九州大学学術情報リポジトリ
Kyushu University Institutional Repository

Dynamic Change of Comparative Advantage in a Small Open Economy

Ikeshita, Kenichiro

https://doi.org/10.15017/3000305

出版情報：経済論究. 113, pp.1-14, 2002-09-10. 九州大学大学院経済学会
バージョン：
権利関係：

Dynamic Change of Comparative Advantage in a Small Open Economy

Kenichiro Ikeshita

1 Introduction

Recently, economic growth theory becomes the central issue of economics and the method of growth theory occupies the basis of modern macroeconomics. Especially, in 1990s, through development of endogenous growth theory, we recognized the importance of human capital, learning by doing and monopolistic competition. Moreover, many studies point out that these engines of endogenous growth can provide complicated dynamics such as multiplicity of steady state, indeterminacy and chaos. Gali (1997) shows that multiplicity of steady states and indeterminacy occur in the model in which there is monopolistic competition in the production of investment goods. Greiner and Semmler (1995) consider the possibility of complicated dynamics in endogenous growth model with learning by doing.

On the other hands, it is not enough to study how externality or monopolistic competition affects dynamic behavior of economy in a situation where international trade exists. However, the pattern of production through trade may have large effect on the externality or increasing return by monopolistic competition. For example, imagine that one country specializes the goods production that has no externality or monopolistic competition,[1] and another country specializes the goods production that has large externality or monopolistic competition; such as, manufacturing industries. It is natural that growth processes of these two economies are much different from each other. Then, it is important to analyze the dynamics of economy where there are trade and imperfect competition (or externality).

The purpose of this paper is to analyze the change of comparative advantage and the equilibrium dynamics in two-sector small open economy. We assume that final goods are produced by intermediate goods and labor. Intermediate goods market is in the circumstance of monopolistic competition. Our model is natural extension of Ciconne and Matsuyama (1996); however, we focus on the change of comparative advantage and pattern of production.

The remaining of this paper is organized as follows. In the section 2, we describe the model and show that the country perfectly specializes one of two kinds of goods. Next, we consider the

1) Imagine a primitive methods of production, such as agriculture.

change of comparative advantage and make conclusion that it only depends on factor intensity. In the section 3, we explore the equilibrium path. Dynamic change of comparative advantage provides complicated dynamics; such as multiplicity of steady state, that causes underdevelopment trap. In the section 4, we treat a case in which production function is Cobb-Douglas. Even in this case, characteristics of our model are preserved.

2 The Model

In this section, we set up the model that is considered in this paper. We consider an economy in a small country. There are two kinds of consumption goods, good 1 and good 2. Good 1 is industrial goods, for example, machinery. On the other hand, good 2 is primitive good. Imagine that agriculture as an example of production of primitive goods. Because of the assumption of small country, the prices of both goods are given exogenously. We set the price of good 2 equal to one, and the price of good 1 is q. This country trades these consumption goods with other countries. First, we start up to describe the behavior of consumer.

2.1 The Behavior of Consumer

In this economy, representative consumer supplies his or her labor force and earns wage and receives the interest from his or her assets. We assume that consumer supplies L units of labor inelastically. Consumer decides the sequence of consumption to maximize their lifetime utility subject to the intertemporal budget constraint. $C_i(t)$ $(i=1,\ 2)$ is the consumption of good i at time t. Then consumer maximizes

$$U(t)=\int_t^{\infty} e^{-\rho(\tau-t)}\log[C_1(\tau)]^{\beta}[C_2(\tau)]^{1-\beta}d\tau, \tag{2.1}$$

where ρ is subjective discount rate. $\log[C_1(t)]^{\beta}[C_2(t)]^{1-\beta}$ is the flow of utility at time t. The intertemporal budget constraint is given by

$$\int_t^{\infty} e^{-\int_t^{\tau} r(s)ds}E(\tau)d\tau \leq \int_t^{\infty} e^{-\int_t^{\tau} r(s)ds}w(\tau)E(\tau)d\tau + W(t). \tag{2.2}$$

$E(t)$ is the expenditure of consumer at time t, $w(t)$ is the wage rate and $W(t)$ is the value of asset holding. The asset consists of the shares of firms that gain profits. Consumer's problem is divided into two stages: the first is how consumer allocates his expenditure between good 1 or good 2. At time t, consumer maximizes $[C_1(t)]^{\beta}[C_2(t)]^{1-\beta}$ subject to $qC_1(t)+C_2(t)\leq E(t)$. From the solution of this problem, we can derive the indirect utility function, that is;

$$[C_1(t)]^{\beta}[C_2(t)]^{1-\beta}=q^{\beta}\beta^{\beta}(1-\beta)^{1-\beta}E(t). \tag{2.3}$$

The second stage is how consumer allocates the income to expenditure or saving. The solution of maximization problem is given by usual Euler condition.

