Congestion, Adjustment Costs, and Public Policy in Small Open Economy

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Congestion, Adjustment Costs, and Public Policy in Small Open Economy*

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1. INTRODUCTION

The AK technology has been the most simple way to deal with the issue of endogenous growth and many researchers have applied this technology to practice various economic analyses. Especially, the AK model provides an instructive framework for analyzing fiscal policy regarding GDP growth. The relationship between fiscal policy and GDP growth is supported empirically. Grier and Tullock (1989) and Barro (1991) found that there was a negative relationship between government expenditure and GDP growth. Barro (1990) also developed a model of endogenous growth in which a government uses tax revenues to finance government expenditure and this expenditure enters the production function as a productive government expenditure. In our model, we consider more realistic features of public goods that are subject to congestion as Barro and Sala-i-Martin (1992), (1995) have argued. This is important when we discuss tax policies because the degree of congestion turns out to be a critical determinant of optimal tax policy. Then, we introduce Judd (1985) type of the production function which involves the degree of congestion. In addition, the assumption that the government spending-GDP ratio is set arbitrary constant transforms the production function in our model into the AK technology type.

We also introduce adjustment costs into the model. The costs of adjustment give some important features of the growth rate of economic variables as Turnovsky (1995), (1996) has mentioned. The rate of the market value to the replacement costs of capital is known as Tobin's q (1969). The q theory tells us that if a firm exhibits a q value of less than one, then there is no incentive for the firm to invest the margin. Conversely, a q value greater than one, indicates a firm has more than a dollar to gain from a dollar worth of capital investment, indicating an incentive for the firm to invest. In our model, we show that an introduction of such costs is a more useful tool to analyze the issue of tax policy in the modified Ramsey model. We assume that our model has constant returns in adjustment costs. That is, doubling both investment and

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capital doubles adjustment costs. Constant returns can be introduced by assuming that adjustment costs take the form of Hayashi (1982). There are two main reasons that we introduce Hayashi type of adjustment costs. First, if the steady-growth equilibrium having ongoing growth is to be sustained, it is necessary that the costs of adjustment have constant returns in both investment and capital. We see that the shadow value of capital with the shadow value of foreign capital determines the growth rate of capital. Second, a well known result, average q is a good proxy for marginal q, enables us to estimate marginal q empirically. This result is due to the assumption of constant returns in the costs of adjustment. Though we do not analyze the model empirically here, we assume his type not only for the sustainable ongoing growth of capital but also for feature empirical research.

Further, the accumulation of foreign bonds is introduced to expand our model into an open economy. The introduction of a perfect foreign bond market with the assumption regarding government expenditure policy turns out to characterize the growth of consumption or total wealth being different from that of capital.

In this paper, we may imagine our economy as a small open economy with positive initial capital stock. For simplicity, initial foreign bond holdings are assumed to be zero and the accumulation of foreign debt is prohibited. It is conceivable that the accumulation of foreign debt is not permitted for a small country if we assume that the country is too young to gain credit in the world economy, and therefore, no foreign countries give a credit loan of foreign bonds. The economy is small in the sense that the world interest rate is exogenously given because the young country is too small to affect the world interest rate through bulk buying of foreign bonds. Then, we see that the rate of time preference for the representative agent in that country must be sufficiently small in order to prevent ever increasing foreign debt. That is, people in that country must be sufficiently patient so that the country can support the growth of consumption without foreign borrowing.

This paper is organized as follows: Section 2 briefly outlines some notations and structures in our model. In section 3, we present the model in a centralized economy; in subsection 3.1 we construct a social planner's instantaneous budget constraint and in subsection 3.2 we solve an optimization problem assuming that the government sets its expenditure rule arbitrary. Subsection 3.3 focuses on the steady growth equilibrium and subsection 3.4 discusses the case in which the government sets its expenditure optimally. In Subsection 3.5, we analyze the effect of a change in government expenditure-GDP ratio on the growth rate of the economy. We consider a decentralized economy corresponding to the centralized economy in Section 4. Section 5 is devoted to explore how changes in producion tax, foreign bond income tax, and consumption tax affect the growth rate of the economy and to find the optimal tax policy that leads the decentralized economy to coincide with the centralized economy. A final section provides some conclusions and discussions.

2. NOTATIONS AND STRUCTURES OF THE MODEL

2.1 *Economic Variables*

The summary of notations regarding economic variables are as follows.

$Y(t)[y(t)]$: aggregate [private] output at time t.

$K(t)[k(t)]$: aggregate [private] capital at time t.

$B(t)[b(t)]$: aggregate [private] foreign bond holdings at time t.

$W(t)[w(t)]$: aggregate [private] wealth at time t.

$I(t)[i(t)]$: aggregate [private] investment at time t.

$c(t)$: private consumption at time t.

With all agents being identical, aggregate economic variables and private ones are related by $y(t)N=Y(t)$, $k(t)N=K(t)$, $b(t)N=B(t)$, $w(t)N=W(t)$, *and* $i(t)N=I(t)$.

2.2 *Households*

Consider the economy that is populated by identical representative agents who consume a consumption good and produce a single commodity. That is, the household and production sectors are consolidated. The agents also accumulate foreign bonds. In our model, the economy is open and small in the sense that the agent accumulates foreign bonds that pay an exogenously given world interest rate.[1] Further, the economy is small enough to maintain growth rates of economic variables that are unrelated to those in the rest of world. For simplicity, we assume that initial holdings of foreign bonds by each agent are zero. That is $b(0)N=B(0)=0$. The accumulation of foreign debt is not permitted since the country has not yet gained credit in the world economy, and therefore, no foreign countries give a credit loan of their bonds. There is no population growth or N is a constant. The identical representative agent's intertemporal utility expressed by the isoelastic utility function is denoted by

$$Z=\int_0^\infty \frac{c(t)^{1-\sigma}-1}{1-\sigma}e^{-\beta t}dt,\ \sigma>0,\ \sigma\neq 1,$$

where β is a positive time discount rate, σ is an inverse of the elasticity of intertemporal substitution. We note that when $\sigma=1$, the utility function is logarithmic.

1) Siegel (1992) reports that the real return on bonds in the U.S. was higher in the 1800's than that in the 1900's. The sample in the 1800's consists of the bond portfolio such as municipal and utility bonds whereas the sample in the 1990's consists of the Treasury bills. However, it is conceivable that this variation may reflect the fact that the bond portfolio is much riskier than the Treasury bills securities. He also reports that the real rate of return on the U.S. Stock Market has remarkably little variation in the sample periods.

