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Innovation, Environment, and Economic Growth*

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1 Intoroduction

At present, environmental problems may be the most serious facing humanity. For instance, many kinds of resources are limited. Vast quantities of pollution which may affect our lives are discharged thorough the industrial process. Some islands may disappear due to global warming in next a few decades. So, when we discuss whether or not growth is sustainable, we must think about problems such as resources, pollution, global warming, green house gases, and so on.

Actually, papers focusing on the relationship between economic growth and the environment have become more widespread recently. For example, Stokey (1998) discusses the relationship between growth rate or per capita income and pollution using an AK model. If an economy is relatively poor, utility from consumption is important. So the policies to regulate pollution are not carried out and the pollution increases with per capita income. However, there is a crucial point where the disutility from pollution becomes so large that such policies must be exercised to decrease pollution. Because of this, pollution is negatively correlated with per capita income where per capita income becomes relatively high. This inverted U-shaped relationship between per capita income and pollution is also pointed out by Grossman and Krueger (1995). One of the problems in the AK model is that growth peters out in the long run. Stokey (1998) also shows if the technology improves continuously, the growth rate of per capita income becomes positive in the long run even if that of pollution is negative. However, in her model, technological improvement is assumed to be exogenous rather than endogenous and how and why the technological progress takes place is not considered.

The endogenous growth theory has been developed to illustrate the determinant of the growth endogenously. It emphasizes the role of the spillover of knowledge capital, R&D, accumulation of human capital, and especially innovation, since these make an economy more productive. One idea of innovation is expanding the number of goods. This type of innovation is described in Romer (1990), Grossman and Helpman (1991, ch3), Barro and Sala-i-Martin (1995, ch6), and so on.

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Another type of innovation is to improve the quality of the goods. This type of innovation is shown by Grossman and Helpman (1991, ch4), Segerstorm (1991), Barro and Sala-i-Martin (1995, ch7), Aghion and Howitt (1998), and so on. If these two types of innovation are combined, the model becomes like that of Young (1998), Dinopoulos and Thompson (1998). In this case, both expansion of the variety and improvement in the quality occur.

Gradus and Smulders (1993) analyze the environmental effects on growth using a neoclassical model, AK model, and the growth model introduced by Lucas (1988). It may be interesting to note that if pollution affects the accumulation of human capital negatively, an economy that values the environment highly grows more rapidly. Ikazaki and Osumi (1998) extend their model in order to analyze the effects of R&D on growth or environmental quality, which Gradus and Smulders (1993) do not consider. One of the interesting results is that the long run growth rate is negatively correlated with pollution. Stokey (1998, sec.4), Aghion and Howitt (1998, ch5), indicate that the growth rate of pollution is negatively correlated with the growth rate of per capita income. However Ikazaki and Osumi (1998) show that the growth rate of per capita income is negatively correlated with the level of the pollution rather than with the growth rate of it.

Among previous papers which analyze the relationship between environment and growth, there are not many which deal with innovation or R&D, even though they are two of the most important topics in the growth theory. So, the environmental problems, especially the problems of pollution, are incorporated into the endogenous growth model to consider how the pollution affects long-run growth rate in this paper.

This paper is organized as follows. In section 2, the endogenous growth model with pollution is introduced and various growth rates will be derived. We will show that sustainable growth, which is defined as being where the growth rate of GDP is positive and that of pollution is not positive, can be attained. In sections 3 and 4, we shall show that sustainable growth is possible even in a market economy if a government introduce a pollution tax or a voucher system appropriately. However, it is not enough to accomplish the socially optimum growth rates since there are other market distortions that make the market equilibrium and optimal outcome diverge. So, we must discuss the conditions to correct these distortions in section 5. Lastly in section 6, the main conclusions of this paper will be described.

