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MEASUREMENT ELASTICITY OF CONSUMPTION GOODS WITH DYNAMIC MODEL

Bao-hua Zhu

1. Introduction

Demand analysis has been paid attention to for a long times because demand plays an important role in economic activity. As in the fields of demand analysis the consumption analysis is an important part in econometric analysis, we shall focus our attention on dynamic consumption analysis in this paper. Until now we attached importance to static demand analysis and didn't discriminate long run from short run in measuring elasticity. For we have to consider time element in economic analysis, it is necessary for us to allow for the difference between long run and short run in econometric analysis. We shall inquire about the relation between prices and quantities demanded with dynamic elements considered the difference between long run and short run.

We know there exists minimum required quantity (intercept) for each goods in linear expenditure system. In this paper we try to develop a new dynamic model basing on linear expenditure system. We think that minimum required quantity changes as time goes on and induce dynamic elements into the intercept of linear expenditure system. We should explain the concepts of long run and short run before working. We define short run as a state where there is a adjustment process or disequilibrium point and long run as a state where adjustment process is finished or equilibrium point.

Section 2 introduces linear expenditure system with dynamic elements. Section 3 presents a new dynamic linear expenditure system combining the habit-formation hypothesis with the stock adjustment hypothesis. Section 4 gives an application of Japan. Section 5 summarizes our conclusion.

2. Dynamic Linear Expenditure System

Stone (1954) suggested the first empirical consumption analysis system satisfying all general restrictions with respect to consumer demand function. This system is known as linear expenditure system and has been widely used in applied econometric analysis, especially in the static consumption analysis.

Linear expenditure system is

$$x_i=\gamma_i+\frac{\beta_i}{p_i}\left(y-\sum_j^n p_j\gamma_j\right). \qquad (2\text{-}1)$$

where x_i : quantity consumed

y: income

p_i: commodity price

γ_i: parameter

β_i: parameter

We should note that γ_i is only a parameter which is either positive or negative and there is no reason why γ_i must be positive in fact, though we usually hope for a positive γ_i in the positive analysis.

The income and price elasticity for linear expenditure system is as follows

$$\begin{cases} \dfrac{y}{x_i}\dfrac{\partial x_i}{\partial y}=\dfrac{\beta_i}{p_i}\dfrac{y}{x_i} \\ \dfrac{p_i}{x_i}\dfrac{\partial x_i}{\partial p_i}=-1+\dfrac{\gamma_i(1-\beta_i)}{x_i} \\ \dfrac{p_j}{x_i}\dfrac{\partial x_i}{\partial p_j}=-\dfrac{\gamma_i\beta_i p_j}{x_i p_i} \end{cases} \tag{2-2}$$

The utility function in according with the linear expenditure system is known as Klein-Rubin one:

$$U=\sum_i \beta_i \ln(x_i-\gamma_i) \tag{2-3}$$

where $x_i > \gamma_i$ for each goods.

Samuelson (1947-1948) suggested a nice interpretation of the parameter γ_i. Under the condition of the positive γ_i's, the expenditure on *i*th goods is divided into minimum expenditure and supernumerary expenditure. The minimum expenditure on *i*th goods is the one which the consumer commits himself to attain a subsistence level and γ_i can be interpreted as the minimum required quantity.

Because past consumption pattern would be an important determinant of present consumption pattern, the dynamic elements need to be introduced into the utility function. Corresponding to the utility function with dynamic elements, consumer demand function ought to be distinguished between long run and short run. There are a few reasons why the demand function is different between long run and short run.

(a) The consumer may have contractually fixed commitments which prevent him from adjusting his consumption behavior quickly in response to changes in prices or income. When the fixed commitments lapse, he is able to adjust to his long run equilibrium point.

(b) Consumer's adjustment to the long run equilibrium corresponding to a new price-income situation will involve a quiet adjustment process and it depends on his past consumption pattern.

Because it is difficult for us to maximize the utility function with dynamic elements, we usually put dynamic elements into the demand function derived from utility maximization without dynamic elements instead of the utility function.

Stone (1966) suggested three change types by which parameters of linear expenditure system accommodate as follows to make linear expenditure system dynamic

$$\begin{cases} \beta_{it}=\beta_i+\beta_i^* t \\ \gamma_{it}=\gamma_i+\gamma_i^* t \end{cases} \tag{2-4}$$

$$\begin{cases} \beta_{it}=\beta_i+\beta_i^* t+\beta_i^{**} t^2 \\ \gamma_{it}=\gamma_i+\gamma_i^* t+\gamma_i^{**} t^2 \end{cases} \tag{2-5}$$

$$\begin{cases} \beta_{it}=\beta_i+\beta_i^* e_t \\ \gamma_{it}=\gamma_i+\gamma_i^* e_t \end{cases} \tag{2-6}$$

where e_t is moving average ending in period t-1 of the past consumption demand.

