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Abstract

The Lorenz curve is a most powerful tool in the analysis of the size distribution of income and wealth. In the past, many authors have been proposed different functional forms for estimating the Lorenz curve. The problem of indirect approach for estimating the Lorenz curve is that it is difficult to find any one hypothetical statistical distribution to serve as a good approximation over the entire range of an actual batch of income data. That is why, the principle objective of this paper is to present direct parametric approximations of a new functional form for estimating the Lorenz curve. We can compare these alternative forms by goodness of fit, F-tests of nested forms, and measurement of the Gini coefficients, Kakwani and Chakravarty inequality indices[1).

1 Introduction

Theoretical consideration of the distribution of income has changed dramatically since Pareto's (1897) seminal work almost one hundread years ago. Concern with the functional distribution of income diminished are researchers became interested in questions about the personal size distribution of income. The major areas of interested have been the specifation of an appropriate hypothetical statistical distribution to approximate the empirical income distribution and choice of one summary measure of inequality that best describes the level of inequality in a given statistical distribution. Research in this area began as an attempt to explain how the observed personal income distribution was generated, but over time it has become primarily a statistical exercise in searching for a good fitting statistical distribution. Pareto (1897), Aitchison and Brown (1957), Champernowne (1953), Fisk (1961), Salem and Mount (1974), Singh and Maddla (1976), Basmen, Molina and Slottje (1984) and McDonald (1984) -among others- have all suggested alternative functional forms to approximate the observed income distribution. All of these forms have problems. Some forms fit well at the tails but not over the lower portion of the empirical distribution, while other forms have the opposite problem. Some forms are not

1) I like to express my thanks to my honorable teacher, Chikayoshi Saeki, Professor, Faculty of Economics, Department of Economic Engineering, Kyushu University, for his sincere co-operations.

flexible enough to allow for multidimentional analysis [see slottje (1987)], while other forms require a large number of parameters to be able to estimate them. The search continues but no definite functional form has captured a consensus. Concurrently, many summary measures of inequality have been suggested. Begining with Gini's (1913) measure, Theil (1967) measure, Atkinson (1970), Kakwani (1980), and Chakravarty's measures. Basman and Slottje (1987, 1988) suggested a functional form which is log-linear and can be estimated the parameters value by using the linear least squares method. Also, this functional form provides very good fits with compare to other functional forms. For a particular case, the specification contains the Gamma distribution, and allows one to compute easily, with the provided formula, the Gini, Kakwani and Chakravarty inequality indices, have all formulated different measures. No single measure appears to satisfy all the criteria that theorists have set down as reasonable descriptions of inequality [cf. Shorrowks (1980, 1982, 1983)]. Given the problems of specifying a functional form and finding an adequate summary measure of inequality, another approach to measuring inequality has been to look directly at the observed income distribution and an individual's relative positions there in. For instrance, by looking directly at income shares one avoids many of these problems. By graduating these shares into a cumulative distribution function one gets closer. Lorenz (1905) started this approach, but Gastwirth (1971, 1972), and Kakwani and Podder (1973), have been among those to refine it. Research along this vein has been promising, but it has been primarily statistical in nature and rather sterile from an economic policy perspective.

The principle purpose of this paper is to present several descriptive approximations of a new functional form for estimating the Lorenz curve. We compre these alternative forms by goodness of fit, F-tests of nested forms, and measurement of the Gini coefficient, Kakwani's and Chakravertays inequality indices.

In section 2, we discuss about the characteristics of the Lorenz curve and in section 3, we presented the descriptive approximations of a general functional form for estimating the Lorenz curve. In section 4, we presented tests of alternative hypothesis. Finally, in section 5, we presented the concluding remarks.

Next we will discuss about the characteristics of the Lorenz Curve;

2 A Characterization of the Lorenz Curve

Lorenz (1905) was the first to present the graphical relationship between the cumulative distribution of income units (earners) ordered by income and the size distribution of income by the same units. Mathematically, the Lorenz curve is defined as follows;

Suppose income X of a unit is a random variable with the probability density function $f(x)$. Then the function $F(x)$ is defined as;

$$F(x)=Prob(X\leq x)$$
$$=\int_0^x f(x)dx \tag{1}$$

where, $F(x)$ can be interpreted as the proportion of units having an income less than or equal to x. $F(x)$ obviously varies from 0 to 1. Further if it is assumed that the mean income μ of the distribution exists, which is given by;

$$\mu=\int Xf(x)dx \tag{2}$$

Then the first moment distribution function of X is defined as;

$$F_1(x)=\frac{1}{\mu}\int_0^x Xf(x)dx \tag{3}$$

where $F_1(x)$ also varies from 0 to 1. It follows that $F_1(x)$ is interpreted as the proportional share of the total income of the units having an income less than or equal to x. Then the Lorenz curve is the relationship between the variables $F(x)$ and $F_1(x)$ and it can be obtained by inverting functions (1) and (3) and eliminating x, if the functions are conveniently invertible. Alternatively, the curve can be plotted by generating the values of $F(x)$ and $F_1(x)$ from (1) and (3) by considering the arbitrary values of x. The curve is represented in a unit square.