2 A Growth Model with Environment

In this section, a model considered in this paper will be described. We will begin by thinking about the final goods sector. A final good is assumed to be homogenous in our model. It is consumed or invested to accumulate capital. The production function is specified as

$$Y(t)=AK(t)^{\alpha}\left[\int_0^{n(t)} x_i(t)^{\xi}di\right]^{\frac{1-\alpha}{\xi}} z(t), \tag{2.1}$$

where $Y(t)$ is actual output at time t, A is productivity parameter, $K(t)$ is capital stock at time t, $n(t)$ denotes measure (number)[1] of the available intermediate goods at time t, $x_i(t)(i\in[0, n(t)])$ is the quantities of ith intermediate goods used at production activities at time t, $z(t)(z(t)\in[0, 1])$ is a technological index at time t and α, ξ is the parameter and $0<\alpha, \xi<1$. The notation (t) denotes the level of time t throughout this paper.

The intermediate goods are horizontally differentiated and the innovation is interpreted as expanding the number of them in this paper. To obtain a blueprint of a new intermediate good, the devotion of resources (in our model, labor) to an R&D sector is needed. The production function of the R&D sector takes the form

$$\dot{n}(t)=\varepsilon n(t)L_R(t), \tag{2.2}$$

where the dot means differentiation with respect to time such as $\dot{n}\equiv\frac{dn}{dt}$, ε is productivity parameter and $L_R(t)$ is labor input in this sector.

Each intermediate good is produced by a single input, labor. Suppose that for any $i(i\in[0, n(t)])$, one unit of labor is needed to produce one unit of intermediate good. It implies that the labor demand in this sector, $L_X(t)$, is equal to $\int_0^{n(t)} x_i(t)di(\equiv X(t))$.

Next, we shall deal with consumers. They have utility over an infinite horizon. We specify the objective function of the representative consumer as

$$U=\int_0^{\infty} e^{-\rho t}\left(\frac{c(t)^{1-\sigma}-1}{1-\sigma}-BD(t)\right)dt, \tag{2.3}$$

where $\rho(>0)$ is the subjective discount rate, $c(t)$ is per capita consumption, $B(>0)$ is the parameter which shows how much each individual suffers from pollution, $D(t)$ is the pollution level. We also assume $\sigma>0$. Let us assume the pollution is represented as

$$D(t)=AK(t)^{\alpha}\left[\int_0^{n(t)} x_i(t)^{\xi}di\right]^{\frac{1-\alpha}{\xi}} z(t)^{\beta}, \tag{2.4}$$

where $\beta>1$.[2] Equations (2.1) and (2.4) imply that pollution increases drastically as output increases for given $AK(t)^{\alpha}\left[\int_0^{n(t)} x_i(t)^{\xi}di\right]^{\frac{1-\alpha}{\xi}}$.

The social planner's problem is to maximize (2.3), subject to

$$\dot{K}(t)=AK(t)^{\alpha}\left[\int_0^{n(t)} x_i(t)^{\xi}di\right]^{\frac{1-\alpha}{\xi}} z(t)-C(t), \tag{2.5}$$

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1 We take the product space of the intermediate goods to be continuous rather than discrete and ignore integer constriant on the number of goods.

2 We assume the pollution is caused only by the production process. For simplicity, pollution that originates in consumption or other actibities is not taken into consideration.

$$\frac{1}{\varepsilon}\frac{\dot{n}(t)}{n(t)}+\int_0^{n(t)} x_i(t)di=L, \tag{2.6}$$

and $K(0)=K_0$, $n(0)=n_0$, where $C(t)\equiv c(t)\,L$ is total consumption and L represents the total labor force and also the population in this economy[3] and is constant over time.