2.3 *Production and Costs of Adjustment*

The production function for each agent is Judd (1985) type which involves a state of congestion of public facilities. The private production function is given by

$$y(t)=Ak(t)\left(\frac{G(t)}{K(t)}\right)^{\delta},\ 0<\delta<1,$$

where δ is the degree of congestion, A is exogenously given productivity, and $G(t)$ is the public facilities or the productive government expenditure at time t. The production function describes a state of congestion in the sense that the expansion of private capital, and therefore, private output leads to the expansion of aggregate capital, thereby deteriorating the government service for other agents for given G. On the other hand, the investment in our economy involves the costs of adjustment. Then we assume the Hayashi (1982) type costs of investment;

$$\Phi(I(t), K(t))=I(t)\left(1+\phi\left(\frac{I(t)}{K(t)}\right)\right),$$

where $\phi(I/K)I$ is adjustment costs with properties of $\phi(0)=0$, $\phi'(I/K)>0$, and $\phi''(I/K)\geq 0$. Thus we have the costs of adjustment with constant returns in both investment and capital. That is, doubling both I and K doubles the costs of adjustment. In this paper, for simplicity, $\phi(I/K)$ is given by more explicit form,

$$\phi\left(\frac{I(t)}{K(t)}\right)=\frac{h}{2}\frac{I(t)}{K(t)},$$

where h is a positive constant. In the same fashion, private costs of investment are given by

$$i(t)\left(1+\frac{h}{2}\frac{i(t)}{k(t)}\right).$$

2.4 *The Government*

We assume that the government always runs a balanced budget, or the tax revenue is spent to finance the supply of the productive government expenditure. In a centralized economy, we regard the government as a social planner.

3. THE MODEL OF A CENTRALIZED ECONOMY

3.1 *An instantaneous budget constraint*

We begin by constructing a social planner's instantaneous budget constraint. Since the social planner takes the relation, $y(t)N=Y(t)$ and $k(t)N=K(t)$ into account when determining his decisions, the private agent's production function can be converted into the following aggregate production function

$$Y(t)=AK(t)^{1-\delta}G(t)^{\delta}. \tag{c.1}$$

Then the social planner's instantaneous budget constraint is given by

$$\dot{B}(t)=AK(t)^{1-\delta}G(t)^{\delta}+rB(t)-I(t)\left(1+\frac{h}{2}\frac{I(t)}{K(t)}\right)-G(t)-Nc(t) \text{ where } I(t)=\dot{K}(t).^{2)} \quad \text{(c.2)}$$

In order to sustain an equilibrium with steady growth, $G(t)$ cannot be fixed at some exogenous level, but rather must be linked to the scale of the economy in some way. Then we assume the social planner ties his expenditure to the aggregate output (Gross Domestic Product, GDP) in accordance with the rule,

$$\frac{G(t)}{Y(t)}=g, \quad 0<g<1, \quad \text{(c.3)}$$

where g is a time invariant. The assumption of g being a time invariant can be supported empirically. Table 1 indicates that the real government share of GDP in the U.S., Canada, West Germany, Switzerland, and Australia has fluctuated remarkably little for the past 40 years.

As far as the behavior of a social planner is concerned, we assume two possible cases. One is that the social planner sets g arbitrary. The other is that the social planner sets g optimally. We consider these two cases because, whether the social planner sets g arbitrary or optimally depends upon his or her policy. Japan used to hold fast to the policy that the expense for the Self-Defense Forces were limited within the 1% of GNP. This policy is not a result of the Japanese government setting g optimally. Rather, the Japanese government tried to spread propaganda for the renunciation of war over the world community. Thus, we may regard this policy as the government setting g arbitrary. In reality, we believe that these two policies are intermingled.

Table 1 Real Government Share of GDP (G/Y), (%)

Period	U.S.A.	Canada	W. Germany	Switzerland	Australia
1950-1956	12.51	13.53	16.09	8.64	11.26
1957-1963	13.04	12.83	14.56	8.06	10.26
1964-1970	14.34	13.11	14.80	8.07	10.80
1971-1977	13.90	13.43	15.06	8.39	11.34
1978-1984	13.26	12.86	15.59	8.97	12.40
1985-1991	13.20	12.03	15.00	9.31	12.73

Source: Penn-World Tables, Mark 5.6

3.2 *The social planner sets g arbitrary*

When the social planner sets g arbitrary, by substituting eq.(c.3) into eq.(c.1), we may rewrite eq.(c.1) as

$$Y(t)=A^{\frac{1}{1-\delta}}g^{\frac{\delta}{1-\delta}}K(t), \quad \text{(c.4)}$$

2) For implicitly, we assume that capital does not depreciate.

which is considered to be the modified aggregate production function under the assumption of g being set arbitrary. Thus eq.(c.2), eq.(c.2) can be rewritten as

$$\dot{B}(t)=A^{\frac{1}{1-\delta}}g^{\frac{\delta}{1-\delta}}(1-g)K(t)+rB(t)-I(t)\left(1+\frac{h}{2}\frac{I(t)}{K(t)}\right)-Nc(t)\text{where}I(t)=\dot{K}(t). \qquad \text{(c.3')}$$

Then the social planner chooses c, K, B, and I to maximize Z subject to eq.(c.3'), initial capital stock, and initial foreign bond holdings. Thus we may set up the current value Hamiltonian,

$$H_c(K, B, c, \lambda, \eta, t)=\frac{c^{1-\sigma}-1}{1-\sigma}+\lambda\left[A^{\frac{1}{1-\delta}}g^{\frac{\delta}{1-\delta}}(1-g)K+rB-I\left(1+\frac{h}{2}\frac{I}{K}\right)-Nc\right]+\eta I,$$

where λ and η[3] are shadow values of foreign bonds and that of capital, respectively. Then the necessary conditions derived from "Maximum Principle" should involve,

$$c^{-\sigma}=N\lambda, \qquad \text{(c.5a)}$$

$$-\dot{\eta}+\eta\beta=\left[A^{\frac{1}{1-\delta}}g^{\frac{\delta}{1-\delta}}(1-g)+\frac{h}{2}\left(\frac{I}{K}\right)^2\right]\lambda, \qquad \text{(c.5b)}$$

$$-\dot{\lambda}+\lambda\beta=r\lambda, \qquad \text{(c.5c)}$$

$$\frac{I}{K}=\frac{\dot{K}}{K}=\frac{(\eta/\lambda)-1}{h}, \qquad \text{(c.5d)}$$

and eq.(c.3'). To ensure that the social planner's budget constraint is met, the optimal conditions for the centralized problem also involve the transversality conditions,

$$\lim_{t\to\infty}\lambda(t)B(t)e^{-\beta t}=0 \qquad \text{(c.5e)}$$

and

$$\lim_{t\to\infty}\eta(t)K(t)e^{-\beta t}=0. \qquad \text{(c.5f)}$$

Let $q\equiv\eta/\lambda$ be defined as the shadow value of capital in terms of the shadow value of foreign bonds. Then we may rewrite eq.(c.5d) as

$$\frac{\dot{K}}{K}=\frac{q-1}{h}. \qquad \text{(c.5d')}$$

We take the time derivative of q in order to obtain,

$$\frac{\dot{q}}{q}=\frac{\dot{\eta}}{\eta}-\frac{\dot{\lambda}}{\lambda}. \qquad \text{(c.6)}$$