$$\frac{\dot{E}(t)}{E(t)}=r(t)-\rho. \tag{2.4}$$

At the same time, consumer's budget constraint is binding, that is,

$$\int_t^\infty e^{-\int_t^\tau r(s)ds}(E(\tau)-w(\tau)L)d\tau=W(t). \tag{2.5}$$

2.2 Firm

The final consumption goods are produced by perfectly competitive firms. The production function of good i is given by

$$Y_i(t)=F_i(X_i(t),\ H_i(t)), \tag{2.6}$$

where $H_i(t)$ is labor input into production of good i and $X_i(t)$ is composite of intermediate goods used for production of good i. We call X intermediate-composite. $X_i(t)$ has a form of CES type.

$$X_i(t)=\left(\int_0^{n(t)}[x_i(j,\ t)]^{1-1/\sigma}dj\right)^{\sigma/(\sigma-1)},\ \sigma>1, \tag{2.7}$$

where $x_i(j,\ t)$ is the quantity of intermediate good j for production of good i at time t. Conveniently, We take the variety of intermediate goods to be real number. We here assume that the elasticity of substitution between two kinds of intermediate goods is greater than one. This setting causes increasing return to scale. Finally, we assume that the production of good 1 is intermediate-composite intensive relative to the production of good 2, On the other hand, the production of good 2 is labor intensive relative to the production of good 1.

Next, we mention to the production of intermediate goods. Each kind of intermediate goods is supplied by single firm monopolistically. The firm producing some kind of intermediate goods supplies its product to the production of good 1 and good 2. The number of firms that produce intermediate goods is $n(t)$. We assume that production of one unit of intermediate goods requires one units of labor. Using these assumption, we can derive the social quantity of labor to produce one unit of good i. The problem that the firms face is to minimize

$$wH_i+\int_0^n p(j)x_i(j)dj \tag{2.8}$$

subject to

$$F_i(X_i,\ H_i)=1,\ X_i=\left(\int_0^n[x_i(j)]^{1-1/\sigma}dj\right)^{\sigma/(\sigma-1)}.$$

$p(j)$ is the price of intermediate good j.[2] We set the solution to this problem $(\{x_i^*(j)|j\leq n\}$, $H_i^*)$ and define $X_i^*=\left(\int_0^n[x_i^*(j)]^{1-1/\sigma}dj\right)^{\sigma/(\sigma-1)}$. This problem can be decomposed into two stages. The first one is to minimize the expenditure to intermediate goods $\int_0^n p(j)x_i(j)dj$ to achieve the

2) we omit subscript t because this problem is simply static.

quantity of intermediate-composite $X_i^*=\left(\int_0^n[x_i^*(j)]^{1-1/\sigma}dj\right)^{\sigma/(\sigma-1)}$. The solution of this problem is given by

$$x_i^*(j)=\frac{X_i^*p(j)^{-\sigma}}{\left(\int_0^n[p(j)]^{1-\sigma}dj\right)^{\sigma/(\sigma-1)}}, \tag{2.9}$$

which is the well-known demand function of intermediate good j. We introduce the new price index $P=\left(\int_0^n[p(j)]^{1-\sigma}dj\right)^{1-/(\sigma-1)}$ and we can get the following relation,

$$\int_0^n p(j)x_i^*(j)dj=PX_i^*.$$

This means that the solution $(\{x_i^*(j)|j\leq n,\ H_i^*)$ minimizes wH_i+PX_i and $(X_i^*,\ H_i^*)$ minimizes wH_i+PX_i subject to $F_i(X_i,\ H_i)=1$, which requires usual minimization condition.

$$\frac{\dfrac{\partial F_i(X_i^*,\ H_i^*)}{\partial X_i}}{P}=\frac{\dfrac{\partial F_i(X_i^*,\ H_i^*)}{\partial H_i}}{w}. \tag{2.10}$$

Moreover, because of the linear homogeneity of production function, Euler's Theorem is required,

$$\frac{\partial F_i(X_i^*,\ H_i^*)}{\partial X_i}X_i^*+\frac{\partial F_i(X_i^*,\ H_i^*)}{\partial H_i}H_i^*=1. \tag{2.11}$$

The producers of intermediate goods set the monopoly prices. Because one unit of intermediate good needs one unit of labor, all of the firm set the same price p given by

$$p(j)=p=\left(1-\frac{1}{\sigma}\right)^{-1}w. \tag{2.12}$$

Using the price index $P=\left(\int_0^n[p(j)]^{1-\sigma}dj\right)^{1/(\sigma-1)}$ and the equation of monopoly pricing, we derive the relative price between intermediate-composite and labor as a function of n.

$$\frac{P}{w}=\left(1-\frac{1}{\sigma}\right)^{-1}n^{\frac{1}{1-\sigma}}. \tag{2.13}$$

Because $\sigma>1$, relative price $\frac{P}{w}$ is decreasing with n. Let a_i be the factor share of intermediate composite. The homogeneity of production function implies that a_i is expressed by relative price $\frac{P}{w}$ only, not depends on the level of production. Moreover, we can express the factor share as a function of the number of intermediate goods,

$$a_i=a_i(P/w)=A_i(n). \tag{2.14}$$

The change of $A_i(n)$ with n is determined by the elasticity of substitution between labor and intermediate-composite. If the elasticity of substitution is larger than one, $A_i(n)$ is increasing function of n. On the other hand, if the elasticity of substitution is smaller than one, $A_i(n)$ is decreasing function of n. Finally, if the production function is Cobb-Douglas type, $A_i(n)$ is independent of n because the elasticity of substitution is one.