The current value Hamiltonian for this problem takes the form

$$H=\frac{c^{1-\sigma}-1}{1-\sigma}-BAK^{\alpha}n^{\frac{1-\alpha}{\xi}}x^{1-\alpha}z^{\beta}+\mu_1(AK^{\alpha}n^{\frac{1-\alpha}{\xi}}x^{1-\alpha}z-cL)+\mu_2(\varepsilon n(L-nx)), \tag{2.7}$$

where x is the quantity of each intermediate good, μ_1 and μ_2 are shadow prices of capital and the measure of the intermediate goods respectively.[4]

The conditions for the maximum can be written as

$$c(t)^{-\sigma}=\mu_1(t)L, \tag{2.8}$$

$$z(t)=\begin{cases}1 & (\text{if } \mu_1(\mathrm{t})\geq\beta B),\\ \left(\dfrac{\mu_1(t)}{\beta B}\right)^{\frac{1}{\beta-1}} & (\text{if } \mu_1(\mathrm{t})<\beta B),\end{cases} \tag{2.9}$$

$$X(t)=\begin{cases}(1-\alpha)\left(1-\dfrac{B}{\mu_1(t)}\right)\dfrac{Y(t)\mu_1(t)}{\varepsilon\mu_2(t)n(t)} & (\text{if } z(t)=1),\\ (1-\alpha)\left(\dfrac{\beta-1}{\beta}\right)\dfrac{Y(t)\mu_1(t)}{\varepsilon\mu_2(t)n(t)} & (\text{if } z(t)<1),\end{cases} \tag{2.10}$$

$$-\frac{\dot{\mu}_1(t)}{\mu_1(t)}=\begin{cases}\alpha\left(1-\dfrac{B}{\mu_1(t)}\right)\dfrac{Y(t)}{K(t)}-\rho & (\text{if } z(t)=1),\\ \alpha\left(\dfrac{\beta-1}{\beta}\right)\dfrac{Y(t)}{K(t)}-\rho & (\text{if } z(t)<1),\end{cases} \tag{2.11}$$

$$\frac{\dot{\mu}_2(t)}{\mu_2(t)}=\begin{cases}\rho-\dfrac{1-\alpha}{\xi}\left(1-\dfrac{B}{\mu_1(t)}\right)\dfrac{Y(t)\mu_1(t)}{n(t)\mu_2(t)}-\varepsilon(L-2X(t)) & (\text{if } z(t)=1),\\ \rho-\dfrac{1-\alpha}{\xi}\dfrac{\beta-1}{\beta}\dfrac{Y(t)\mu_1(t)}{n(t)\mu_2(t)}-\varepsilon(L-2X(t)) & (\text{if } z(t)<1),\end{cases} \tag{2.12}$$

and the transversality conditions

$$\lim_{t\to\infty}e^{-\rho t}\,\mu_1(t)\,K(t)=0, \tag{2.13}$$

$$\lim_{t\to\infty}e^{-\rho t}\,\mu_2(t)\,n(t)=0. \tag{2.14}$$

In the steady state, each variable grows at a constant rate, which in turn implies $Y(t)$, $K(t)$ and $C(t)$ grow at the same rate. This rate is given by

$$g_Y^*=\frac{1}{\sigma}(1+\Gamma)^{-1}\left(\frac{1-\xi}{\xi}\varepsilon L-\rho\right), \tag{2.15}$$

3 We assume that each person has one unit of labor.

4 Note that the conditions, $\frac{1}{\varepsilon}\frac{\dot{n}(t)}{n(t)}+\int_0^{n(t)}x_i(t)di=L$ and $x_i(t)=x(t)$ are already used. From the previous discussion, $L_X=\int_0^{n(t)}x_i(t)di$. But according to the problem to maximize $\left(\int_0^{n(t)}x_i(t)^{\xi}di\right)^{\frac{1-\alpha}{\xi}}$, subject to the constraint $L_X=\int_0^{n(t)}x_i(t)di$, we can show that $x_i(t)=x(t)$ and $L_X(t)=n(t)x(t)$.

where g_j denotes the growth rate of variable j, and $\Gamma \equiv \frac{1}{(1-\alpha)(\beta-1)}$.[5]

We are also interested in the dynamic behavior of the pollution. Suppose that the economy is relatively poor at first, that is, $K(0)$, $Y(0)$, $n(0)$, $c(0)$ are relatively small so that $z(0)=1$. If so, at the first stage of the development, $K(t)$, $Y(t)$, $n(t)$, $c(t)$ and also the pollution increase over time. However, $z(t)$ becomes less than 1 eventually and after that it falls over time, which tends to make the pollution decrease. In the steady state, the growth rate of the pollution is given by

$$g_D^* = (1-\sigma) g_Y^*. \tag{2.16}$$

That is, the pollution declines in the steady state if and only if $\sigma > 1$.[6] It means that if $\sigma > 1$, the long run growth rate is negatively correlated with the pollution. For the derivation of (2.15) and (2.16), see Appendix 7.1.

