ . Let  $M'(*)$  denote  $M'_1(*) \cup M'_2(*) \cup M(P)$ . By Proposition 2, it suffices to prove that

$$T(P)(\{C\})(M'(*) \subseteq M'(*) \text{ for any rule } C \text{ in } S_1PS_2.$$

We assume, for simplicity, that each rule in  $P_i$  has at most one literal in the body. We have three cases.

(a)  $C$  in  $copy(S_i)$ .

By Proposition 1, we have  $T(P)(\{C\})(M'_1(*) \subseteq M'_1(*)$ . Since  $T(P)(\{C\})(M'_1(*) = T(P)(\{C\})(M'(*)$ ,  $T(P)(\{C\})(M'(*) \subseteq M'(*)$  for any rule  $C$  in  $copy(S_1)$ . Similarly, it follows that  $T(P)(\{C\})(M'(*) \subseteq M'(*)$  for any rule  $C$  in  $copy(S_2)$ .

(b)  $C$  in  $tran(S_i)$ .

Suppose that  $C \in tran(S_1)$  has a form:

$$C: p_1(W_1, \dots, W_n) \leftarrow$$

$$W_1 \sim t_1, \dots, W_n \sim t_n, V_1 \sim s_1, \dots, V_m \sim s_m, q_2(s_1, \dots, s_m), q_1(V_1, \dots, V_m).$$

We show that  $T(P)(\{C\})(M'(*) \subseteq M'(*)$ . From the definition of analogical union, the rule  $p(t_1, \dots, t_n) \leftarrow q(s_1, \dots, s_m)$  is in  $S_2$ .

Put  $A \in T(P)(\{C\})(M'(*)$ . Thus there exists a substitution  $\theta$  such that

$$A = p_1(W_1\theta, \dots, W_n\theta),$$

$$\{W_1\theta \sim t_1\theta, \dots, W_n\theta \sim t_n\theta, V_1\theta \sim s_1\theta, \dots, V_m\theta \sim s_m\theta, q_2(s_1\theta, \dots, s_m\theta), q_1(V_1\theta, \dots, V_m\theta)\} \subseteq M'(*).$$

Clearly,

$$\{W_1\theta \sim t_1\theta, \dots, W_n\theta \sim t_n\theta, V_1\theta \sim s_1\theta, \dots, V_m\theta \sim s_m\theta\} \subseteq M(P),$$

$$q_2(s_1\theta, \dots, s_m\theta) \in M'_2(*), \quad q_1(V_1\theta, \dots, V_m\theta) \in M'_1(*).$$

Hence, there exists a natural number  $N$  such that

$$q_2(s_1\theta, \dots, s_m\theta) \in M'_2(N) \text{ and } q_1(V_1\theta, \dots, V_m\theta) \in M'_1(N).$$

So  $R_1(N)$  has a rule  $p(W_1\theta, \dots, W_n\theta) \leftarrow q(V_1\theta, \dots, V_m\theta)$ . Thus  $p(W_1\theta, \dots, W_n\theta) \in M_1(N+1)$ . Hence  $A = p_1(W_1\theta, \dots, W_n\theta) \in M'_1(N+1) \subseteq M'_1(*) \subseteq M'(*)$ . Thus we have shown that  $T(P)(\{C\})(M'(*) \subseteq M'(*)$  for any rule  $C$  in  $tran(S_1)$ . Similarly, it follows that  $T(P)(\{C\})(M'(*) \subseteq M'(*)$  for any rule  $C$  in  $tran(S_2)$ .

(c)  $C$  in  $P$ .

Clearly,  $T(P)(\{C\})(M(P)) \subseteq M(P)$  for any rule  $C$  in  $M(P)$ . Since  $T(P)(\{C\})(M(P)) = T(P)(\{C\})(M'(*))$ , it follows that  $T(P)(\{C\})(M'(*)) \subseteq M'(*)$  for any rule  $C$  in  $P$ . Thus we have shown that (2) implies (1).