Combining eq.(c.6) with eqs.(c.5b) and (c.5c) yields

$$\frac{\dot{q}}{q}+\frac{A^{\frac{1}{1-\delta}}g^{\frac{\delta}{1-\delta}}(1-g)}{q}+\frac{(q-1)^2}{2hq}=r. \qquad \text{(c.7)}$$

Eq.(c.7) states that the rate of capital gain plus the marginal product of capital, deflated by q, plus the marginal reduction in adjustment costs, deflated by q, equal to the exogenously given rate of return on foreign bonds. In other words, the rate of return on domestic capital must be set such that it is indifferent between investing in domestic capital and investing in foreign bonds. In absence of adjustment costs or in eq.(c.3'), eq.(c.7) reduces to

3) In this paper, η is also considered to be marginal q.

$$A^{\frac{1}{1-\delta}} g^{\frac{\delta}{1-\delta}}(1-g)=r,$$

which says that marginal product of capital with no costs of adjustment should be equal to the rate of return on foreign bonds.

3.3 *Steady-growth equilibrium*

In characterizing the steady state of the model, we begin by defining the steady-growth equilibrium. The steady-growth equilibrium is the state where all economic variables grow at positive constant rates and the possibility of different economic variables to grow at different rates is allowed. From now on, we assume the existence of the steady-growth equilibrium and focus on such an equilibrium. First of all, let the constant growth rate of capital be denoted by γ_c. Then there must be a stationary state of q which satisfies $\dot{q}=0$ so that the stationary state ensures that capital grows at a constant rate, γ_c. This can be seen by taking the time derivative of $\gamma_c=(q-1)/h$. Setting $\dot{q}=0$ in eq.(c.7) implies that the stationary state of q should be a solution to the following quadratic equation,

$$rq-\frac{(q-1)^2}{2h}-A^{\frac{1}{1-\delta}} g^{\frac{\delta}{1-\delta}}(1-g)=0. \tag{c.8}$$

Denote the stationary state of q by $\tilde{q}$. Then the candidates for $\tilde{q}$ are

$$\tilde{q}_a=(hr+1)+\sqrt{(hr+1)^2-\left[2hA^{\frac{1}{1-\delta}} g^{\frac{\delta}{1-\delta}}(1-g)+1\right]} \tag{c.9a}$$

and

$$\tilde{q}_b=(hr+1)-\sqrt{(hr+1)^2-\left[2hA^{\frac{1}{1-\delta}} g^{\frac{\delta}{1-\delta}}(1-g)+1\right]}. \tag{c.9b}$$

In order for eqs.(c.9a) and (c.9b) to be real values, insides of the roots of eqs.(c.9a) and (c.9b) must be a positive or zero. That is, $(hr+1)^2 \geq \left[2hA^{\frac{1}{1-\delta}} g^{\frac{\delta}{1-\delta}}(1-g)+1\right]$ or

$$A^{\frac{1}{1-\delta}} g^{\frac{\delta}{1-\delta}}(1-g) \leqq r\left(1+\frac{hr}{2}\right) \tag{c.10}$$

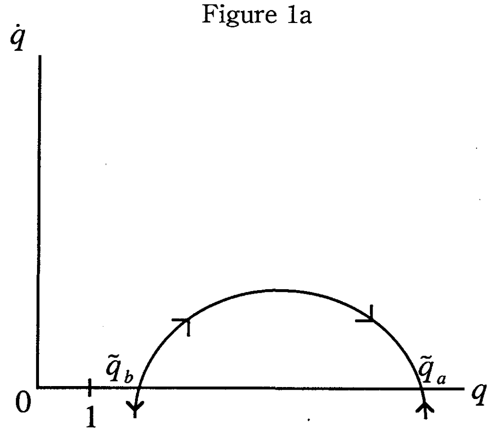
must hold. Then eq.(c.10) implies the following two possible cases.

(case 1):

$$r<A^{\frac{1}{1-\delta}} g^{\frac{\delta}{1-\delta}}(1-g).$$

In this case eqs.(c.9a) and (c.9b) imply that $\tilde{q}_a>\tilde{q}_b>1$. As figure 1a shows $\tilde{q}_a$ is locally stable. That is, if we take the neighborhood of $\tilde{q}_a$ to be anywhere to the right of the stationary state, $\tilde{q}_a$ then q which starts from its initial value will converge to $\tilde{q}_a$. On the other hand, $\tilde{q}_b$ is an unstable stationary state value.

Figure 1a

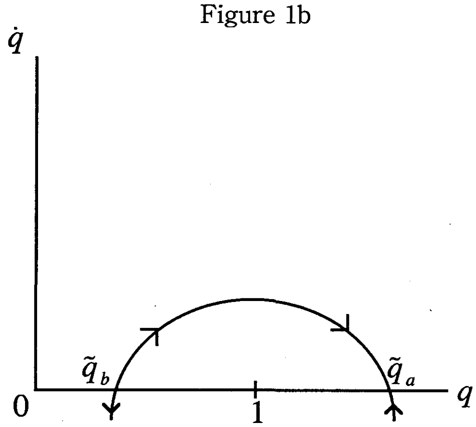


(case 2):

$$r > A^{\frac{1}{1-\delta}} g^{\frac{\delta}{1-\delta}}(1-g).$$

In this case eqs.(c.9a) and (c.9b) imply that $\tilde{q}_a > 1 > \tilde{q}_b > 0$. Figure 1b shows this second case. In the same fashion as case 1, $\tilde{q}_a$ is locally stable and $\tilde{q}_b$ is unstable.