3 The Market Economy

In this section, a decentralized economy shall be considered. In a market economy, we assume that there exists a government setting a pollution tax.

First of all, we shall analyze the final good sector. The market for the final good is assumed to be perfectly competitive. A lot of firms manufacture homogenous final goods subject to the same technology given by (2.1). We aggregate these firms to the industry level and use (2.4) to get[7]

$$Y(t) = A^{\frac{\beta-1}{\beta}} K(t)^{\alpha\frac{\beta-1}{\beta}} \left[\int_0^{n(t)} x_i(t)^{\xi} di \right]^{\frac{1-\alpha}{\xi}\frac{\beta-1}{\beta}} D(t)^{\frac{1}{\beta}}. \tag{3.1}$$

Note that the final good is in effect produced with capital stock ($K(t)$), intermediate goods ($x_i(t) \in [0, n(t)]$) and pollution ($D(t)$) as inputs.[8]

Firms maximize their profits at each date, taking the interest rate, $r(t)$, the number of the intermediate goods, $n(t)$, the prices of intermediate goods, $p_i(t)$ ($i \in [0, n(t)]$), and the tax rate, $\tau(t)$, as given. A profit function[9] is given by

$$\Pi(t) = A^{\frac{\beta-1}{\beta}} K(t)^{\alpha\frac{\beta-1}{\beta}} \left[\int_0^{n(t)} x_i(t)^{\xi} di \right]^{\frac{1-\alpha}{\xi}\frac{\beta-1}{\beta}} D^{\frac{1}{\beta}} - r(t) K(t) - \int_0^{n(t)} p_i(t) x_i(t) di - \tau(t) D(t). \tag{3.2}$$

From the firms' profit maximization and the break-even condition, we can get

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5 We use the asterisk to denote the socially optimum growth rate of each variables. We also assume $\frac{1-\xi}{\xi} \varepsilon L - \rho > 0$ so that $g_Y^* > 0$.

6 This result is also derived by Stokey (1998), Aghion and Howitt (1998, ch5).

7 Note that $D(t) \leq Y(t)$ since $z(t) \in [0, 1]$.

8 The production function in which the pollution can be interpreted as a primary factor of production is also specified in Copeland and Taylor (1994). However the consumption goods are produced with labor and pollution as inputs in their model.

9 The price of the final good is normalized to 1.

$$r(t) \geq \alpha \frac{\beta-1}{\beta} \frac{Y(t)}{K(t)}, \tag{3.3}$$

$$x_i(t) = \frac{\int_0^{n(t)} p_i(t) x_i(t) di}{\int_0^{n(t)} p_i(t)^{-\frac{\xi}{1-\xi}} di} p_i(t)^{-\frac{1}{1-\xi}}, \tag{3.4}$$

$$\tau(t) \leq \frac{1}{\beta} \frac{Y(t)}{D(t)}. \tag{3.5}$$

In the R & D sector, firms may enter freely into R & D. They finance the cost by issuing equity and employ workers to obtain blueprints of new intermediate goods. If they succeed in inventing, they can produce monopotically over time. The production function in this sector is again given by (2.2).