Before deriving the equation of the Lorenz curve for several well known distribution functions, it would be useful to express the relationship as;

$$L(z)=F_1(x) \ \ and \ \ z=F(x), \ \ where\ 0\leq z\leq 1 \tag{4}$$

The functional form $L(z)$ obtained by eliminating x from (3) is interpreted as the fraction of the total income received by the lowest zth fraction of the families.

Kakwani and Podder (1973, 1976) noted that the Lorenz curve $L(z)$ should exhibit the following properties;

(i) $L(z)=0, \ if\ z=0;$ (5)

(ii) $L(z)=1, \ if\ z=1$ (6)

(iii) $L(z)\leq z\ for\ 0\leq z\leq 1$ (7)

and

(iv) $L(z)$ should be twice continuously differentiable and the slope of the curve should increase monotonically (ie. $L'(z)\geq 0, \ L''(z)\geq 0$)

In addition Kakwani (1980a) has noted a number of other properties that the Lorenz curve possesses. He lists these as lemmas in his comprehensive discussion of the Lorenz curve in his 1980 book. We present these lemmas without comment as follows

(1) The distance between the Lorenz curve and the egalitarian line is a maximum at income level $X=\mu$

(2) Dividing the population into two groups so in the first group all the income units have income less than μ the proportion of income that should be transferred so both groups have

the same income is given by the maximum distance between the Lorenz curve and the egalitarian line

(3) The Lorenz curve $q=L(z)$ is symmetric iff $1-z=L(1-q)$

(4) If the Lorenz curve $q=L(z)$ is symmetric, the point $(z_\mu, L(z_\mu))$ corresponding to mean income μ lies on the diagonal perpendicular to the egalitarian line

(5) The necessary and sufficient condition for the Lorenz curve to be symmetric is $\frac{f(\mu^2/x)}{f(x)}=\left(\frac{x}{z}\right)^3$ for a density $f(x)$ for all X

(6) The Lorenz curve for the Lognormal distribution is symmetric

(7) The Lorenz curve $q=L(z)$ is skewed toward (0, 0) iff $z_\mu+L(z_\mu)>1$

(8) The Lorenz curve for the Pareto distribution is skewed toward (0, 0)

The interest reader is encouraged to review Kakwani (1980a) for a complete discussion of these properties. It can readily be seen that the shape of the Lorenz curve depends on the underlying hypothetical statistical distribution $F(x)$. While a number of hypothetical forms have been specified as the distribution function $F(x)$, cf McDonald (1984). Some of the more popular forms in the literature have been the Pareto, Lognormal, generalized gamma and generalized beta distributions. Arnold (1983, 1986) and Villasenor and Arnold (1989) suggested other functional form approaches. These forms were based on quadratic and hyperbolic functions. By imposs-ing a particular functional form, the researcher is implicitly assuming that these hypothetical statistical distributions approximate (or more accurately fit) the actual observed income data well. Thus one meaning of "parametric" approximation of a Lorenz curve is the process whereby an hypothetical statistical distribution is presumed to approximate the income data. Furthermore, a functional form of the Lorenz curve can be derived which depends explicitly on the parameters of the underlying hypothetical statistical distribution. These parameters have an explicit interpretation (cf Esteban (1986) and sometimes do not. The same is true of the Lorenz curve parameters. A number of statistical distribution and the attendant Lorenz curves which can be derived from them are also presented with the following table.

It is here that Kakwani's (1980) definition of the Lorenz curve is presented. His derivation is very useful in understanding the approximation approach to constructing empirical Lorenz curve. Aigner and Goldberger (1970) also discussed the use of the Pareto distribution for income inequality analysis in considerable detail. Chotikapanich (1993, 1994a, b) has presented some more recent forms of explicit functions. Her forms rely on the work of Rasche et al (1980) whose work was in turn based on that of Kakwani and Podder (1973, 1976) as we discuss below.

The problem of indirect approach for estimating the Lorenz curve is that it is difficult to find any one hypothetical statistical distribution to serve as a good approximation over the entire range of an actual batch of income data. That is why in this chapter, another attempt has been

Table 1: A Number of Statistical Distributions and the Attendant Lorenz Curves

Distribution	C.D.F.	Lorenz Curve
Equal	$F(x)=0$, if $x<\mu$, $F(x)=1$, if $x\geq\mu$	$L(z)=z$
Exponential	$F(x)=1-e^{-\lambda x}$, $x>0$	$z+(1-z)ln(1-z)$
Shifted Exponential	$F(x)=1-e^{-\lambda(x-a)}$; $a<x<a+\theta$	$z+(1-\lambda a)^{-1}(1-z)ln(1-z)$
General Uniform	$F(x)=\frac{x-a}{\theta}$; $a<x<a+\theta$	$\frac{az+\theta z^2/z}{z+\theta/2}$
Pareto	$F(x)=1-\left(\frac{x}{x_0}\right)^{-a}$; $x>x_0$, $a>1$	$1-(1-z)^{\frac{(a-1)}{a}}$
Lognormal	$F(x)=\frac{1}{x\sigma\sqrt{(2\pi)}}e^{-\frac{1}{2\sigma^2}}(\log x-\mu)^2$	$\frac{\log z-\mu}{\sigma}-\sigma$
Pareto Second Kind	$1-F(x)=K(c+x)^a$	$1-\alpha(1-z)^{\alpha-1/2}+(\alpha-1)(1-z)$
Burr	$1-F(x)=K(c^\beta+x^\beta)^{-\frac{a}{\beta}}$	$\beta_2\left(\frac{1}{\beta}+1,\ \frac{\alpha-1}{\beta}\right)$
Fisk	$1-F(x)=\frac{K}{c^a+x^a}$	$\beta_2\left(\frac{1}{\alpha}+1,\ \frac{\alpha-1}{\alpha}\right)$
Generalized Gamma	See McDonald	
Generalized Beta	See McDonald	

made for the direct parametric approximations of a new functional form for estimating the Lorenz curve.