Figure 1b



However, the transversality condition that is given by eq.(c.5f) eliminates the locally stable stationary state value, $\tilde{q}_a$. To see this, we substitute the solutions to eqs.(c.5c) and (c.5d) into the transversality condition. The solutions to eqs.(c.5c) and (c.5d) are denoted by

$$\lambda(t) = \lambda(0)e^{(\beta-r)t}$$

and

$$K(t) = K(0)exp\left[\int_0^t \frac{q(s)-1}{h} ds\right],$$

respectively. Substituting these solutions into eq.(c.5f) remembering $\eta = q\lambda$ yields

$$\lim_{t\to\infty} q(t)\lambda(t)K(t)e^{-\beta t}=\lim_{t\to\infty} q(0)\lambda(0)K(0)exp\left[\sqrt{h^2r^2+2hr-2hA^{\frac{1}{1-\delta}}g^{\frac{\delta}{1-\delta}}(1-g)}\,t\right]=0. \qquad \text{(c.5f')}$$

Eq. (c.10) implies that the inside of the root in eq.(c.5f') is a positived, and therefore, $\tilde{q}_a$ violates the transversality condition. Thus, we may conclude that our choice of the stationary state value is $\tilde{q}_b$. Consequently, (case 2) is eliminated since, by definition, the growth rate of capital is a positive, and therefore, the stationary state value of q must be greater than one as is clear from eq.(c.5d'). Hence, we may summarize our findings concerning the growth rate of capital in steady growth equilibrium.

Proposition 1

In order for capital to be in steady-growth equilibrium, our choice of the stationary state of q must be $\tilde{q}_b$ given by eq.(c.9b) that is a real value and is consistent with the transversality condition. The corresponding growth rate of capital is expressed by

$$\frac{\dot{K}}{K}=\frac{\tilde{q}_b-1}{h}=\gamma_c. \qquad \text{(c.5d'')}$$

Furthermore, by definition, the growth rate of capital is a positive, and therefore,

$$r<A^{\frac{1}{1-\delta}}g^{\frac{\delta}{1-\delta}}(1-g)$$

or return on foreign bonds must be less than the marginal product of capital. Consequently, we have the condition,

$$r<A^{\frac{1}{1-\delta}}g^{\frac{\delta}{1-\delta}}(1-g)\leqq r\left(1+\frac{hr}{2}\right).$$

Based on the definition of steady-growth equilibrium, denote the growth rates of personal consumption and aggregate total wealth by Ψ_{cc} and Ψ_{cw}, respectively. Then, taking the time derivative of condition (c.5a) yields $-\sigma\dot{c}/c=\dot{\lambda}/\lambda$. By substituting this into condition (c.5c), we obtain

$$\frac{\dot{c}(t)}{c(t)}=\frac{r-\beta}{\sigma}=\Psi_{cc}. \qquad \text{(c.11)}$$

We now define the aggregate total wealth at time t as

$$W(t)\equiv B(t)+q(t)K(t). \qquad \text{(c.12)}$$

Then, by taking the time derivative of eq.(c.12), we get

$$\dot{W}(t)=\dot{B}(t)+\dot{q}(t)K(t)+q(t)\dot{K}(t). \qquad \text{(c.12')}$$

Substituting eqs.(c.3'), (c.5d'), and (c.7) into eq.(c.12') yields

$$\dot{W}(t)=rW(t)-Nc(t). \qquad \text{(c.12'')}$$

Then, by definition,

$$\frac{\dot{W}(t)}{W(t)}=r-\frac{Nc(t)}{W(t)}=\Psi_{cw}. \qquad \text{(c.13)}$$

Eq.(c.13) implies that $Nc(t)/W(t)$ must be a constant, say μ_c, and therefore,

$$\frac{\dot{c}(t)}{c(t)}=\Psi_{cc}=\frac{r-\beta}{\sigma}=\Psi_{cw}=\frac{\dot{W}(t)}{W(t)}=\Psi_c. \tag{c.14}$$

Thus, we combine eqs.(c.13) and (c.14) in order to obtain,

$$\frac{Nc(t)}{W(t)}=\frac{\beta-(1-\sigma)r}{\sigma}=\mu_c. \tag{c.15}$$

We note that the transversality condition guarantees that μ_c is a positive constant. For proof see Appendix I.

Finally, we need to examine the feature of the accumulation of foreign bond holdings. First, we now know that the solutions to eqs.(c.5d") and (c.11) are denoted by

$$K(t)=K(0)e^{\gamma_c t} \tag{c.16}$$

and

$$c(t)=c(0)e^{\Psi_c t}, \tag{c.17}$$

respectively. We also know that

$$I(t)=\gamma_c K(t)=\gamma_c K(0)e^{\gamma_c t}. \tag{c.18}$$

Then, substituting eqs.(c.16), (c.17), and (c.18) into (c.3') yields

$$\dot{B}(t)=rB(t)+\left[A^{\frac{1}{1-\delta}}g^{\frac{\delta}{1-\delta}}(1-g)-\gamma_c\left(1+\frac{h}{2}\gamma_c\right)\right]K(0)e^{\gamma_c t}-Nc(0)e^{\Psi_c t}. \tag{c.3"}$$

However, eq.(c.8) implies that eq.(c.3") reduces to

$$\dot{B}(t)=rB(t)+(r-\gamma_c)\tilde{q}_b K(0)e^{\gamma_c t}-Nc(0)e^{\Psi_c t}. \tag{c.19}$$

Then, the solution to eq.(c.19) is given by

$$B(t)=e^{rt}\left\{B(0)+\int_0^t(r-\gamma_c)\tilde{q}_b K(0)e^{(\gamma_c-r)t}-Nc(0)e^{(\Psi_c-r)t}dt\right\}. \tag{c.20}$$

Remembering $B(0)=0$, eq.(c.20) can be rewritten as

$$B(t)=\left[\tilde{q}_b K(0)+\frac{Nc(0)}{\Psi_c-r}\right]e^{rt}-\tilde{q}_b K(0)e^{\gamma_c t}-\left(\frac{Nc(0)}{\Psi_c-r}\right)e^{\Psi_c t}. \tag{c.21}$$

Substituting eq.(c.21) and $\lambda(t)=\lambda(0)e^{(\beta-r)t}$ into the transversality condition, eq.(c.5e) yields

$$\lim_{t\to\infty}\lambda(0)\left\{\tilde{q}_b K(0)-\frac{Nc(0)}{r-\Psi_c}-\tilde{q}_b K(0)e^{(\gamma_c-r)t}+\frac{Nc(0)}{r-\Psi_c}e^{(\Psi_c-r)t}\right\}=0.$$