Let M_i^* denote the quantity of labor used to produce X_i^*. Because of demand function of intermediate goods, the same price $p(j)=p$ implies that $x_i(j)^*=x_i^*$. This means that $X_i^*=n^{\frac{1}{1-\sigma}}M_i^*$ because $M_i^*=nx_i^*$. This equation expresses the effect through increasing return. Moreover, using $PX_i^*=npx_i^*$ we can derive the following relation,

$$PX_i^*=\left(1-\frac{1}{\sigma}\right)^{-1}wM_i^*. \tag{2.15}$$

Dividing this equation by $PX_i^*+wH_i^*$, using the definition of $A_i(n)$, then,

$$A_i(n)=\frac{1}{PX_i^*+wH_i^*}\frac{w}{\left(1-\frac{1}{\sigma}\right)}M_i^*. \tag{2.16}$$

Similaly,

$$1-A_i(n)=\frac{1}{PX_i^*+wH_i^*}wH_i^*. \tag{2.17}$$

Then, we derive the social quantity of labor to produce one unit of good i as

$$M_i^*+H_i^*=\frac{PX_i^*+wH_i^*}{w}\left(1-\frac{A_i(n)}{\sigma}\right). \tag{2.18}$$

Using the definition of X_i^*, M_i^* and H_i^*, $1=F_i(n^{\frac{1}{1-\sigma}}M_i^*,\ H_i^*)$ and honogeneity of $F_i(X_i,\ H_i)$,

$$\frac{PX_i^*+wH_i^*}{w}=\frac{1}{F_i\left(n^{\frac{1}{\sigma-1}}\left(1-\frac{1}{\sigma}\right)A_i(n),\ 1-A_i(n)\right)}. \tag{2.19}$$

This means that $M_i^*+H_i^*$ depends on n only, that is

$$M_i^*+H_i^*=\frac{1}{F_i\left(n^{\frac{1}{\sigma-1}}\left(1-\frac{1}{\sigma}\right)A_i(n),\ 1-A_i(n)\right)}\left(1-\frac{A_i(n)}{\sigma}\right). \tag{2.20}$$

Here we define $\Gamma_i(n)$ as

$$\Gamma_i(n)=F_i\left(n^{\frac{1}{\sigma-1}}\left(1-\frac{1}{\sigma}\right)A_i(n),\ 1-A_i(n)\right). \tag{2.21}$$

Then, the firm that produces Y_i units of good i needs $\left(1-\frac{A_i(n)}{\sigma}\right)\frac{Y_i}{\Gamma_i(n)}$ units of labor. Next, we consider the cost to produce one unit of good i. The unit cost is easily derived as

$$PX_i^*+wH_i^*=\frac{w}{\Gamma_i(n)}. \tag{2.22}$$

When good i is produced in this country, zero-profit condition requires that the unit cost of goods is equal to it price. Then, we can determine the pattern of production by comparing costs of two goods. If q is larger than $\Gamma_2(n)/\Gamma_1(n)$, this country will specialize the production of good 1. On the other hand, if q is smaller than $\Gamma_2(n)/\Gamma_1(n)$, the country will specialize the production of good 2. We summarize this implication as proposition.

proposition 1 *The patterns of production are determined as follows.*

(1) *The economy specializes the production of good 1 when* $q>\Gamma_2(n)/\Gamma_1(n)$.

(2) *The economy produces both kinds of goods when* $q=\Gamma_2(n)/\Gamma_1(n)$.

(3) *The economy specializes the production of good 2 when* $q<\Gamma_2(n)/\Gamma_1(n)$.

2.3 R & D and Labor Constraint

Next, we describe the technology of R & D sector. We assume that one unit of blueprint to produce new goods needs a_n units of labor. Thus, when $\dot{n}(t)$ units of new kinds of goods are provided from R & D sector, $a_n\dot{n}(t)$ units of labor are employed. By considering the production of consumption goods, full-employment condition is given by

$$a_n\dot{n}(t)+\left(1-\frac{A_1(n(t))}{\sigma}\right)\frac{Y_1(t)}{\Gamma_1(n(t))}+\left(1-\frac{A_2(n(t))}{\sigma}\right)\frac{Y_2(t)}{\Gamma_2(n(t))}=L. \tag{2.23}$$

We can consider this constraint as the production possibility frontier. Once the increase of variety $n(t)$ and terms of trade are determined, we can easily know the level of production. Moreover, we assume that there is no international transfer of assets. In other words, R & D activity is financed through domestic saving only. Then, the revenue from producing consumption goods is equal to the expenditure at every period t.