3 Parametric Approximations of a New Functional Form for Estimating the Lorenz Curve

To circumvent this "goodness-of-fit" problem from using indirect approximations, it is possible to specify a functional form for the Lorenz curve directly. This method for attacking the problem was formulated by Kakwani and Podder in 1973 and 1976 papers. Kakwani and Podder specified the functional form of the Lorenz curve directly as;

$$L(z)=ze^{-h(1-z)}\ ;\ 0\leq z\leq 1,\ \ h>0 \tag{8}$$

Kakwani and podder also considered the more general functional form which is given as,

$$L(z)=z^a e^{-h(1-z)}\ ;\ 0\leq z\leq 1 \tag{9}$$

when $a=1$, then (9) reduces to (8). Let, $L(z)$ be a nonnegative real valued function of the real variable z defined on and possessing second derivatives of every point of the domain $0\leq z\leq 1$. Now, consider a general maintained hypothesis of the Lorenz curve $L(z)$ to characterize inequality.

$$H_m: L(z)=z^a\left[1+(z-1)e^{(z-1)}\right]^{(bz+c)} \tag{10}$$

From equation (10) it has been found that when $z=0$, then $L(0)=0$, $z=1$, then $L(1)=1$, thus we can say that the general functional form satiesfies property (1) and (2).

The first derivative of the general functional form is given by;

$$L'(z)=az^{(a-1)}\Big[1+(z-1)e^{(z-1)}\Big]^{(bz+c)}+bz^{(a+2)}e^{(z-1)}\Big[1+(z-1)e^{(z-1)}\Big]^{(bz+c-1)}$$
$$+bz^{a}\Big[1+(z-1)e^{(z-1)}\Big]^{(bz+c)}\Big[\log(1+z-1)e^{(z-1)}\Big]$$
$$+cz^{(a+1)}e^{(z-1)}\Big[1+(z-1)e^{(z-1)}\Big]^{(bz+c-1)} \tag{11}$$

which indicated that $L'(z)\geq 0$, for $0\leq z\leq 1$. Also the second derivative of the general functional form is given by;

$$L''(z)=az^{(a-1)}\{(bz+c)ze^{(z-1)}\Big(1+(z-1)e^{(z-1)}\Big)^{(bz+c-1)}+b\Big(1+(z-1)e^{(z-1)}\Big)^{(bz+c)}$$
$$\log(1+(z-1)e^{(z-1)})\}+a(a-1)z^{(a-2)}\Big[1+(z-1)e^{(z-1)}\Big]^{(bz+c)}$$
$$+bz^{(a+2)}e^{(z-1)}\{(bz+c-1)ze^{(z-1)}\Big(1+(z-1)e^{(z-1)}\Big)^{(bz+c-2)}+$$
$$b\Big(1+(z-1)e^{(z-1)}\Big)^{(bz+c-1)}\log(1+(z-1)e^{(z-1)})\}$$
$$+\Big[1+(z-1)e^{(z-1)}\Big]^{(bz+c-1)}\Big[b(a+2)e^{(z-1)}z^{(a+1)}+bz^{(a+2)}e^{(z-1)}\Big]$$
$$+bz^{a}\log(1+(z-1)e^{(z-1)})\{(bz+c)ze^{(z-1)}\Big(1+(z-1)e^{(z-1)}\Big)^{(bz+c-1)}+$$
$$b\Big(1+(z-1)e^{(z-1)}\Big)^{(bz+c)}\log(1+(z-1)e^{(z-1)})\}+\Big[1+(z-1)e^{(z-1)}\Big]^{(bz+c)}$$
$$\Big[bz^{(a+1)}e^{(z-1)}\Big[1+(z-1)e^{(z-1)}\Big]^{-1}+abz^{(a-1)}\log[1+(z-1)e^{(z-1)}]\Big]$$
$$+cz^{(a+1)}e^{(z-1)}\{(bz+c-1)ze^{(z-1)}\Big(1+(z-1)e^{(z-1)}\Big)^{(bz+c-2)}+b\Big(1+(z-1)e^{(z-1)}\Big)^{(bz+c-1)}$$
$$\log(1+(z-1)e^{(z-1)})\}+[1+(z-1)e^{z-1}]^{(bz+c-1)}\Big[cz^{(a+1)}e^{(z-1)}+c(a+1)z^{a}e^{(z-1)}\Big] \tag{12}$$

which indicates thet $L''(z)\geq 0$, for $0\leq z\leq 1$. Also, the functional form (10) can be written as follows;

$$L(z)=z^{a}\Big[1+\frac{(z-1)}{e^{(1-z)}}\Big]^{(bz+c)} \tag{13}$$

which indicates that $L(z)\leq z$, for $0\leq z\leq 1$. Thus, we can say that the considered functional form satiesfied all the properties of the Lorenz curve. So, our considered functional form is of the Lorenz curve.