Next we show that (1) implies (2). It suffices to prove that

$$M_i(*) = \bigcup_{n \geq 0} M_i(n) \subseteq M(S_1PS_2)_i \quad \text{for } i=1, 2,$$

where

$$M(S_1PS_2)_i = \{a(t_1, \dots, t_n) \mid a_i(t_1, \dots, t_n) \in M(S_1PS_2)\}.$$

We prove that  $M_i(n) \subseteq M(S_1PS_2)_i$  for all  $n \geq 0$  and  $i=1, 2$ , by induction on  $n$ . Suppose first that  $n=0$ . We have  $M_i(0) = M(S_i) \subseteq M(S_1PS_2)_i$  for  $i=1, 2$ . Next suppose that  $M_i(n) \subseteq M(S_1PS_2)_i$  holds for  $i=1, 2$  and some  $n \geq 0$ . We prove that  $M_2(n+1) \subseteq M(S_1PS_2)_2$  holds. By the definition,  $M_2(n+1) = M(R_2(n) \cup M_2(n) \cup S_2)$ . It suffices to prove that  $M(S_1PS_2)_2$  is a model for  $R_2(n) \cup M_2(n) \cup S_2$ . So we show that  $T(P)(\{C\})(M(S_1PS_2)_2) \subseteq M(S_1PS_2)_2$  for any rule  $C$  in  $R_2(n) \cup M_2(n) \cup S_2$ . We consider three cases.

(a)  $C \in R_2(n)$ .

For simplicity, we assume that  $C$  has a form  $p(s') \leftarrow q(t')$ . By the definition of  $R_2(n)$ , there exist a rule  $C_1: p(s) \leftarrow q(t)$  in  $S_1$  and a substitution  $\theta$  such that  $q(t\theta) \in M_1(n)$ ,  $q(t') \in M_2(n)$ ,  $s\theta \sim s'$  and  $t\theta \sim t'$ . Since  $p(s')$  is a ground atom,  $T(P)(\{C\})(M(S_1PS_2)_2) \subseteq \{p(s')\}$ . From the definition,  $S_1PS_2$  has a rule  $C_2: p_2(W) \leftarrow s \sim W, t \sim V, q_1(t), q_2(V)$ , where the variables  $W$  and  $V$  never appear in  $s$  nor  $t$ . Let  $\sigma = \theta \cup \{W \leftarrow s', V \leftarrow t'\}$ . Thus we have

$$s\sigma \sim W\sigma = s\theta \sim s' \in M(S_1PS_2),$$

$$t\sigma \sim V\sigma = t\theta \sim t' \in M(S_1PS_2).$$

Furthermore, by the induction hypothesis, we have

$$q_1(t\sigma) = q_1(t\theta) \in M_1'(n) \subseteq M(S_1PS_2),$$

$$q_2(V\sigma) = q_2(t') \in M_2'(n) \subseteq M(S_1PS_2).$$

Since  $C_2 \in S_1PS_2$ , we have  $p_2(W\sigma) = p_2(s') \in M(S_1PS_2)$ . Thus we have  $T(P)(\{C\})(M(S_1PS_2)_2) \subseteq M(S_1PS_2)_2$ .

(b)  $C \in M_2(n)$ .

By the fact that  $C$  is a ground atom and the induction hypothesis,  $T(P)(\{C\})(M(S_1PS_2)_2) \subseteq \{C\} \subseteq M_2(n) \subseteq M(S_1PS_2)_2$ .

(c)  $C \in S_2$ .

Since  $M(S_2) \subseteq M(S_1PS_2)_2$ , the results holds.