Here we describe the remaining of the model. Especially we treat the case where specialization occurs.[3] When the economy specializes the production of good i, the constraint of labor market (2.23) is given by

$$a_n\dot{n}(t)+\left(1-\frac{A_i(n(t))}{\sigma}\right)\frac{Y_i(t)}{\Gamma_i(n(t))}=L. \tag{2.24}$$

Thus, let $R(t)$ denote the revenue from production of consumption goods and $R(t)$ is given by

$$R(t)=q_iY_i=q_1\Gamma_i(n(t))(L-a_n\dot{n}(t)),\quad q_i=q \text{ or } 1. \tag{2.25}$$

On the other hand, the profit of the firm that produces some kind of intermediate goods $\pi(t)$ is

$$\pi(t)=\frac{A_i(n(t))q_iY_i(t)}{\sigma n(t)}. \tag{2.26}$$

If firms succeed to R & D activities, they enter the intermediate goods market and receive the profit $\pi(t)$ monopolistically. Let $v(t)$ denote the market value of a firm that produces one kind of intermediate good. Because we assume that the intermediate market is in the circumstance of free entry, $v(t)$ never exceeds R & D cost $w(t)a_n$. Moreover, when $\dot{n}(t)$ is positive and finite, $w(t)a_n=v(t)$. Thus, in equilibrium, we have,

$$w(t)a_n\geq v(t),\quad (w(t)a_n-v(t))\dot{n}(t)=0. \tag{2.27}$$

By differentiating the market value of the firm $v(t)$, we obtain the following no-arbitrage condition

$$r(t)=\frac{\pi(t)}{v(t)}+\frac{\dot{v}(t)}{v(t)}. \tag{2.28}$$

This implies that the return to hold stocks is equal to the interest rate. Finally, multiplying (2.

3) We describe the case where both goods are produced with the following procedure.

24) by $w(t)$ and using (2.26), (2.28) and $E(t)=R(t)$, we can derive

$$w(t)L+n(t)\pi(t)=E(t)+v(t)\dot{n}(t), \tag{2.29}$$

which implies that income is divided between expenditure and saving. By integrating (2.29) and using (2.28),

$$\lim_{T\to\infty} n(T)v(T)e^{-\int_t^T r(s)ds}=\int_t^{\infty} e^{-\int_t^{\tau} r(s)ds}(w(\tau)L-E(\tau))d\tau+n(t)v(t). \tag{2.30}$$

From the definition of $W(t)$, $W(t)=n(t)v(t)$ and by using (2.5),

$$\lim_{T\to\infty} n(T)v(T)e^{-\int_t^T r(s)ds}=0. \tag{2.31}$$

2.4 The Change of Comparative Advantage

In this subsection, we consider how comparative productivity $\Gamma_2(n)/\Gamma_1(n)$ changes when n increases. Before we start this analysis, we must mention that the difference of factor-intensity implies that $A_1(n)>A_2(n)$ for any n.[4] Let us return to our interest. We define a function $\Psi(n)$ as follows.

$$\Psi(n)=\frac{\Gamma_2(n)}{\Gamma_1(n)}. \tag{2.32}$$

If $\Psi(n)$ is decreasing with n, the economy tends to possess the comparative advantage in good 1 and the specialization of production may change to good 1. Then the analysis of the behavior of comparative advantage is important for the dynamics of economy. Then, we consider the elasticity of $\Psi(n)$ which is

$$\frac{d\log\Psi(n)}{d\log n}=\frac{d\log\Gamma_2(n)}{d\log n}-\frac{d\log\Gamma_1(n)}{d\log n}. \tag{2.33}$$

The first term of LHS is

$$\frac{d\log\Gamma_2(n)}{d\log n}=\frac{1}{\Gamma(n)}\left\{\frac{\partial F_2(\cdot)}{\partial X_2}\left[A_2(n)\left(1-\frac{1}{\sigma}\right)n^{1/(\sigma-1)}\left[A'_2(n)\frac{n}{A_2(n)}+\frac{1}{\sigma-1}\right]\right]\right.$$
$$\left.+\frac{\partial F_2(\cdot)}{\partial H_2}(1-A_2(n))\frac{1}{1-A_2(n)}(-A'_2(n)n)\right\}. \tag{2.34}$$

From (2.10) and (2.11), and by using

$$X_2^*=\left(1-\frac{1}{\sigma}\right)\frac{1}{\Gamma_2(n)}n^{1/(\sigma-1)}A_2(n) \tag{2.35}$$

and

$$H_2^*=\frac{1}{\Gamma_2(n)}(1-A_2(n)), \tag{2.36}$$

we derive the following relationship,

$$\frac{\partial F_2(\cdot)}{\partial X_2}=\left(1-\frac{1}{\sigma}\right)^{-1}\Gamma_2(n)n^{1/(\sigma-1)} \text{ and } \frac{\partial F_2(\cdot)}{\partial H_2}=\Gamma_2(n). \tag{2.37}$$

By substituting (2.37) into (2.34), we can derive the simple form,

---

4) From the definition of $A_i(n)$, $X_1^*/H_1^*>X_2^*/H_2^*$ implies that $A_1(n)>A_2(n)$.

$$\frac{d\log \Gamma_2(n)}{d\log n}=\frac{A_2(n)}{\sigma-1}. \tag{2.38}$$

Similarly, we can derive $d\log\ \Gamma_1(n)/d\log n=A_1(n)/(\sigma-1)$. Thus, the elasticity of $\Psi(n)$ is given by

$$\frac{d\log \Psi(n)}{d\log n}=\frac{1}{\sigma-1}(A_2(n)-A_1(n)). \tag{2.39}$$

This condition is very clear; the comparative advantage depends on factor shares only. We have already assumed that $A_1(n)$ is larger than $A_2(n)$, we can come to the conclusion that $\Psi(n)$ is decreasing function of n.