There is a another way, in which the minimum required quantity depends on consumed quantity or actual stock existing essentially, to deduce dynamic linear expenditure system.

Pollak (1970) suggested habit-formation hypothesis, which considered minimum required quantity γ_{it} as a variable related to the past consumption quantities. The simplest case in that the minimum required quantity of each goods is proportional to the consumption quantity of that goods in previous period:

$$\begin{cases} \gamma_{it}=b_i x_{it-1} \\ \gamma_{it}=b_i^*+b_i x_{it-1} \end{cases} \tag{2-7}$$

where $0<b_i<1$ for each goods.

It is natural for us to consider the minimum required quantity γ_{it} as the function of all past consumption quantities because the habit-formation is a known well phenomenon of 'inertia'. Then we assume that the present minimum required quantity of each goods depends on a geometrically weighted average of all past consumption quantities of that goods:

$$\begin{cases} \gamma_{it}=b_i q_{it-1} \\ \gamma_{it}=b_i^*+b_i q_{it-1} \end{cases} \tag{2-8}$$

where $q_{it}=(1-\delta)\sum_{j=1}^{\infty}\delta^j x_{it-j}$.

Phlips (1972) proposed the stock adjustment hypothesis, which is nothing but another expression of habit-formation hypothesis in fact. We think the effect of the past consumption behavior is represented by a certain state-variable (psychological stock variable) and let it be s_{it} for ith goods. In other words the state-variable expresses the result of past consumption behavior or the effect of all past consumption demand. In particular we define s_{it} as measuring stock or stock of habit.

We make minimum required quantity γ_{it} a variable related to state-variable s_{it}. To stay within the framework of linear analysis we consider a linear relation between present minimum required quantity γ_{it} and state variable s_{it} as

$$\gamma_{it}=\theta_i+\alpha_i s_{it}. \tag{2-9}$$

When α_i is negative, the taste change is quantity diminishing; when α_i is positive, the taste change is quantity augmenting.

The reason why we make distinction between the habit-formation hypothesis and stock adjustment one is the different expression of the past consumption effect. In the habit-formation hypothesis, the effect of past consumption behavior is expressed by means of the past consumption quantities. On the contrary, the effect of the past consumption behavior is showed by a state-variable in the stock adjustment hypothesis. The proxy of state-variable is usually a stock-variable. The stock-variable is also thought as a psychological stock variable. The advantage of the stock adjustment hypothesis is that the effect of the past consumption behavior can be described in a state-variable easily.

3. Model

Up-dating the effect of the past consumption behavior has been presented either by habit-formation hypothesis or by stock adjustment one. The current consumption behavior is affected not only by the latest consumption quantity but also by all past consumption quantities. If we adopt equation (2-8) to express the effect of all past consumption behavior, it is difficult for us to choose correct distributed lag and estimate it. For this reason we don't directly adopt the hypothesis in equation (2-8), where the effect of all past consumption behavior is presented in a geometrically weighted average of all past consumption quantities. We try to combine habit-formation hypothesis with stock adjustment hypothesis to show the effect of all past consumption consumption behavior in applied consumption analysis. We would like to use a state-variable or proxy of state-variable (stock-variable) to express the effect of all past consumption behavior except the latest period and present the effect of the latest period's consumption behavior in that period's consumer quantity directly.

We assume minimum required quantity in linear expenditure system is expressed by the function in terms of stock-variable and the latest period's consumption quantity. We make the hypothesis as follows.

$$\gamma_{it}=\theta_i+\alpha_i s_{it-1}+\eta_i x_{it-1} \qquad (3\text{-}1)$$

We utilize Klein-Rubin utility function corresponding with our hypotheses (3-1)

$$U=\sum_i \beta_i \ln(x_{it}-\theta_i-\alpha_i s_{it-1}-\eta_i x_{it-1}). \tag{3-2}$$

Under the above utility function, the sum $\sum_i^n \beta_i$ in the utility function is not always equal to one, β_i is generalized marginal budget share.