This shows that  $M_2(n+1) \subseteq M(S_1PS_2)_2$ . Similarly, it follows that  $M_1(n+1) \subseteq M(S_1PS_2)_1$ . Thus we have shown that (1) implies (2).  $\square$

### 3. A Reasoning System using Inductive Inference of Analogical Union

Now consider the following programs:

$$S_1 = \{\text{like}(\text{tom}, \text{fruit}(\text{apple})),$$

$$\text{like}(X, \text{juice}(Y)) \leftarrow \text{like}(X, \text{fruit}(Y))\},$$

$$S_2 = \{\text{like}(\text{jerry}, \text{whole}(\text{orange}))\}.$$

The current ARTS system partially computes a paring

$$\phi = \{\langle \text{juice}(\text{apple}), \text{juice}(\text{orange}) \rangle, \langle \text{tom}, \text{jerry} \rangle, \langle \text{fruit}(\text{apple}), \text{whole}(\text{orange}) \rangle\}$$

and reasons backward a new fact

$$\text{like}(\text{jerry}, \text{juice}(\text{orange}))$$

based on the following basic schema:

$$\frac{\frac{\text{like}(X, \text{juice}(Y)) \leftarrow \text{like}(X, \text{fruit}(Y))}{\text{like}(\text{tom}, \text{juice}(\text{apple})) \leftarrow \text{like}(\text{tom}, \text{fruit}(\text{apple}))}}{\frac{\text{like}(\text{jerry}, \text{whole}(\text{orange})) \quad \text{like}(\text{jerry}, \text{juice}(\text{orange})) \leftarrow \text{like}(\text{jerry}, \text{whole}(\text{orange}))}{\text{like}(\text{jerry}, \text{juice}(\text{orange}))}}$$

where the upper real line, the broken line and the lower real line denote an instantiation, a rule transformation based on  $\phi$  and modus ponens, respectively.

Suppose that *apple*, *orange*, *fruit*(*X*) and *whole*(*X*) denote apple, orange, a fruit of *X* and a whole *X*, respectively. Some identify *apple* with *orange* since they are fruits, and some not since they are different fruits. In this case we cannot decide which way of thinking is true. One may analogically reason based on each way of thinking.

Now we propose an algorithm *MA*. Given the examples by a user which are paired facts about two objects with signs of whether they are analogous or not, it inductively infers a *U*-pairing intended by the user. Therefore it analogically reasons a new fact by deducing the fact from the analogical union constructed by the *U*-pairing.

First we give some necessary definitions and results of inductive inference, especially model inference for logic program [5, 6]. Let *L* be a first order language. Given examples of an unknown model *M*, a model inference algorithm *IA* infers a program *P* such that *M* is the least Herbrand model of *P*. An example given to *IA* is a pair  $\langle \alpha, V \rangle$ , called a *fact* of *M*, where  $\alpha$  is a ground atom of *L* and *V* is a truth value of  $\alpha$  with respect to *M* (i.e.,  $V = \text{true}$  if  $\alpha \in M$ ,  $V = \text{false}$  otherwise). An *enumeration* of a model *M* is an infinite sequence of facts in which every fact of *M* appears at least once. An output of *IA* is a program and called a *conjecture*. A model inference algorithm *IA* is said to *identify M in the limit* if *IA*, given any enumeration of *M*, outputs a same program *P* from some time on and *M* is the least Herbrand model of *P*.

Next we give the concept of *h*-easiness in order to characterize the power of a model inference algorithm. Let *h* be a recursive function, *T* be a program and  $\alpha$  be an atom. We write  $T \vdash_n \alpha$  if  $\alpha$  is deducible from *T* in *n* steps. Let  $a_1, a_2, \dots$  be a fixed effective enumeration of the all ground atoms of *L*. A program *T* is *h*-easy if  $\min \{j \mid T \vdash_j a_n\} \leq h(n)$  holds for all except a finite number of  $n > 0$  such that  $T \vdash a_n$ . An Herbrand model *M* is *h*-easy if there exists an *h*-easy program *T* such that  $M(T) = M$ .

Shapiro gives an enumerative model inference algorithm and characterizes its power [5, 6].

Let  $h$  be a recursive function and  $T_1, T_2, \dots$  be a fixed effective enumeration of all programs in  $L$ .