In the intermediate goods sector, firms produce goods, using the blueprints which they created in the R & D sector. The profit function of firm i is given by

$$\pi_i(t) = p_i(t)\, x_i(t) - w(t)\, x_i(t), \tag{3.6}$$

where $w(t)$ is the wage rate and $\pi_i(t)$ is the profit of firm i. Since demand function for firm i is (3.4), it maximizes the profit by setting $p_i(t)$ as

$$p_i(t) = p(t) = \frac{w(t)}{\xi}. \tag{3.7}$$

Equation (3.7) means that the price of each intermediate good and the profit which each firm can earn are the same in every industry at any moment in time. We will denote $\pi_i(t)$ ($\in [0, n(t)]$) as $\pi(t)$.

Next, the value of each R & D is considered. A firm which succeeds in research activity at a certain time can earn profits after that time by supplying the intermediate good monopotically to the final good sector. So the value of each R & D can be written as

$$v(t) = \int_t^{\infty} e^{-\int_t^{t'} r(\eta) d\eta} \pi(t') dt', \tag{3.8}$$

where $v(t)$ is the value of the equity of each firm. From (3.8), we can get the no-arbitrage condition

$$r(t)\, v(t) = \pi(t) + \dot{v}(t). \tag{3.9}$$

Note that the right hand side of (3.9) is the total return to the owners of each firm. On the other hand, the left-hand side is that to the investors in the riskless loan. Because of the equilibrium in the capital market, they must be equal.

From (2.2), the labor needed to develop one unit of intermediate good is $\frac{1}{n(t)\varepsilon}$. The value created by such activity is $v(t)$. So, by the assumption of free-entry to R & D, we can get

$$v(t) \leq \frac{1}{n(t)\varepsilon} w(t). \tag{3.10}$$

If $\dot{n}(t) > 0$, then (3.10) must hold with equality.[10]

Let us turn to the consumers. Individuals earn wages by supplying the labor force, the interest from their asset, subsidy from the government.[11] They decide how much they will consume and save to maximize their utilities over an infinite horizon, taking the path of $r(t)$, $w(t)$, $\eta(t)$ as given, where $\eta(t)$ is the subsidy from the government and $\eta(t)=\frac{\tau(t)\,D(t)}{L}$.

$$\dot{f}(t)=r(t)\,f(t)+\eta(t)+w(t)-c(t), \tag{3.11}$$

where $f(t)$ is per capita assets and $f(0)$ is given by f_0. The conditions for the maximum are

$$c(t)^{-\sigma}=\mu_3(t), \tag{3.12}$$

$$\dot{\mu}_3(t)-\rho\mu_3(t)=-r(t)\,\mu_3(t), \tag{3.13}$$

and the transversarity condition

$$\lim_{t\to\infty} e^{-\rho t}\mu_3(t)\,f(t)=0, \tag{3.14}$$

where μ_3 is the shadow price of the income.

From (3.12) and (3.13), the growth rate of consumption is given by

$$g_{c(t)}=\frac{1}{\sigma}(r(t)-\rho). \tag{3.15}$$

4 Steady State Equilibrium

The steady state is being focused on in this section again. Suppose that the government can set the tax rate so that the equation (2.16) satisfies. That is, the dynamic behavior of the pollution (relative to $Y(t)$, $K(t)$ and $C(t)$) is the same in both cases. If so, the relationship between g_Y and g_n is also represented as before.

To derive the growth rate of each variable, two equations (the labor market clearing condition and the no-arbitrage condition) will be used.

The labor market clearing condition is again given by (2.3). For the no-arbitrage condition, using (3.7), (3.10), (3.15), we can rewrite (3.9) as

$$\frac{1-\xi}{\xi}\varepsilon X=g_n+(\sigma-1)g_Y+\rho. \tag{4.1}$$

So, the growth rate attained by the market equilibrium, which is denoted by g_Y^d, is given by

$$g_Y^d=\left(\sigma-1+\frac{1}{1-\xi}(1+\sigma\Gamma)\right)^{-1}\left(\frac{1-\xi}{\xi}\varepsilon L-\rho\right). \tag{4.2}$$

Note that (2.15) can be rewritten as

$$g_Y^*=(\sigma-1+(1+\sigma\Gamma))^{-1}\left(\frac{1-\xi}{\xi}\varepsilon L-\rho\right). \tag{4.3}$$

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10 If $v(t)>\frac{1}{n(t)\varepsilon}w(t)$, firms must employ as much labor as possible. However, this situation never occurs in the equilibrium. On the contrary, if $v(t)<\frac{1}{n(t)\varepsilon}w(t)$, firms must choose $\dot{n}(t)=0$ for their optimization.