In equation (10) a, b, and c are the parameters to be estimated. From (10) we considered the following null hypothesis;

$$H_0^1: b=0,\ \ L(z)=z^{a}\Big[1+(z-1)e^{(z-1)}\Big]^{c} \tag{14}$$

$$H_0^2 : c=0, \quad L(z)=z^a\left[1+(z-1)e^{(z-1)}\right]^{bz} \tag{15}$$

$$H_0^3 : a=1, \quad L(z)=z\left[1+(z-1)e^{(z-1)}\right]^{(bz+c)} \tag{16}$$

$$H_0^4 : b=0, \quad c=1, \quad L(z)=z^a\left[1+(z-1)e^{(z-1)}\right] \tag{17}$$

$$H_0^5 : a=1, \quad b=0, \quad L(z)=z\left[1+(z-1)e^{(z-1)}\right]^c \tag{18}$$

All of the specifications are of course subject to empirical examination as to their validity. Actual estimation of the descriptive approximations of the Lorenz curves given in (14)-(18) can be faciliated by a logarithmic transformation. Now taking logarithm of the equation (10), then we have

$$\log L(z)=a\log(z)+bz\log(1+(z-1)e^{(z-1)}+c\log(1+(z-1)e^{(z-1)}) \tag{19}$$

where $L(z)$ is the share of income of the ith class and z is the proportion of the income units of the ith class. As for example, if we looked at (say) quintiles and the 30 percent of the population had 10 percent of the income, then for the year in question $L(z)$ would be log(.10) and z would be .30. This method obviously exhibits increasing degrees of freedom as the income class quantiles increase.

Also the logarithamic transformation of the equations (14)-(18) are as follows;

$$\log L(z)=a\log(z)+c\log(1+(z-1)e^{(z-1)}) \tag{20}$$

$$\log L(z)=a\log(z)+bz\log(1+(z-1)e^{(z-1)}) \tag{21}$$

$$\log L(z)=\log(z)+bz\log(1+(z-1)e^{(z-1)})+c\log(1+(z-1)e^{(z-1)}) \tag{22}$$

$$\log L(z)=a\log(z)+\log(1+(z-1)e^{(z-1)}) \tag{23}$$

To compare among these functional forms we have to do the empirical analysis. Next we will move for empirical analysis.

4 Empirical Ressults

For empirical analysis a real data set has been collected from the BBS (Bangladesh Bureau of Statistics) publications, Household Expenditure Survey for the years 1981-82, 1983-84, 1985-86, 1988-89, 1991-92 and 1995-96. The data set for each year has been divided into several income classes. This allows us to still maintain enough observations in each cell for reasonable estimation and also to be able to present empirical Lorenz curves that are reasonable approximations to the hypothetical forms introduced above. We have also found that at every point in the sample $L(z)$ satisfied the properties (1)-(5). Impossing the restriction implied by H_0^1 through H_0^5, that $L(z)$ satisfied all the properties.

Table 2 presents the results of analyzing each of the five alternative hypotheses nested within

Table 2: F-statistics for H_0^1-H_0^5

Year	1981-82	1983-84	1985-86	1988-89	1991-92	1995-96
H_0^1 $b=0$ R^2	5.93903 (0.03299) 0.9988	7.2408 (0.02099) 0.9991	12.36801 (0.0045) 0.9990	10.59807 (0.00626) 0.9985	8.3031 (0.01084) 0.9993	18.2558 (0.00058) 0.9981
H_0^2 $c=0$ R^2	3.6691 (0.08178) 0.9989	9.74994 (0.00971) 0.9989	28.5353 (0.00023) 0.9970	5.7747 (0.0319) 0.9985	21.54271 (0.00027) 0.9989	8.65202 (0.00958) 0.9986
H_0^3 $a=1$ R^2	58.2432 (0.00001) 0.9591	102.9089 (0.000000) 0.9448	77.595 (0.00000) 0.9152	75.2975 (0.00000) 0.9514	143.4469 (0.00000) 0.9609	125.7859 (0.00000) 0.93311
H_0^4 $b=0,\ c=1$ R^2	14.0345 (0.0009) 0.9964	11.4713 (0.0020) 0.9979	17.5639 (0.0004) 0.9959	23.2667 (0.00005) 0.9952	29.1393 (0.00000) 0.9973	47.7428 (0.00000) 0.9925
H_0^5 $a=1,\ b=0$ R^2	88.0592 (0.0000) 0.8894	128.3086 (0.0000) 0.8705	81.4109 (0.0000) 0.8337	82.7699 (0.0000) 0.9018	203.0118 (0.00000) 0.8966	124.7706 (0.00000) 0.8747

the maintained hypothesis for each year of the data.