proposition 2 *If the production of good 1 is more intermediate-composite intensive than good 2, $\Psi(n)$ is decreasing function of n and $\Psi(n)=q$ has one solution at most.*

If both production functions of consumption goods are Cobb-Douglas type, The factor shares are constant, that is, $A_1(n)=\alpha_1$ and $A_2(n)=\alpha_2$. Then, $\Psi(n)$ is specified to

$$\Psi(n)=\phi n^{\frac{1}{\sigma-1}(\alpha_2-\alpha_1)}, \tag{2.40}$$

where ϕ is positive constant. Because $\lim_{n\to\infty}\Psi(n)=0$, $\Psi(n)=q$ has the unique solution.

Corollary 1 *If both production functions are Cobb-Douglas type, $\Psi(n)=q$ has the unique solution $n=\bar{n}$.*

Proposition 2 has a clear implication. The increase of n causes two effects. The first one is substitution between labor and intermediate-composite, that is, the demand for intermediate-composite increases and the demand for labor decreases because relative factor price P/w is the decreasing function of n. This substitution occurs in both sectors. Moreover, Higher demand for intermediate-composite and lower demand of labor offset each other. The second one is productivity effect, that is, the increase of n raises the productivity of intermediate-composite. The productivity in production of good 1 goes up higher than good 2 because the production of good 1 is more intermediate-composite intensive. Then, comparative advantage moves from good 2 to good 1 when n increases.

In the following analysis, we focus on the case in which $\Psi(n)=q$ has a solution. That may cause a change of specialization during the economy moves.

3 The Dynamic Behavior of the Economy

Now we start to reduce the model to the model into two variables. We define new variable $V(t)=v(t)/E(t)$. By using this definition and (2.4), the dynamic change of $V(t)$ is derived as

$$\dot{V}(t)=\rho V(t)-\frac{A(n(t))}{\sigma n(t)},\quad A(n)\begin{cases}=A_2(n) & (n<\bar{n}),\\ \in[A_2(\bar{n}),\ A_1(\bar{n})] & (n=\bar{n}),\\ =A_1(n) & (n>\bar{n}).\end{cases}\tag{3.1}$$

When n is sufficiently small, the economy specializes good 2 and $A(n)=A_2(n)$. However, if the economy grows and derives the high productivity of good 1, the pattern of trade will change and the economy will specializes the production of good 1, then $A(n)=A_1(n)$ Moreover, from (2.24) and (2.27), we also derive the dynamic change of $n(t)$, which is given by

$$\dot{n}(t)=\max\left\{\frac{L}{a_n}-\left(1-\frac{A(n(t))}{\sigma}\right)\frac{1}{V(t)},\ 0\right\},\quad A(n)\begin{cases}=A_2(n) & (n<\bar{n}),\\ \in[A_2(\bar{n}),\ A_1(\bar{n})] & (n=\bar{n}),\\ =A_1(n) & (n>\bar{n}).\end{cases}\tag{3.2}$$

Finally, from (2.4) and (2.31),

$$\lim_{t\to\infty}V(t)n(t)e^{-\rho t}=0.\tag{3.3}$$

For initial number of firms the economy inherits, n_0, a market equilibrium of this economy is a path of $\{V(t),\ n(t)\}$ that satisfies (3.1), (3.2) and (3.3). This reduction of model is same as Ciccone and Matsuyama (1996). Therefore, the dynamics of economy is similar to their analysis. Here we attempts to analyze the property of equilibrium dynamics. $\dot{V}=0$ implies that

$$V=\frac{A(n)}{\rho\sigma n},\quad A(n)\begin{cases}=A_2(n) & (n<\bar{n}),\\ \in[A_2(\bar{n}),\ A_1(\bar{n})] & (n=\bar{n}),\\ =A_1(n) & (n>\bar{n}).\end{cases}\tag{3.4}$$

This is a relation between V and n. We call this relation VV-curve. The shape of VV-curve depends on behavior of $A(n)/n$. On the other hands, $\dot{n}=0$ implies that

$$V=\frac{a_n}{L}\left(1-\frac{A(n)}{\sigma}\right),\quad A(n)\begin{cases}=A_2(n) & (n<\bar{n}),\\ \in[A_2(\bar{n}),\ A_1(\bar{n})] & (n=\bar{n}),\\ =A_1(n) & (n>\bar{n}).\end{cases}\tag{3.5}$$