11 We assume that the government distributes the tax revenue to the households.

Only difference between these two growth rates is the term $\frac{1}{1-\xi}$ in the denominator. Note that this reflects the market distortions.[12]

5 Industiral Policies

In this section, a policy that corrects the market distortions will be described. Even in the previous section, the government already exercises power by taking tax from firms producing final goods. However, it is not enough to achieve the social optimum. It means that the government should use (at least) two measures to attain the socially optimal growth rate.[13]

Suppose that the government pays a fraction ψ of the research cost. If such a policy being carried out, free-entry conditions becomes

$$\varepsilon nv = w(1-\psi). \tag{5.1}$$

Then, the no-arbitrage condition becomes

$$\frac{\frac{1-\xi}{\xi}\varepsilon X}{1-\psi} = g_n + (\sigma-1)\, g_Y + \rho. \tag{5.2}$$

The labor market clearing condition and the relationship between g_n and g_Y are as before. By a simple calculation, an optimal subsidy rate (which is denoted by ψ^*) to achieve g_Y^* can be shown as

$$\psi^* = \frac{g_n^*}{g_n^* + (\sigma-1)\, g_Y^* + \rho}. \tag{5.3}$$

If the R & D policy described above is carried out appropriately, various variables grow at the socially optimum rate.

Next, we shall argue the effects of a subsidy to the intermediate goods sector. Let ψ_x denote the ad calorem rate of subsidy so that the firm can receive $p(1+\psi_x)$ for every unit of intermediate good which they sell. In this case, the profit function of firm i becomes

$$\pi_i = p_i(1+\psi_x)\, x_i - w x_i. \tag{5.4}$$

Firms maximize their profit by setting p_i as

$$p_i = p = \frac{w}{\xi(1+\psi_x)}. \tag{5.5}$$

Since $pnx = \frac{(1-\alpha)(1-\beta)}{\beta} Y$, x and $\frac{\pi}{v}$ are

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12 It is also interesting to compare these growth rates with that in the economy which does not value the environment at all, that is, $B=0$. In that case, $z(t)$ is always equal to unity and the growth rates of the social optimum (g_Y^{**}), and that of the market economy (g_Y^{dd}), are $\frac{1}{\sigma}\left(\frac{1-\xi}{\xi}\varepsilon L-\rho\right)$ and $\left(\sigma+\frac{\xi}{1-\xi}\right)^{-1}\left(\frac{1-\xi}{\xi}\varepsilon L-\rho\right)$ respectively. It is easy to show $g_Y^* < g_Y^{dd} < g_Y^{**}$.

13 Note that this result is consistent with that indicated by Timbergen.

$$x=\frac{\frac{(1-\alpha)(\beta-1)}{\beta}\xi(1+\psi_x)Y}{nw}, \tag{5.6}$$

$$\frac{\pi}{v}=\frac{\frac{(1-\alpha)(\beta-1)}{\beta}(1-\xi)(1+\psi_x)\varepsilon Y}{w}, \tag{5.7}$$

respectively. Although, w is equal to $\frac{\frac{(1-\alpha)(\beta-1)}{\beta}\xi(1+\psi_x)\varepsilon Y}{X}$, $\frac{\pi}{v}$ can be rewritten as

$$\frac{\pi}{v}=\frac{(1-\xi)\varepsilon X}{\xi}. \tag{5.8}$$

That is, a subsidy to the intermediate goods does not have any effect on the no-arbitrage condition and growth can not be promoted by this policy.