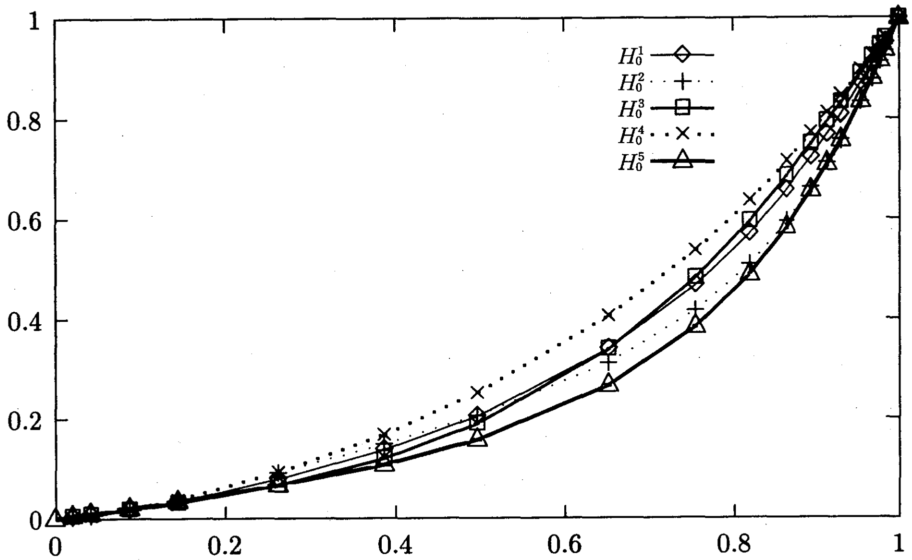
[2)]From the tabulated value it has been found that the R^2 value significantly drops for different years for the null hypothesis H_0^3, and H_0^5. But for the null hypothesis H_0^5, R^2 value drops dramatically. So, based on the F-statistic given in Table 2, it is clear we can reject the null hypothesis H_0^3, and H_0^5 at any reasonable significance level. Also, from the tabulated values it can be concluded that, except the null hypothesis H_0^2 for the years 1981-82, all of the null hypothesis are rejected at 5% level of significance. Finally, from the tabulated values it can be concluded that all of the parameters are most important for describing the Lorenz curve in the considered functional form. From thes tabulated values and also from the null hypothesis, H_0^1, and H_0^5, it can be concluded that the null hypothesis, $a=1$, is so restrictive.

We can pictorally compare the differences of these forms from the graphical representaion on the basis of the estimated value of the Lorenz curve for each null hypothesis.

From the graphical representation it can be concluded that the H_0^5 is always below the other empirical curves and H_0^4 is always above the other curves over most of the range 0-1. This indicates that the H_0^5 demonstrates more inequality and H_0^4 is demonstrating less inequality over most of the distribution. These results hold for all of the years. Following the suggestion of a referee we only report the results for 1995-96, here since the results are similar for the other years.

2) Number in the perentheses represents $Pr[F>F_{a,k}]$

Figure 1: Graphical Representation of the Different Forms on the Basis of the Estimated Values



Using numerical integration techniques, Gini coefficients, Kakwani's and Chakravartay inequality indices are calculated on the basis of the real data set which is collected from the Household Expenditure Survey for different years in Bangladesh. These calculated values are reported with the following tables.

Table 3: Gini Coefficients Estimated Under Various Lorenz Curve Hypothesis [3]

Years	1981-82	1983-84	1985-86	1988-89	1991-92	1995-96
H_0^m	0.38	0.35	0.38	0.38	0.38	0.43
H_0^1	0.37	0.34	0.36	0.37	0.37	0.41
H_0^2	0.38	0.36	0.39	0.39	0.39	0.44
H_0^3	0.37	0.33	0.35	0.37	0.35	0.41
H_0^4	0.33	0.31	0.31	0.32	0.34	0.34
H_0^5	0.45	0.41	0.45	0.43	0.45	0.49

3) The mathematical form of the Gini coefficient is given by;

$$GC=1-2\int_0^1 L(z)\,dz$$

Table 4: Estimated Kakwani's Inequality Measure Under Various Lorenz Curve Hypothesis

Year	Kakwani (r=0.5)						Kakwani (r=1.5)					
	H_0^m	H_0^1	H_0^2	H_0^3	H_0^4	H_0^5	H_0^m	H_0^1	H_0^2	H_0^3	H_0^4	H_0^5
1981-82	0.25	0.24	0.26	0.23	0.21	0.30	0.46	0.42	0.46	0.46	0.41	0.53
1983-84	0.23	0.22	0.24	0.20	0.20	0.27	0.43	0.42	0.43	0.42	0.39	0.49
1985-86	0.25	0.23	0.27	0.22	0.19	0.30	0.45	0.44	0.46	0.44	0.38	0.53
1988-89	0.25	0.24	0.26	0.23	0.20	0.29	0.46	0.45	0.46	0.46	0.40	0.51
1991-92	0.25	0.24	0.26	0.22	0.20	0.30	0.45	0.45	0.46	0.45	0.40	0.53
1995-96	0.29	0.27	0.30	0.26	0.21	0.33	0.51	0.50	0.51	0.51	0.42	0.57