We call this relation NN-curve. The slope of this curve depends on how $A(n)$ changes with n. Therefore, the elasticities of substitution of final goods are crucial for its change. If the elasticity of substitution of final good i is greater than one, NN-curve is downward-sloping in the area where good i is produced. Because we would like to know the shape of VV-curve, we start to analyze the elasticities of subsutitution of final goods. Let ε_i denote the elasticity of substitution of production function F_i. From this definition,

$$\varepsilon_i(P/w)=-\frac{d\log(X_i/H_i)}{d\log(P/w)}.\tag{3.6}$$

By integrating (3.6),

$$\frac{X_i}{H_i}=\beta\exp\left[\int_{(1-\frac{1}{\sigma})^{1-\sigma}}^{n}\frac{\bar{\varepsilon}_i(s^{1/(1-\sigma)})}{(1-\sigma)s}ds\right],\tag{3.7}$$

where $\bar{\varepsilon}_i(s^{1/(1-\sigma)})=\varepsilon_i((1-1/\sigma)^{-1}s^{1/(1-\sigma)})$. β is a positive constant. From $1/A_i(n)=1+wH/PX$, we derive

$$\frac{n}{A_i(n)} = n + \beta' \exp\left[\int_{(1-\frac{1}{\sigma})^{1-\sigma}}^{n} \frac{\bar{\varepsilon}_i(s^{1/(1-\sigma)}) - \sigma}{(1-\sigma)s} ds\right], \tag{3.8}$$

where $\beta' = (1-1/\sigma)^{1-\sigma}\beta$. This equation implies that, if $\varepsilon_i(P/w) < \sigma$, $A_i(n)/n$ is the decreasing function of n. Then the VV-curve is downward-sloping curve. If $\varepsilon_i(P/w) > \sigma$, VV may have a part where its slope is positive.

proposition 3 *If $\varepsilon_i(P/w) \leq \sigma$ for all P/w, $V = A_i(n)/(\rho\sigma n)$ is downward-sloping and intersects with $V = (a_n/L)(1 - A_i(n)/\sigma)$ at most once.*

This proposition suggests that elasticity of substitution in final goods production ε_i must be limited by σ in the case where $V = A_i(n)/(\rho\sigma n)$ is downward-sloping. When we interpret V as the market value of firm, V is a sum of discounted value of profit gained by a firm in intermediate sector.[5] When n increases, the profit per firm in intermediate sector decreases. This effect become more strong with σ. Moreover, the increase of n also makes production of final goods more intermediate-composite intensive. Then, if the substitution is not difficult in final goods sector, the demand for each intermediate good and profit per firm will increase. $\varepsilon_i \leq \sigma$ means that this substitution effect is not so large, therefore, the profit per firm will decreases with n and the VV-curve is downward-sloping curve. In Grossman and Helpman (1991), $A(n) = 1$ and we could ignore this substitution effect.

In Figure 1, we depict down-sloping VV-curve and the vector field. the VV-curve in Figure 1 is characterized by $\varepsilon_1(P/w) < \sigma$ and $\varepsilon_2(P/w) < \sigma$. Then, VV-curve is down-ward sloping curve except for $n = \bar{n}$. Similarly, we depict the NN-curve in Figure 2. The NN-curve in Figure 2 is also downward-sloping, that is ε_1 and ε_2 are greater than 1.

The dynamics of the economy is much complicated. In Figure 3, we show one example of the equilibrium path $\{V(t), n(t)\}$. In the case of Figure 3, there exist two steady state in this economy, one is upper steady state n_1^*, the other is lower steady state n_2^*. In this case, equilibrium path is unique for initial condition n_0. If $\bar{n} < n_0 < n_1^*$, the economy converges to the upper steady state. If $n_2^* \leq n_0 < \bar{n}$ or $n_1^* \leq n_0$, no entry takes place and the economy remains on VV-curve. If $n_0 < n_2^*$, the economy will converges to the lower steady state. Obviously, this model shows some kind of threshold condition. This implies that if the economy starts with sufficiently small variety and specializes the production of good 1, it converges to lower steady state and cannot change the pattern of production. Moreover it never reaches the higher steady state. Initial condition is crucial for dynamic behavior of the economy. We can say that this

5) Exactly, the profit is expressed by (2.26) and includes expenditure E because $R = E$. We also interpret V is a sum of discounted value of profit measured by utility. See Ciccone and Matsuyama (1996).

case generates underdevelopment trap.[6]

Moreover, threshold level of variety $\bar{n}$ is determined by relative price q. If the economy inherits lower variety initially and the price of good 2 increases, the terms of trade for this economy improves and flow of revenue will be increase. However, the higher price of good 2 implies q decreases relatively. Because $\Psi(n)$ is decreasing function of n, the threshold level of n will goes up. This example shows that the economy with lower variety may be more difficult to take off when the terms of trade improves.

In the case of figure 4, it is possible to exist two equilibrium path. We assume that the initial condition n_0 is slightly smaller than $\bar{n}$. In such a situation, there may be two equilibrium paths. In one of them, no entry occurs and the economy remains on VV-curve because people expect the economy in the future pessimistically. In the other equilibrium, people choose high value of V and entrepreneurs start R & D activities. Then, the economy can take off and change the pattern of production from good 2 to good 1. Finally it will converge to the upper steady state. This phenomenon is one kind of indeterminacy.

4 An Example : Cobb-Douglas type

In this section, we focus on the case in which production functions of final goods are Cobb-Douglas type. Needless to say, Cobb-Douglas production function is the most popular for economists because it is easy to treat when we consider economic models. Although many studies of underdevelopment trap assume externality or peculiar production function, we can show the characteristics of model and possibility of underdevelopment trap by using Cobb-Douglas production function.