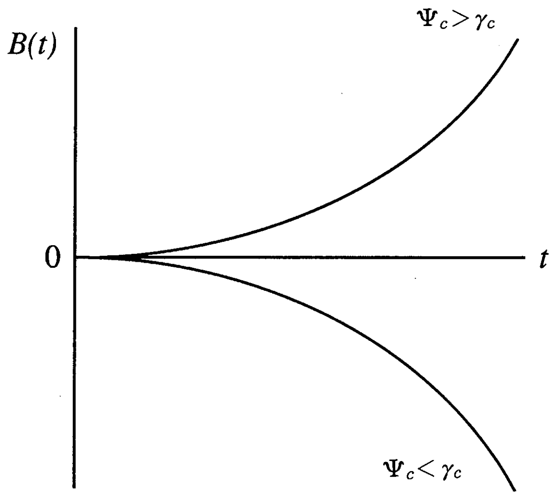
Thus, in order to satisfy the transversality condition, $\gamma_c<r$[4] ; (c.22a), $\Psi_c<r$; (c.22b), and $Nc(0)=(r-\Psi_c)\tilde{q}_b K(0)$; (c.22c) must hold. Then, substituting condition (c.22c) into eq.(c.21) gives

$$B(t)=\tilde{q}_b K(0)\left(e^{\Psi_c t}-e^{\gamma_c t}\right). \tag{c.23}$$

4) The condition, $\gamma_c<r$ is ensured by the stationary state, $\tilde{q}_b$.
Since $\tilde{q}_b=1+hr-\sqrt{(1+hr)^2-\left[1+2hA^{\frac{1}{1-\delta}}g^{\frac{\delta}{1-\delta}}(1-g)\right]}$, we have
$\gamma_c=\frac{\tilde{q}_b-1}{h}=r-\left\{\sqrt{(1+hr)^2-\left[1+2hA^{\frac{1}{1-\delta}}g^{\frac{\delta}{1-\delta}}(1-g)\right]}\right\}/h<r.$

Figure 3 shows the path of foreign bond holdings.

Figure 2



We see that the growth rate of aggregate foreign bond holdings asymptotically converges to the rate of either Ψ_c or γ_c in the steady-growth equilibrium. However, by definition, the growth rate of foreign bond holdings must converge to a positive constant rate. As figure 2 shows, if $\Psi_c < \gamma_c$, then the state of ever increasing foreign debts occurs. Therefore, the condition, $\Psi_c > \gamma_c$ needs to be held in order to eliminate such an unrealistic state and this condition implies that a rate of time preference needs to be small enough to satisfy

$$\beta < (1-\sigma)r + \frac{\sigma}{h}\sqrt{(1+hr)^2 - \left[1 + 2hA^{\frac{1}{1-\delta}} g^{\frac{\delta}{1-\delta}}(1-g)\right]}. \tag{c.24}$$

Then, we obtain proposition 2.

Proposition 2

In order to prevent ever increasing foreign debts, the agent's rate of time preference must be sufficiently small such that eq.(c.24) holds. In this case, the growth rate of aggragate foreign bond holdings asymptotically converses to the growth rate of aggregate total wealth (consumption) in the steady-growth equilibrium. Consequently, we have the condition, $\gamma_c < \Psi_c < r$.

It is now clear that, when condition (c.24) holds, the income from foreign bond holdings enables us to sustain the steady growth of consumption in excess of the steady growth of capital.

3.4 *The social planner sets g optimally*

When the social planner sets g optimally, he or she maximizes Z subject to eq.(c.2), initial capital stock, and initial bond holdings. Then, we set up the current value Hamiltonian,

$$H_c^{\circ}(K, B, c, \lambda, t)=\frac{c^{1-\sigma}-1}{1-\sigma}+\lambda\left[AK^{1-\delta}G^{\delta}+rB-I\left(1+\frac{h}{2}\frac{I}{K}\right)-Nc-G\right]+\eta I.$$

The necessary conditions for an optimum involve the repetitions of eqs.(c.5a), (c.5c), (c.5d), (c.5e), and (c.5f). Since the social planner chooses G at this time, it is straightforward to show that the optimal choice of g leads to the additional necessary condition, $\partial H_c^{\circ}/\partial G=0$. Then we have

$$\frac{G(t)}{K(t)}=(A\delta)^{\frac{1}{1-\delta}}. \tag{c.25}$$

Thus, the condition corresponding to eq.(c.5d) here is denoted by

$$-\dot{\eta}+\eta\beta=\left[A(1-\delta)K^{-\delta}G^{\delta}+\frac{h}{2}\left(\frac{I}{K}\right)^2\right]\lambda. \tag{c.26}$$

Then, substituting eq.(c.25) into eq.(c.26) yields

$$-\dot{\eta}+\eta\beta=\left[A^{\frac{1}{1-\delta}}\,\delta^{\frac{\delta}{1-\delta}}(1-\delta)+\frac{h}{2}\left(\frac{I}{K}\right)^2\right]\lambda. \tag{c.26'}$$

We combine eqs.(c.26') and (c.5c) in order to obtain

$$\frac{\dot{q}}{q}+\frac{A^{\frac{1}{1-\delta}}\,\delta^{\frac{\delta}{1-\delta}}(1-\delta)}{q}+\frac{(q-1)^2}{2hq}=r. \tag{c.27}$$

It is obvious, by the direct comparison between eqs.(c.7) and (c.27), that $g=\delta$ or government expenditure-GDP ratio equals the degree of congestion when the social planner chooses g optimally. Thus, by setting, $g=\delta$ propositions 1 and 2 automatically hold here. Empirically speaking, Table 1 tells us that the degree of congestion in the U.S. from 1985 to 1991 was 13.2%. On the other hand, the degree of congestion in Switzerland during that period was 9.31%.

3.5 *Impact of an increase in g on the equilibrium growth rate of capital*

We begin by rewriting eq.(c.8) in terms of the growth rate of aggregate capital, γ_c in steady-growth equilibrium. By substituting $q=\tilde{q}_b$ into eq.(c.8) and remembering $\gamma_c=(\tilde{q}_b-1)/h$, eq.(c.8) can be rewritten as

$$(\gamma_c h+1)r-\frac{\gamma_c^2 h}{2}-A^{\frac{1}{1-\delta}}\,g^{\frac{\delta}{1-\delta}}(1-g)=0. \tag{c.28}$$

Differentiating eq.(c.28) with respect to g yields

$$\frac{d\gamma_c}{dg}=\frac{A^{\frac{1}{1-\delta}}\,g^{\frac{2\delta-1}{1-\delta}}(\delta-g)}{h(1-\delta)(r-\gamma_c)}. \tag{c.29}$$

Since stationary value, $\tilde{q}_b$ guarantees that $\gamma_c<r$, the sign of eq.(c.29) depends upon the sign of $(\delta-g)$. Then, we may conclude that if $g<\delta$, then $d\gamma_c/dg>0$ and if $g>\delta$, then $d\gamma_c/dg<0$. When g is set optimally or $g=\delta$, $d\gamma_c/dg=0$. Since $\tilde{q}_b$ is concave, γ_d is also concave. Thus, we reconfirm that γ_d reaches its maximum value at $g=\delta$. Then, we obtain proposition 3.