Year	Kakwani (r=2)						Kakwani (r=2.5)					
	H_0^m	H_0^1	H_0^2	H_0^3	H_0^4	H_0^5	H_0^m	H_0^1	H_0^2	H_0^3	H_0^4	H_0^5
1981-82	0.51	0.51	0.51	0.52	0.46	0.58	0.55	0.55	0.54	0.56	0.51	0.61
1983-84	0.47	0.47	0.47	0.48	0.44	0.54	0.51	0.51	0.50	0.52	0.48	0.57
1985-86	0.50	0.50	0.50	0.50	0.43	0.58	0.53	0.53	0.52	0.55	0.46	0.61
1988-89	0.51	0.50	0.51	0.52	0.46	0.56	0.54	0.54	0.54	0.59	0.50	0.60
1991-92	0.50	0.50	0.51	0.51	0.46	0.58	0.54	0.54	0.54	0.55	0.50	0.61
1995-96	0.56	0.55	0.56	0.57	0.48	0.63	0.54	0.59	0.59	0.62	0.52	0.67

Table 5: Estimated Chakravarty's Inequality Measure Under Various Lorenz Curve Hypothesis [4]

Year	Chakravarty (r=0.5)						Chakravarty (r=1.5)					
	H_0^m	H_0^1	H_0^2	H_0^3	H_0^4	H_0^5	H_0^m	H_0^1	H_0^2	H_0^3	H_0^4	H_0^5
1981-82	0.35	0.34	0.36	0.33	0.31	0.41	0.40	0.39	0.41	0.39	0.35	0.47
1983-84	0.33	0.31	0.33	0.30	0.29	0.38	0.37	0.36	0.38	0.35	0.33	0.44
1985-86	0.35	0.34	0.36	0.32	0.28	0.41	0.40	0.38	0.42	0.37	0.32	0.47
1988-89	0.35	0.34	0.36	0.34	0.30	0.40	0.40	0.39	0.41	0.39	0.34	0.46
1991-92	0.35	0.34	0.36	0.32	0.30	0.41	0.40	0.39	0.41	0.38	0.34	0.47
1995-96	0.40	0.38	0.40	0.38	0.31	0.45	0.46	0.43	0.46	0.44	0.36	0.52

Year	Chakravarty (r=2)						Chakravarty (r=2.5)					
	H_0^m	H_0^1	H_0^2	H_0^3	H_0^4	H_0^5	H_0^m	H_0^1	H_0^2	H_0^3	H_0^4	H_0^5
1981-82	0.42	0.41	0.42	0.40	0.36	0.49	0.43	0.42	0.44	0.42	0.37	0.51
1983-84	0.39	0.37	0.40	0.36	0.34	0.46	0.40	0.38	0.41	0.38	0.35	0.47
1985-86	0.41	0.40	0.43	0.40	0.33	0.49	0.43	0.41	0.45	0.40	0.34	0.51
1988-89	0.42	0.40	0.43	0.41	0.35	0.48	0.43	0.42	0.44	0.42	0.36	0.50
1991-92	0.41	0.40	0.43	0.39	0.35	0.49	0.43	0.42	0.44	0.41	0.36	0.51
1995-96	0.48	0.45	0.48	0.46	0.37	0.54	0.43	0.46	0.50	0.47	0.38	0.56

4) The mathematical forms of the Kakwani's and Chakravarty's measures are as;

$$K_r=1-r(r+1)\int_0^1 L(z)(1-z)^{(r-1)}dz,\quad C_r=2\left[\int_0^1 (z-L(z))^r dz\right]^{1/r}$$

From table 3, we see that the Gini coefficient for H_0^5 is larger and for H_0^4 is smaller for each year. Also from table 4 and 5 we see that the Kakwani and Chakravarty inequality indices for H_0^5 are larger and for H_0^4 are smaller for each year. So, it can be concluded that the functional form (18) is so restrictive, that it probably leads to an upward bias of estimated inequality, against the maintained hypothesis. We also, see that the estimated values of the inequality indices are higher for the year 1995-96 comparing to the previous years. Thus, it can be concluded that the income distribution of Bangladesh is going to be worsened and it may continue to worsen in the years ahead. We also found that the Kakwani and Chakravarty's inequality measures are highly correlated with the Gini index. The correlation coefficient between Gini index and the Kakwani measure is 0.9979 and between Gini index and Chakravarty's measure is 0.9909. Also the predicted values of $L(z)$, for different years under alternative hypothesis are given below.