6 Concluding Remarks

In this paper, environmental problems, especially problems of pollution, are incorporated into a simple endogenous growth model to consider the sustainability of growth. First, a socially optimal growth rate and the dynamic behavior of pollution are derived. One of the most important results is that the pollution increases over time at first, but if the economy develops substantially, it decreases over time because energy-saving activity becomes more important.

Then, we consider whether or not sustainable growth is attained in a decentralized economy. In a decentralized economy, the private incentives of the entrepreneurs to engage in research activities become the engine of the growth. However, each innovator does not internalize his or her contribution to the knowledge capital, and as this causes the failure of the market, the growth rate in a market economy tends to be lower than that in the centralized economy.

Lastly, the policy which makes the market economy grow at the socially optimal rate is derived. Since the growth rate in a market economy is too low, a policy that promotes research activities is necessary.

The results here suggest several areas for further research. One simple idea is that innovation is interpreted so as to improve the quality of the goods rather than to expand the number of the goods. It may be also interesting to extend this model to the framework of the international economy. We can consider the effects of trade, or pollution which spill over across national boundaries. Another idea that might be interesting is to introduce Joint Implementation, in which a developed country and a less developed country cooperate to reduce pollution.

In addition, one may think that if we focus on the relationship between growth and the environment, innovation that aims to reduce pollution should be introduced. The problem of whether or not entrepreneurs in the economy have enough incentive to engage in environmental R & D must be considered.

7 Appendix

7.1 Derivation of the Growth Rate

Here, the growth rates of various variables are derived. Since $Y(t)$, $K(t)$ and $C(t)$ grow at the same rate in the steady state, (2.8), (2.9) and (2.10) imply

$$g_Y=-\frac{1}{\sigma}g_{\mu_1}, \tag{7.1}$$

$$g_z=\frac{1}{\beta-1}g_{\mu_1}=-\frac{\sigma}{\beta-1}g_Y, \tag{7.2}$$

$$g_Y+g_{\mu_1}-g_n-g_{\mu_2}=0. \tag{7.3}$$

Since, $Y=AK^{\alpha}n^{\frac{1-\alpha}{\xi}}\left(\frac{X}{n}\right)^{1-\alpha}z$, equation (7.1) and (7.2) can be used to derive

$$g_n=\frac{\xi}{1-\xi}(1+\sigma\Gamma)\,g_Y. \tag{7.4}$$

From (7.1), (7.3), and (7.4), g_{μ_2} becomes

$$g_{\mu_2}=\left(1-\sigma-\frac{\xi}{1-\xi}(1+\sigma\Gamma)\right)g_Y. \tag{7.5}$$

Although, from (2.6), (2.10), (2.11) and (7.4), g_{μ_2} is also represented as

$$g_{\mu_2}=\rho-\frac{1-\xi}{\xi}\varepsilon L+\left(\frac{1}{\xi}-2\right)\frac{\xi}{1-\xi}(1+\sigma\Gamma)\,g_Y. \tag{7.6}$$

If we use (7.5) and (7.6), the growth rate can be shown as

$$g_Y^*=\frac{1}{\sigma}(1+\Gamma)^{-1}\left(\frac{1-\xi}{\xi}\varepsilon L-\rho\right). \tag{7.7}$$

For the dynamic behavior of pollution, (2.4), (7.2) and (7.4) imply

$$g_D^*=(1-\sigma)\,g_Y^*. \tag{7.8}$$

7.2 Stability of the Steady State

Here, the stability of the steady state is argued. The production function (2.1) implies

$$g_Y=\alpha g_K+\frac{1-\alpha}{\xi}g_n+(1-\alpha)\,g_x+\frac{1}{\beta-1}g_{\mu_1}. \tag{7.9}$$