Proposition 3

When the social planner sets g arbitrary, an increase in g lowers the growth rate of capital in steady-growth equilibrium as long as the government expenditure-GDP ratio is more than the degree of congestion. Conversely, an increase in g expands the growth rate of capital as long as the government expenditure-GDP ratio is less than the degree of congestion. When the social planner sets g optimally a change in g does not have any impact on the growth rate of capital.

4. THE MODEL OF A DECENTRALIZED ECONOMY

We turn now to analyze the model of a decentralized economy. Since the household and production sectors are consolidated, the representative agent chooses c, k, b, and i to maximize his intertemporal utility function, given by Z, subject to his wealth accumulation equation denoted by

$$\dot{b}(t)=(1-\tau_k)Ak(t)\left(\frac{G(t)}{K(t)}\right)^{\delta}+(1-\tau_b)rb(t)-i(t)\left(1+\frac{h}{2}\frac{i(t)}{k(t)}\right)-(1+\tau_c)c(t) \tag{d.1}$$

where $i(t)=\dot{k}(t)$,

initial holdings of private capital, and initial foreign bonds. Production tax, foreign bond income tax, and consumption tax rates are represented by τ_k, τ_b, and τ_c, respectively. We keep the assumption, $G(t)/Y(t)=g$ and the agent takes g as given when he solves his optimization problem. Then, we set up the current value Hamiltonian in the decentralized economy,

$$H_d(k, b, c, \lambda, \eta, t)=\frac{c^{1-\sigma}-1}{1-\sigma}+\lambda\left[(1-\tau_k)A\left(\frac{G}{K}\right)^{\delta}k+(1-\tau_b)rb-i\left(1+\frac{h}{2}\frac{i}{k}\right)-(1+\tau_c)c\right]+\eta i.$$

The necessary conditions for the optimum should involve the following,

$$c^{-\sigma}=(1+\tau_c)\lambda, \tag{d.2a}$$

$$-\dot{\eta}+\eta\beta=\left[(1-\tau_k)A\left(\frac{G}{K}\right)^{\delta}+\frac{h}{2}\left(\frac{i}{k}\right)^{2}\right]\lambda, \tag{d.2b}$$

$$-\dot{\lambda}+\lambda\beta=(1-\tau_b)r\lambda, \tag{d.2c}$$

$$\frac{i}{k}=\frac{\dot{k}}{k}=\frac{(\eta/\lambda)-1}{h}, \tag{d.2d}$$

and the transversality conditions denoted by

$$\lim_{t\to\infty}\lambda(t)b(t)e^{-\beta t}=0, \tag{d.2e}$$

and

$$\lim_{t\to\infty}\eta(t)k(t)e^{-\beta t}=0. \tag{d.2f}$$

In the world as seen by the representative agent, he thinks that he has no effect on aggregate output and aggregate capital. Therefore, he ignores the aggregate linkage between capital and productive government expenditure. Since we know that $G/Y=g$, we have $G/K=A^{\frac{1}{1-\delta}}g^{\frac{\delta}{1-\delta}}$. Then, eq.(d.2b) can be expressed as

$$-\dot{\eta}+\eta\beta=\left[(1-\tau_k)A^{\frac{1}{1-\delta}}g^{\frac{\delta}{1-\delta}}+\frac{h}{2}\left(\frac{i}{k}\right)^2\right]\lambda. \tag{d.3}$$

Combining eqs.(d.2b), (d.2c), and (d.2d) after substituting eq.(d.3') into eq.(d.2b) and remembering $q=\eta/\lambda$ yield

$$\frac{\dot{q}}{q}+\frac{(1-\tau_k)A^{\frac{1}{1-\delta}}g^{\frac{\delta}{1-\delta}}}{q}+\frac{(q-1)^2}{2hq}=(1-\tau_b)r. \tag{d.4}$$

We also rewrite eqs.(d.1) and (d.2d) as

$$\dot{b}(t)=(1-\tau_k)A^{\frac{1}{1-\delta}}g^{\frac{\delta}{1-\delta}}k(t)+(1-\tau_b)rb(t)-i(t)\left(1+\frac{h}{2}\frac{i(t)}{k(t)}\right)-(1+\tau_c)c(t) \tag{d.1'}$$

where $i(t)=\dot{k}(t)$ and

$$\frac{\dot{k}}{k}=\frac{q-1}{h}, \tag{d.2d'}$$

respectively.

We again assume the existence of steady-growth equilibrium and focus on such an equilibrium in a decentralized economy. The similar definition of steady-growth equilibrium in the centralized economy is applied here. We denote positive and constant growth rates of consumption, private total wealth, and private capital stock by Ψ_{dc}, Ψ_{dw}, and γ_d, respectively. On the other hand, in order to find the stationary state value of q, we follow the same procedures we carried in the centralized economy case. The quadratic equation corresponding to the eq.(c.8) in the decentralized economy is denoted by

$$(1-\tau_b)rq-\frac{(q-1)^2}{2h}-(1-\tau_k)A^{\frac{1}{1-\delta}}g^{\frac{\delta}{1-\delta}}=0. \tag{d.4'}$$

Then, we obtain an unstable stationary state value of q,

$$\tilde{q}^{\circ}_b=\left[1+(1-\tau_b)rh\right]-\sqrt{\left[1+(1-\tau_b)rh\right]^2-\left[2hA^{\frac{1}{1-\delta}}g^{\frac{\delta}{1-\delta}}(1-\tau_k)+1\right]}. \tag{d.5}$$

As a result, the condition given by

$$r(1-\tau_b)<(1-\tau_k)A^{\frac{1}{1-\delta}}g^{\frac{\delta}{1-\delta}}\leqq r(1-\tau_b)\left[1+\frac{hr(1-\tau_b)}{2}\right]$$

must hold in order to satisfy the transversality condition as well as a positive growth rate of private capital. Thus, we may rewrite eq.(d.2d') as

$$\frac{\dot{k}}{k}=\frac{\dot{K}}{K}=\frac{\tilde{q}^{\circ}_b-1}{h}=\gamma_d. \tag{d.2d''}$$