Table 6: The observed and predicted values of $L(z)$ for Different Years

Year	z	$L(z)$	H_0^m	H_0^1	H_0^2	H_0^3	H_0^4	H_0^5
1981-1982	0.0416	0.0076	0.0074	0.0078	0.0071	0.0095	0.0071	0.0119
	0.0984	0.0241	0.0249	0.0242	0.0254	0.0239	0.0241	0.0284
	0.1696	0.0504	0.0528	0.0499	0.0555	0.0449	0.0522	0.0499
	0.3880	0.1702	0.1637	0.1582	0.1700	0.1416	0.1757	0.1294
	0.5541	0.2879	0.2764	0.2791	0.2780	0.2687	0.3107	0.2218
	0.6838	0.4062	0.3963	0.4107	0.3902	0.4176	0.4502	0.3345
	0.7676	0.4997	0.4995	0.5215	0.4881	0.5422	0.5618	0.4408
	0.8719	0.6461	0.6734	0.6988	0.6585	0.7296	0.7313	0.6295
	0.9213	0.7357	0.7809	0.8022	0.7679	0.8297	0.8260	0.7499
	0.9488	0.7969	0.8499	0.8664	0.8397	0.8879	0.8834	0.8283
	0.9758	0.8726	0.9253	0.9344	0.9195	0.9465	0.9432	0.9143
	0.9880	0.9169	0.9620	0.9669	0.9589	0.9734	0.9715	0.9564
	0.9935	0.9413	0.9792	0.9819	0.9775	0.9856	0.9844	0.9761
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1983-1984	0.0300	0.0060	0.0059	0.0061	0.0056	0.0078	0.0057	0.0099
	0.1139	0.0343	0.0354	0.0341	0.0370	0.0323	0.0344	0.0379
	0.2312	0.0878	0.0898	0.0859	0.0954	0.0759	0.0902	0.0797
	0.3641	0.1655	0.1632	0.1599	0.1700	0.1446	0.1710	0.1349
	0.4871	0.2532	0.2440	0.2458	0.2468	0.2343	0.2640	0.2006
	0.6834	0.4315	0.4176	0.4351	0.4058	0.4507	0.4617	0.3625
	0.7987	0.5648	0.5665	0.5919	0.5463	0.6267	0.6179	0.5170
	0.8687	0.6650	0.6854	0.7109	0.6639	0.7498	0.7326	0.6462
	0.9339	0.7816	0.8236	0.8424	0.8071	0.8724	0.8558	0.8002
	0.9607	0.8429	0.8901	0.9030	0.8785	0.9239	0.9118	0.8753
	0.9761	0.8863	0.9313	0.9399	0.9236	0.9538	0.9455	0.9219
	0.9839	0.9123	0.9531	0.9591	0.9476	0.9688	0.9629	0.9467
	0.9900	0.9359	0.9705	0.9744	0.9669	0.9807	0.9769	0.9665
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

continue of table 6

Year	z	$L(z)$	H_0^m	H_0^1	H_0^2	H_0^3	H_0^4	H_0^5
	0.0070	0.0011	0.0011	0.0011	0.0009	0.0016	0.0009	0.0020
	0.0458	0.0106	0.0112	0.0109	0.0121	0.0108	0.0110	0.0132
	0.1189	0.0352	0.0368	0.0355	0.0429	0.0307	0.0387	0.0346
	0.2072	0.0734	0.0735	0.0712	0.0856	0.0510	0.0811	0.0619
	0.3253	0.1359	0.1303	0.1286	0.1447	0.1120	0.1501	0.1035
	0.5244	0.2704	0.2533	0.2587	0.2528	0.2516	0.3016	0.2022
1985-	0.6663	0.3937	0.3786	0.3926	0.3563	0.4092	0.4464	0.31670
1986	0.7686	0.5024	0.5038	0.5235	0.4657	0.5600	0.5777	0.4427
	0.8718	0.6394	0.6777	0.6979	0.6336	0.7431	0.7408	0.6296
	0.9183	0.7195	0.7780	0.7948	0.7399	0.8338	0.8267	0.7422
	0.9442	0.7736	0.8414	0.8546	0.8107	0.8859	0.8784	0.8147
	0.9638	0.8224	0.8935	0.9029	0.8709	0.9258	0.9194	0.8750
	0.9715	0.8443	0.9150	0.9228	0.8963	0.9415	0.9361	0.9000
	1.000	1.000	0.9997	0.9997	0.9996	0.9998	0.9998	0.9999
	0.0380	0.0074	0.0072	0.0077	0.0068	0.0095	0.0068	0.0115
	0.1010	0.0268	0.0276	0.0267	0.0282	0.0268	0.0264	0.0309
	0.1864	0.0605	0.0625	0.0589	0.0657	0.0540	0.0622	0.0583
	0.2770	0.1040	0.1053	0.0997	0.1110	0.0896	0.1094	0.0900
	0.4536	0.2117	0.2040	0.2023	0.2085	0.1886	0.2284	0.1687
	0.5972	0.3239	0.3084	0.3184	0.3062	0.3129	0.3575	0.2641
1988-	0.6937	0.4163	0.4012	0.4217	0.3926	0.4275	0.4664	0.3569
1989	0.8239	0.5725	0.5779	0.6093	0.5619	0.6310	0.6516	0.5438
	0.8892	0.6741	0.7017	0.7318	0.6855	0.7557	0.7654	0.6777
	0.9234	0.7393	0.7780	0.8056	0.7658	0.8269	0.8318	0.7626
	0.9457	0.7895	0.8371	0.8578	0.8253	0.8755	0.8779	0.8244
	0.9596	0.8260	0.8753	0.8921	0.8658	0.9066	0.9078	0.8658
	0.9692	0.8546	0.9031	0.9166	0.8953	0.9284	0.9289	0.8957
	0.9761	0.8774	0.9237	0.9346	0.9174	0.9442	0.9444	0.9179
	0.9842	0.9086	0.9487	0.9562	0.9443	0.9629	0.9629	0.9449
	1.000	1.000	0.9926	0.9938	0.9920	0.9948	0.9948	0.9921