In the case of Cobb-Douglas type, factor share is constant. Thus, we set $A_1(n)=\alpha_1$ and $A_2(n)=\alpha_2$. Moreover, from corollary 1, there exists $\bar{n}$. then, the factor share becomes

$$A(n)\begin{cases} =\alpha_2 & (n<\bar{n}), \\ \in[\alpha_2,\ \alpha_1] & (n=\bar{n}), \\ =\alpha_1 & (n>\bar{n}). \end{cases} \tag{4.1}$$

Because the elasticity of substitution is one, VV-curve is downward-sloping except for $n=\bar{n}$. On the other hands, NN-curve becomes horizontal line except for $n=\bar{n}$. Then the phase diagram is given by figure 6. In this case, the equilibrium path is unique for any initial condition. However, like Figure 4, the possibility of underdevelopment trap remains. Figure 7 corresponds to Figure 4. If n_0 is slightly smaller than $\bar{n}$, two equilibrium paths may exist and realized path in this economy depends on people's expectation.

6) This case occurs if the VV-curve and the NN-curve intersect in area where $n<\bar{n}$.

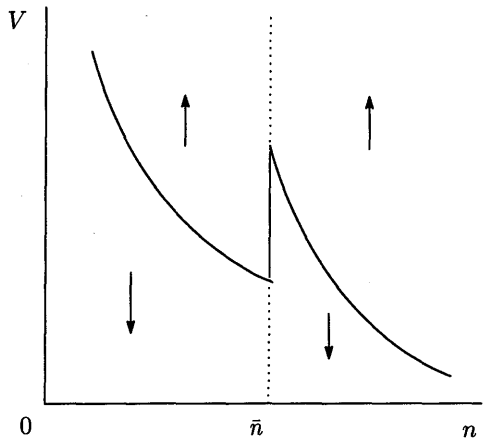

Figure 1　VV-curve and change of V
($\varepsilon_1 < \sigma$ and $\varepsilon_2 < \sigma$)

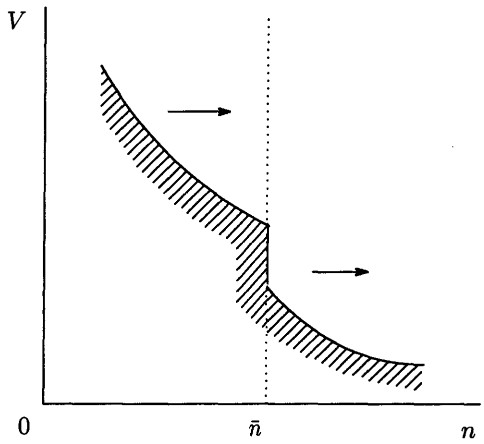

Figure 2　NN-curve and change of n
($1 < \varepsilon_1$ and $1 < \varepsilon_2$)

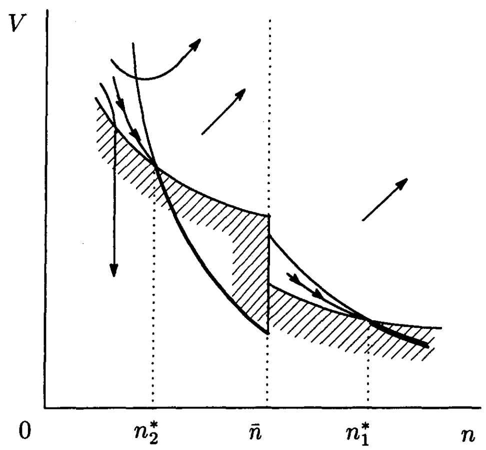

Figure 3　dynamic behavior of the economy
($1 < \varepsilon_1 < \sigma$ and $1 < \varepsilon_2 < \sigma$)

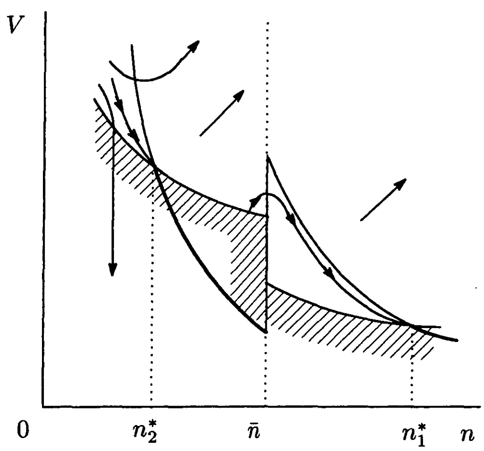

Figure 4　equilibrium path is not unique
($1 < \varepsilon_1 < \sigma$ and $1 < \varepsilon_2 < \sigma$)

5　Concluding Remarks

In this paper, we constructed the two-sector economic growth model and analyzed the change of comparative advantage as well as the dynamic change of the economy. One of the characteristics of the production is perfect specialization, which leads industry changing dramatically. Furthermore, we explored how comparative advantage changes with a number of intermediate goods. The conclusion is entirely simple; the change of comparative advantage depends only on relative magnitude of factor share that is related to factor intensities.