$S_{false} := \{\square\}, S_{true} := \{ \}, k=1.$

**repeat**

  read the next fact  $F_n = \langle a, V \rangle.$

$S_V := S_V \cup \{a\}.$

**while** there exist  $a \in S_{false}$  such that  $T_k \vdash a$   
     or there exist  $a_i \in S_{true}$  such that  $T_k \not\models_{h(i)} a_i$  **do**

$k := k + 1.$

  output  $T_k.$

**forever**

### An Enumerative Model Inference Algorithm

PROPOSITION 3. [5]

(1) *Let  $h$  be a recursive function,  $M$  be an  $h$ -easy Herbrand model of  $L$ . Then an enumerative model inference algorithm identifies  $M$  in the limit.*

(2) *Let  $I$  be an enumerative model inference algorithm. Then there exists a recursive function  $h$  uniform in  $I$  such that if  $I$  identifies an Herbrand model  $M$  then  $M$  is  $h$ -easy.*

Let  $S_1$  and  $S_2$  be programs, and  $\phi$  be a  $U$ -paring. We give an algorithm  $MA$ , proposed in Section 1, which receives examples of corresponding terms and inductively infers a program representing the correspondence, and reasons by analogy using a deducibility from the analogical union. Algorithm  $MA$  receives as an input an enumeration  $\langle s_1 \sim t_1, V_1 \rangle, \langle s_2 \sim t_2, V_2 \rangle, \dots$  of  $\{s \sim t \mid \langle s, t \rangle \in \phi\}$  and calculates a conjecture for the program of  $\phi$ , by using an enumerative model inference algorithm, and outputs conjectures  $P_1, P_2, \dots$ . When the algorithm is asked at point  $n$  whether a ground atom  $a \in B(S_i)$  is a fact reasoned by a analogy based on  $\phi$ , it will check the following deducibility holds or not:

$$copy(S_1) \cup copy(S_2) \cup tran(S_1) \cup tran(S_2) \cup P_n \vdash (a)_i$$

and if it confirms the above deducibility it say that  $a$  is a fact reasoned by analogy.

Taking Theorem 1 into consideration, we may define the correctness of algorithm  $MA$  as follows: Algorithm  $MA$  based on  $\phi$  *correctly reasons in the limit* if the model inference algorithm incorporated in  $MA$  identifies a model  $\{s \sim t \mid \langle s, t \rangle \in \phi\}$  in the limit.

Let  $h$  be a recursive function. A  $U$ -paring  $\phi$  is  $h$ -easy if there exists an  $h$ -easy Herbrand model for  $\phi$ .

In this framework, the correctness of  $MA$  and the characterization of it's power are reduced to those of  $\phi$  by a model inference algorithm.

THEOREM 2.

(1) *Let  $h$  be a recursive function,  $\phi$  be an  $h$ -easy  $U$ -paring. Then algorithm  $MA$  correctly reasons in the limit.*

(2) *Let  $\phi$  be a  $U$ -paring. There exists a recursive function  $h$  uniform in  $MA$  such that if  $MA$  correctly reasons in the limit based on  $\phi$  then  $\phi$  is  $h$ -easy.*

PROOF OF THEOREM 2. The results follow from Proposition 3 and the definition of the correctly reasoning in the limit.  $\square$

We think *MA* is realizable by adopting Shapiro's *MIS* as the model inference algorithm.

#### 4. Concluding Remarks

We have proposed a reasoning system *MA* using inductive inference of analogical union and characterized its power. *MA* is a simple algorithm constructed by serially connecting two algorithms of a model inference and an analogical reasoning. Moreover it is powerful since it needs to receive infinitely many examples from a theoretical point of view. Results in Section 3 say that even such powerful *MA* can analogically reason only between two worlds which have the similar structures whose correspondences can be represented by *h*-easy Herbrand model. In this sense the results show a limitation of the power of an analogical reasoning system by a computer.

We need to extend a current *ARTS* system and construct an algorithm which, given a ground atom, backward and interactively finds an analogy intended by a user and reasons based on the analogy. Such algorithm is applicable to a transference of knowledges between two worlds and conjectured to be as powerful as *MA*.

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*Received September 30, 1988*

*Communicated by S. Arikawa*