continue of table 6

Year	z	$L(z)$	H_0^m	H_0^1	H_0^2	H_0^3	H_0^4	H_0^5
	0.0283	0.0049	0.0048	0.0050	0.0044	0.0064	0.0045	0.0081
	0.0647	0.0145	0.0147	0.0146	0.0148	0.0153	0.0141	0.0187
	0.1306	0.0367	0.0378	0.0364	0.0404	0.0333	0.0376	0.0381
	0.2005	0.0655	0.0670	0.0643	0.0725	0.0563	0.0688	0.0597
	0.3534	0.1451	0.1435	0.1404	0.1519	0.1250	0.1560	0.1148
	0.4806	0.2305	0.2223	0.2232	0.2266	0.2108	0.2499	0.1759
	0.5854	0.3164	0.3030	0.3103	0.2994	0.3091	0.3453	0.2453
	0.7469	0.4844	0.4753	0.4947	0.4556	0.5222	0.5358	0.4121
	0.8351	0.6018	0.6098	0.6327	0.5840	0.6723	0.6698	0.5545
1991-	0.8943	0.6984	0.7244	0.7456	0.6995	0.7848	0.7749	0.6816
1992	0.9283	0.7638	0.8018	0.8193	0.7806	0.8526	0.8417	0.7694
	0.9473	0.8062	0.8493	0.8637	0.8317	0.8912	0.8812	0.8240
	0.9583	0.8342	0.8784	0.8905	0.8634	0.9138	0.9049	0.8577
	0.9689	0.8643	0.9075	0.9171	0.8956	0.9356	0.9282	0.8915
	0.9837	0.9331	0.9502	0.9556	0.9433	0.9662	0.9618	0.9414
	0.9901	0.9394	0.9694	0.9728	0.9650	0.9795	0.9766	0.9639
	0.9946	0.9611	0.9832	0.9851	0.9807	0.9888	0.9872	0.9801
	0.9959	0.9684	0.9872	0.9886	0.9853	0.9915	0.9903	0.9849
	1.000	1.000	0.9994	0.9994	0.9993	0.9996	0.9995	0.9993
	0.0206	0.0026	0.0025	0.0028	0.0023	0.0038	0.0023	0.0048
	0.0422	0.0070	0.0070	0.0071	0.0070	0.0080	0.0065	0.0099
	0.0876	0.0188	0.0195	0.0183	0.0204	0.0176	0.0188	0.0206
	0.1447	0.0368	0.0387	0.0353	0.0416	0.0311	0.0389	0.0346
	0.2633	0.0843	0.0858	0.0791	0.0922	0.0669	0.0941	0.0664
	0.3872	0.1481	0.1430	0.1382	0.1499	0.1209	0.1696	0.1078
	0.4970	0.2169	0.2035	0.2061	0.2070	0.1899	0.2533	0.1576
	0.6523	0.3401	0.3190	0.3406	0.3127	0.3390	0.4069	0.2671
	0.7551	0.4450	0.4322	0.4679	0.4179	0.4826	0.5386	0.3848
1995-	0.8196	0.5258	0.5289	0.5706	0.5106	0.5945	0.6372	0.4887
1996	0.8654	0.5938	0.6149	0.6571	0.5954	0.6849	0.7160	0.5821
	0.8959	0.6461	0.6824	0.7221	0.6635	0.7499	0.7731	0.6558
	0.9153	0.6837	0.7304	0.7670	0.7127	0.7934	0.8114	0.7081
	0.9318	0.7197	0.7747	0.8075	0.7587	0.8316	0.8454	0.7564
	0.9556	0.7801	0.8451	0.8698	0.8327	0.8885	0.8967	0.8329
	0.9689	0.8215	0.8880	0.9068	0.8784	0.9212	0.9266	0.8794
	0.9783	0.8563	0.9201	0.9339	0.9129	0.9447	0.9482	0.9140
	0.9840	0.8807	0.9402	0.9508	0.9348	0.9591	0.9616	0.9358
	1.000	1.000	0.9992	0.9993	0.9992	0.9994	0.9995	0.9992

Next we will move for overall discussion and conclusion.

5 Discussion and Conclusion

In this paper we presented a new functional form for the Lorenz curve which we have tested as the maintained hypothesis against five alternatives. From the estimated values of the F-statistic it can be concluded that all the parameters that are included in the general functional form are most important for describing the Lorenz curve. From the graphical representation, it can be concluded that the H_0^5 demonstrated more inequality and H_0^4 is less inequality for the entire range of the income distribution. From table 3, we see that the Gini coefficient for H_0^5 is larger and for H_0^4 is smaller for each year. Same conclusion can be drawn on the basis of the Kakwani's and Chakravarty's inequality measures. Thus, it can be concluded that the functional form (18) is so restrictive, that probably leads to an upward bias of estimated inequality, against the maintained hypothesis. We found that the Kakwani's and Chakravarty's inequality measures are highly correlated with the Gini index. The correlation coeffciient between Gini index and Kakwani measure is 0.9979 and bewteen Gini index and Chakravarty's measure is 0.9909. From the estimated values of the inequality indices, it can be concluded that in Bangladesh the overall income distribution is going to be worsened and it may continue to worsen in the years ahead.

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