Using (7.3) and (7.9), we can show

$$g_x=g_K+\frac{1-\alpha-2\xi}{\alpha\xi}g_n+\frac{\beta}{\alpha(\beta-1)}g_{\mu_1}-\frac{1}{\alpha}g_{\mu_2}, \tag{7.10}$$

$$g_Y=g_K+\frac{(1-\alpha)(1-2\xi)}{\alpha\xi}g_n+\left(\frac{\beta}{\alpha(\beta-1)}-1\right)g_{\mu_1}-\frac{\alpha-1}{\alpha}g_{\mu_2}. \tag{7.11}$$

Let us define

$$\frac{Y}{K}\equiv M_1,\ \frac{C}{K}\equiv M_2,\ \frac{Y\mu_1}{n\mu_2}\equiv M_3,$$

and

$$m_i \equiv \log \frac{M_i}{M_{iss}},\ (i=1, 2, 3),$$

where M_{iss} is a steady state value of M_i. From (2.5), (2.6), (2.11) and (2.12), the log-linear approximation to the law of the notion is written by

$$\begin{pmatrix} \dot{m}_1 \\ \dot{m}_2 \\ \dot{m}_3 \end{pmatrix} \approx \begin{pmatrix} \left(\frac{\alpha(\beta-1)}{\beta}-1\right) M_{1ss} & 0 & \frac{(1-\alpha)^2(\beta-1)(\xi-1)}{\alpha\beta\xi} M_{3ss} \\ \left(\frac{\alpha(\beta-1)}{\sigma\beta}-1\right) M_{1ss} & M_{2ss} & 0 \\ 0 & -M_{2ss} & \frac{(1-\alpha)(\beta-1)(2-\alpha+\alpha\xi)}{\alpha\beta\xi} M_{3ss} \end{pmatrix} \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix}.$$

The characteristic roots are solutions to

$$-x^3+\nu_1 x^2+\nu_2 x+\nu_3=0, \tag{7.12}$$

where

$$\nu_1 \equiv \frac{(1-\alpha)(\beta-1)(2-\alpha+\alpha\xi)}{\alpha\beta\xi} M_{3ss}+\left(\frac{\alpha(\beta-1)}{\beta}-1\right) M_{1ss}+M_{2ss},$$

$$\nu_2 \equiv \left(1-\frac{\alpha(\beta-1)(1+M_{2ss})}{\beta}\right) M_{1ss}-M_{2ss},$$

$$\nu_3 \equiv M_{1ss}\, M_{2ss}\, M_{3ss} \frac{(1-\alpha)(\beta-1)}{\alpha\beta\xi}\left[\left(\frac{\alpha(\beta-1)}{\beta}-1\right)(2-\alpha+\alpha\xi)+(1-\alpha)(1-\xi)\left(\frac{\alpha(\beta-1)}{\sigma\beta}-1\right)\right].$$

Suppose that $\sigma>1$. Then $\nu_3<0$ and this equation has at least one negative root. This result implies the steady state is (at least) locally saddle-path stable. If all three roots are negative, then it becomes stable.

7.3 Demand Function of the Intermediate Goods Sector

Here, the deviation of (3.4) is examined. Since the producers maximize their profits, they must maximize

$$\left(\int_0^n x_i^{\xi}\, di\right)^{\frac{1-\alpha}{\xi}}, \tag{7.13}$$

subject to the constraint $\int_0^n p_i x_i di \le E_x$, where E_x is the expenditure on the intermediate goods. Since $\frac{1-\alpha}{\xi}>0$, this problem becomes to maximize

$$\int_0^n x_i^{\xi}\, di, \tag{7.14}$$

subject to the constraint. So from the Eular equation,

$$\xi x_i^{\xi-1} - \lambda p_i=0, \tag{7.15}$$

where λ is Lagrange multiplier. That is, $x_i=\left(\frac{\lambda p_i}{\xi}\right)^{\frac{1}{\xi-1}}$ and the constraint becomes

$$\frac{\lambda}{\xi}\int_0^n p_i^{\frac{\xi}{\xi-1}}\, di=E_x. \tag{7.16}$$

If (7.15) and (7.16) are combined, the relationship between p_i and x_i is given by (3.4).

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