On the other hand, combining eqs.(d.2a) and (d.2c) after taking the time derivative of eq.(d.2a) yields

$$\frac{\dot{c}}{c}=\frac{(1-\tau_b)r-\beta}{\sigma}=\Psi_{dc}. \tag{d.6}$$

Denote the private total wealth as

$$w(t)\equiv b(t)+q(t)k(t). \tag{d.7}$$

We substitute eqs.(d.1), (d.2d') and (d.4) into eq.(d.7) after taking the time derivative of (d.7). Then we rewrite the outcome as the aggretate term remembering that aggregate economic variables and private ones are related by $Nk(t)=K(t)$, $Ny(t)=Y(t)$, $Nb(t)=B(t)$, and $Nw(t)=W(t)$ under the assumption of identical agents. Then, we obtain

$$\dot{W}(t)=(1-\tau_k)A^{\frac{1}{1-\delta}}g^{\frac{\delta}{1-\delta}}K(t)+(1-\tau_b)rB(t)-(1+\tau_c)Nc(t)+(1-\tau_b)rq(t)K(t)$$
$$-(1-\tau_k)A^{\frac{1}{1-\delta}}g^{\frac{\delta}{1-\delta}}K(t). \tag{d.8}$$

We assume that the government runs a continuously balanced budget, and therefore, the government budget constraint is given by

$$\tau_k A^{\frac{1}{1-\delta}}g^{\frac{\delta}{1-\delta}}K(t)+\tau_b rB(t)+\tau_c Nc(t)=G(t)=gY(t). \tag{d.9}$$

By substituting eq.(d.9) into eq.(d.8), we obtain

$$\dot{W}(t)=rW(t)-Nc(t)+\left[\tau_k A^{\frac{1}{1-\delta}}g^{\frac{\delta}{1-\delta}}-A^{\frac{1}{1-\delta}}g^{\frac{1}{1-\delta}}-\tau_b rq\right]K(t). \tag{d.10}$$

It is clear from eq. (d.10) that in order for the accumulation of aggregate total wealth in a decentralized economy to have the same characteristic as that in a centralized economy, the evolution of W must be independent of K. This can be seen by comparing eqs.(d.10) and eq.(c.12"). Then, the condition that needs to be met is

$$\tau_k A^{\frac{1}{1-\delta}}g^{\frac{\delta}{1-\delta}}-A^{\frac{1}{1-\delta}}g^{\frac{1}{1-\delta}}=\tau_b rq.\text{[5]} \tag{d.11}$$

If the condition (d.11) holds, then eq.(d.10) reduces to

$$\dot{W}(t)=rW(t)-Nc(t). \tag{d.9'}$$

Since the growth rate of W in steady-growth equilibrium is given by Ψ_{dw}, eq.(d.10') implies that the $Nc(t)$-$W(t)$ ratio must be a constant, say μ_d. Consequently, private consumption and aggregate total wealth grow at the same rate or $\dot{c}/c=\Psi_{dc}=\Psi_{dw}=\dot{W}/W=\Psi_d$ and $Nc(t)$-$W(t)$ ratio can be expressed as

$$\frac{Nc(t)}{W(t)}=\frac{\beta-(1-\sigma)r+\tau_b r}{\sigma}. \tag{d.12}$$

The accumulation of private foreign bond holdings is characterized by following the same argument as we described in the centralized economy. The conditions corresponding to eqs. (c.22a), (c.22b), and (c.22c) which ensure to satisfy the transversality condition are denoted by

5) Substituting condition (d.11) into eq.(d.4) yields eq.(c.7). This ensures that shadow price of capital in terms of shadow price of foreign bonds in the decentralized economy coincides with that in the centralized economy, thereby making the growth rate of capital in the decentralized economy also coincide with that in the centralized economy.

$\gamma_d < r(1-\tau_b)$; (d.13a), $\Psi_d < r(1-\tau_b)$; (d.13b), and $c(0)=\left[(1-\tau_b)r-\Psi_d\right]\mathring{\tilde{q}}_b k(0)$; (d.13c), respectively. Consequently, the behavior of private foreign bond holdings is denoted by

$$b(t)=\mathring{\tilde{q}}_b k(0)[e^{\Psi_d t}-e^{\gamma_d t}]. \tag{d.14}$$

Therefore, the growth rate of private foreign bond holdings (aggregate foreign bond holdings[6]) also asymptotically converges to the growth rate of Ψ_d in steady-growth equilibrium as long as we assume no foreign debt.

5 . TAXES, GROWTH, AND OPTIMAL TAX POLICY

It is easy to see how changes in three kinds of tax rates influence the growth rate of consumption or equivalently the growth rate of total wealth. If we take partial derivatives of Ψ_d with respect to τ_b, τ_c, and τ_k, we obtain $\partial\Psi_d/\partial\tau_b<0$, $\partial\Psi_d/\partial\tau_c=0$, and $\partial\Psi_d/\partial\tau_k=0$, respectively. On the other hand, to see how changes in tax rates affect the growth rate of capital, we rewrite the quadratic equation given by eq.(d.4') in terms of γ_d. Then, we obtain

$$(1-\tau_b)(1+h\gamma_d)r-\frac{h}{2}\gamma_d^2-(1-\tau_k)A^{\frac{1}{1-\delta}}g^{\frac{\delta}{1-\delta}}=0. \tag{d.15}$$

Thus, differentiating eq.(d.15) with respect to τ_b yields

$$\frac{\partial\gamma_d}{\partial\tau_b}=\frac{r(1+h\gamma_d)}{h[(1-\tau_b)r-\gamma_d]}>0^{7)}.$$

We also differentiate eq.(d.15) with respect to τ_k in order to get

$$\frac{\partial\gamma_d}{\partial\tau_k}=-\frac{A^{\frac{1}{1-\delta}}g^{\frac{\delta}{1-\delta}}}{h[(1-\tau_b)r-\gamma_d]}<0.$$

We note that an unstable stationary state value, $\mathring{\tilde{q}}_d$ ensures that $(1-\tau_b)r>\gamma_d$ in the same fashion as we have argued in footnote 4. As it is clear from eq.(d.12) that an increase in the rate of tax on foreign bond holdings increases μ_d. We may now summarize our observations by proposition 4.

Proposition 4

An increase in the rate of tax on foreign bond holdings ameliorates the growth rate of capital. However, it deteriorates the growth rate of consumption (total wealth). On the other hand, an increase in the rate of tax on capital deteriorates the growth rate of capital but has no effect on the growth rate of consumption (total wealth). A

6) Since $Nb(t)=B(t)$ and $Nk(t)=K(t)$ eq.(d.14) can be converted into the aggregate term denoted by $B(t)=\mathring{\tilde{q}}_b K(0)[e^{\Psi_d t}-e^{\gamma_d t}]$.

change in consumption tax has no effect on the growth rate of both consumption (total wealth) and capital.