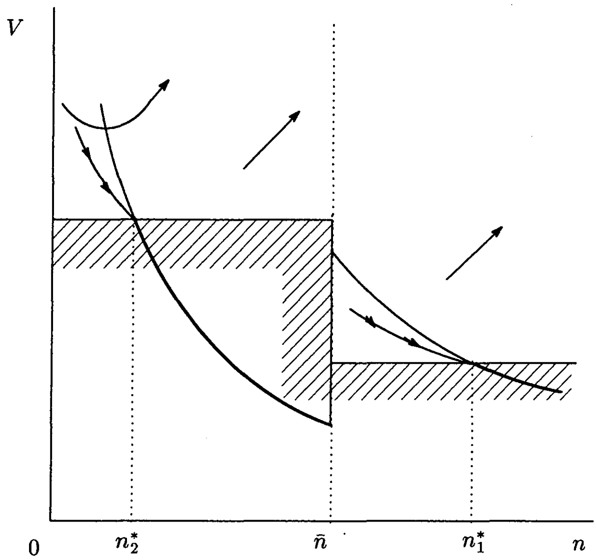

Figure 5 Cobb-Dougras case (1)
Equilibrium path is unique
($\varepsilon_1=1$ and $\varepsilon_2=1$)

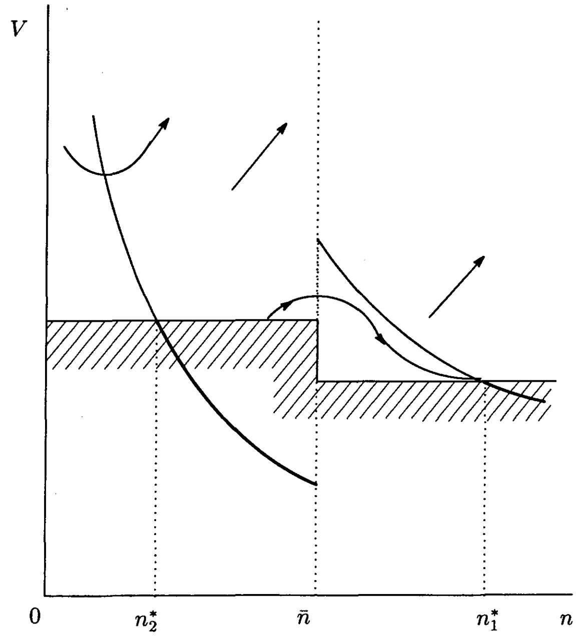

Figure 6 Cobb-Dougras case (2)
In this figure, two pathes from the same initial value exist
($\varepsilon_1=1$ and $\varepsilon_2=1$)

Model is reduced to the system of variety n and new variable V. Because of perfect specialization and difference of factor share, the phase diagram of this model includes two kinked curve, which are VV-curve and NN-curve. These peculiar shapes of curves provide complicated equilibrium path.

Mainly we focused on the cases in which multiple steady state occurs. This means that the growth path of economy critically depends on its initial condition. If economy starts with low number of intermediate goods, economy will converge to lower steady state and never reach higher steady state. On the other hands, we showed the case in which equilibrium path is not unique. One of these paths is to remain on the VV-curve and increasing of variety cannot occur. The other equilibrium path is to increase the variety and reach higher steady state. Which path the economy chooses is dependent on people's expectation. This phenomenon can be regarded as indeterminacy.

We have some extensions of this model for future research. The resource of underdevelopment trap is perfect specialization. However perfect specialization is considerably unrealistic when we think about real international trade. We speculate that multiple production factor case will recover this shortage because our model will become Heckscher-Ohlin framework when the model is reduced. Moreover, the growth rate in steady state is zero in our model. Endogenizing growth rate by introducing knowledge level into the economy may affect on the start-up cost. That will be an interesting extension.

References

(1) Aghion, P. and P. Howitt (1998), *Endogenous Growth Theory*, MIT Press.

(2) Barro, R. J. and X. Sala-i-Martin (1995), *Economic Growth*, McGraw-Hill.

(3) Ciccone, A. and K. Matsuyama (1996), "Start-up Costs and Pecuniary Externalities as Barriers to Economic Development," *Journal of Development Economics*, vol.49, 33-59.

(4) Ciccone, A. and K. Matsuyama (1999), "Efficiency and Equilibrium with Dynamic Increasing Aggregate Returns due to Demand Complementarities," *Econometrica*, vol.67, No.3, 499-525.

(5) Gali, J (1996), "Multiple Equilibria in a Growth Model with Monopolistic Competition," *Economic Theory*, vol.8, 251-266.

(6) Greiner, A. and W. Semmler (1996), "Multiple Steady States, Indeterminacy, and Cycles in a Basic Model of Endogenous Growth," *Journal of Economics*, vol.63, No.1, 79-99.

(7) Grossman, G. M. and E. Helpman (1991), *Innovation and Growth in the Global Economy*, MIT Press.

(8) Romer, P. M (1986), "Increasing Returns and Long-Run Growth," *Journal of Political Economy*, vol.94, 1002-1037.