We finally turn to find the optimal tax policy in our model. The social planner's job is to make the rate of economic growth in the decentralized economy coincide with that in the centralized economy. Then, depending on the government expenditure rules, the social planner levies a proper combination of prodution tax, consumption tax, and foreign bond income tax as an economic intervention. Thus, the key question to be addressed is the extent to which, through a proper tax policy, we can lead the decentralized economy to the centralized economy. In order for the decentralized economy to replicate the centralized economy, we compare eqs.(c.14) and (d. 6) or equivalently eqs.(c.15) and (d.12). We also compare eqs.(c.5d") and (d.2d"). Then we obtain the following three relations to describe the state in which the decentralized economy coincides with the centralized economy.

$$\Psi_c=\frac{r-\beta}{\sigma}=\frac{(1-\tau_b)r-\beta}{\sigma}=\Psi_d, \tag{r.1}$$

$$\mu_c=\frac{\beta-(1-\sigma)r}{\sigma}=\frac{\beta-(1-\sigma)r+\tau_b r}{\sigma}=\mu_d, \tag{r.2}$$

and

$$\gamma_c=\frac{\tilde{q}_b-1}{h}=\frac{\mathring{\tilde{q}}_b-1}{h}=\gamma_d. \tag{r.3}$$

The relations (r.1) and (r.2) imply that the social planner should impose no tax on foreign bond holdings. Therefore, under the assumption that the government sets g arbitrary, eq.(d.11) reduces to $\tau_k A^{\frac{1}{1-\delta}} g^{\frac{\delta}{1-\delta}}-A^{\frac{1}{1-\delta}} g^{\frac{1}{1-\delta}}=0$. Thus, $\tau_k=g$ holds. Further, if we substitutc $\tau_k=g$ and $\tau_b=0$ into the government budget constraint, then eq.(d.9) implies $\tau_c=0$. When $\tau_k=g$ and $\tau_b=0$, we see that eq.(d.4) coincides with eq.(c.7). We also see that $\tilde{q}_b=\mathring{\tilde{q}}_b$ is accomplished, thereby satisfying relation (r.3). On the other hand, $\tau_k=\delta$ with no consumption and foreign bond income taxes are the optimal tax policy when the government sets g optimally. Therefore, we may summarize our findings regarding the optimal tax policy by the following proposition 5.

Proposition 5

Assuming that eq.(d.11) holds, the optimal tax policy when the government sets g arbitrary is denoted by $\tau_k=g$ and $\tau_b=\tau_c=0$. On the other hand, the optimal tax policy when the government sets g optimally is denoted by $\tau_k=\delta$ and $\tau_b=\tau_c=0$. In both cases, government revenue should be covered by production tax only.

6. CONCLUDING REMARKS

In this paper, we introduced two important aspects of economic activities, namely, congestions and costs of adjustment into the Ramsey model. Further, we expanded our model to the small open economy and found five noteworthy propositions. Our primary purpose in this paper was to find the optimal tax policy for our model. In Section 5 we concluded that the optimal tax policy was a production tax with consumption and foreign bond holdings being untaxed if the government set its expenditure rule arbitrary. The rate of production tax is equivalent to the government expenditure-GDP ratio, g. If the government set its expenditure rule optimally, then the government expenditure-GDP ratio was equivalent to the degree of congestion, thereby determining the rate of tax on production. The optimal tax policy in this case consists of the tax rate of production being the degree of congestion with no tax on consumption and foreign bond holdings.

We also found that, in steady-growth equilibrium, if it exists, total wealth and consumption shared the same growth rate. On the other hand, the growth rate of domestic capital had a different rate from the other two economic variables. This result comes from two aspects of the model. One is the introduction of adjustment costs into the model. The other is the assumption regarding the government expenditure rule. If the government has set the government expenditure rule in accordance with the government expenditure-aggregate consumption ($Nc(t)$) ratio being a positive constant, then the three economic variables could have grown at the same rate in steady-growth equilibrium.

On the other hand, as long as the time preference of the agent exhibits a sufficiently small value, the growth rate of foreign bond holdings asymptotically converges to the growth rate of total wealth (consumption). Further, under the assumption of initial foreign bond holdings being zero, foreign bond holdings are non zero at any moment in time except time zero. Thus, a drawback of our model is that if the agent is not patient enough or he or she has an extremely large value of β, then ever increasing foreign debt occurs since domestic capital grows faster than total wealth (consumption). This is one point that may need more consideration. However, our observation regarding an agent's time preference gives an interesting insight; a country with a thrifty population (small β) may grow faster than a country with an extravagant population (large β). The wide estrangement of growth rates of economies across countries is one of the most puzzling economic phenomena. Romer (1991) and Jones and Manuelli (1990) argued that the source of this result is due to cross-country differences in government policies. On the other hand, our model shows that this cross-country estrangement results from the characteristics of a nation represented by β. Then we may argue that the country who sees dark clouds of economy should have a policy of austerity which urges the population to save money by economizing. This policy may be one requirement for a country with no credit loan of foreign bonds to grow faster.

APPENDIX I

The solutions to $\dot{c}/c=\Psi_c$ and $\dot{W}/W=\Psi_c$ are given by $c(t)=c(0)e^{\Psi_c t}$ and $W(t)=W(0)e^{\Psi_c t}$, respectively. On the other hand, $c(t)/W(t)=\mu_c$ implies that $c(0)=\mu_c W(0)$. In addition, we have eq.(c.5a) or $c(t)^{-\sigma}=N\lambda(t)$. We know that aggregate total wealth is given by eq.(c.12). By taking limits of both sides of eq.(c.12) after multiplying both sides of eq.(c.12) by $\lambda(t)e^{-\beta t}$, we get the transversality condition regarding aggregate total wealth,

$$\lim_{t\to\infty}\lambda(t)W(t)e^{-\beta t}=\lim_{t\to\infty}\lambda(t)B(t)e^{-\beta t}+\lim_{t\to\infty}\eta(t)K(t)e^{-\beta t}=0. \tag{a.1}$$

Substituting $N\lambda(t)=c(t)^{-\sigma}=c(0)^{-\sigma}e^{-\sigma\Psi_c t}$ and $W(t)=W(0)e^{\Psi_c t}=(c(0)/\mu_c)e^{\Psi_c t}$ into eq.(a.1) yields

$$\lim_{t\to\infty}\frac{c(0)^{1-\sigma}}{N\mu_c}e^{-(\beta-(1-\sigma)\Psi_c)t}=\lim_{t\to\infty}\lambda(t)W(t)e^{-\beta t}=0.$$

Thus, $\beta-(1-\sigma)\Psi_c>0$ or $\beta>(1-\sigma)r$ must hold in order to satisfy the transversality condition.

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