As stock-variable is usually not observable, we must take some transformation to substitute other observable variable for stock-variable s_{it-1}. We use average stock in one period instead of stock at the beginning of that period and define it as

$$\overline{s_{it-1}}=(s_{it-1}+s_{it-2})/2 \tag{3-3}$$

where $\overline{s_{it-1}}$ is the average stock. After $\overline{s_{it-1}}$ takes the place of s_{it-1}, the hypotheses (3-1) is written as

$$\gamma_{it}=\theta_i+\alpha_i\overline{s_{it-1}}+\eta_i x_{it-1} \tag{3-4}$$

From the first-order conditions of utility maximization, we hold

$$x_{it}=\theta_i+\alpha_i\overline{s_{it-1}}+\eta_i x_{it-1}+\frac{\beta_i}{\lambda_t p_{it}} \tag{3-5}$$

We define $\pi_{it}=1/\lambda_t p_{it}$ and substitute π_{it} for $\lambda_t p_{it}$ in equation (3-5). Then we write equation (3-5) as

$$x_{it}=\theta_i+\alpha_i\overline{s_{it-1}}+\eta_i x_{it-1}+\beta_i\pi_{it} \tag{3-6}$$

Usually the stock adjustment formulation is

$$\dot{s}_{it-1}=x_{it-1}-\delta_i s_{it-1}. \tag{3-7}$$

Here we adopt a discrete time model and write stock adjustment equation (3-7) as

$$s_{it-1}-s_{it-2}=x_{it-1}-\delta_i\overline{s_{it-1}}. \tag{3-8}$$

Furthermore from equations (3-7) and (3-8) we find:

$$\begin{aligned}2(\overline{s_{it-1}}-\overline{s_{it-2}})&=(s_{it-1}+s_{it-2})-(s_{it-2}+s_{it-3})\\&=(s_{it-1}-s_{it-2})+(s_{it-2}-s_{it-3})\\&=(x_{it-1}+x_{it-2})-\delta_i(\overline{s_{it-1}}+\overline{s_{it-2}}).\end{aligned} \tag{3-9}$$

Using equation (3-6)

$$2(x_{it}-x_{it-1})=\alpha_i(x_{it-1}+x_{it-2})-\alpha_i\delta_i(\overline{s_{it-1}}+\overline{s_{it-2}})$$
$$+2\eta_i(x_{it-1}-x_{it-2})+2\beta_i(\pi_{it}-\pi_{it-1}). \tag{3-10}$$

From equation (3-6)

$$-\alpha_i\delta_i(\overline{s_{it-1}}+\overline{s_{it-2}})$$
$$=2\delta_i\theta_i+\eta_i\delta_i(x_{it-1}+x_{it-2})+\beta_i\delta_i(\pi_{it}+\pi_{it-1})-\delta_i(x_{it}+x_{it-1}), \tag{3-11}$$

on substituting equation (3-11) into equation (3-10)

$$(2x_{it}-x_{it-1})$$
$$=\alpha_i(x_{it-1}+x_{it-2})+2\delta_i\theta_i+\eta_i\delta_i(x_{it-1}+x_{it-2})+\beta_i\delta_i(\pi_{it}+\pi_{it-1})$$
$$-\delta_i(x_{it}+x_{it-1})+2\eta_i(x_{it-1}-x_{it-2})+2\beta_i(\pi_{it}-\pi_{it-1}) \tag{3-12}$$

We expand equation (3-12) and get

$$2x_{it}-2x_{it-1}-\alpha_i x_{it-1}-\eta_i\delta_i x_{it-1}+\delta_i x_{it}+\delta_i x_{it-1}-2\eta_i x_{it-1}$$
$$=2\delta_i\theta_i+\eta_i\delta_i x_{it-2}-2\delta x_{it-2}+\alpha_i x_{it-2}+2\beta_i\pi_{it}-2\beta_i\pi_{it-1}$$
$$+\beta_i\delta_i\pi_{it}+\beta_i\delta_i\pi_{it-1} \tag{3-13}$$

that is

$$(\delta_i+2)x_{it}-(\alpha_i-\delta_i+\eta_i\delta_i+2\eta_i+2)x_{it-1}$$
$$=2\delta_i\theta_i+(\eta_i\delta_i-2\eta_i+\alpha_i)x_{it-2}+\beta_i(\delta_i+2)\pi_{it}+\beta_i(\delta_i-2)\pi_{it-1}. \tag{3-14}$$

Finally

$$x_{it}=K_{i1}+K_{i2}x_{it-1}+K_{i3}x_{it-2}+K_{i4}\pi_{it}+K_{i5}\pi_{it-1}$$

and it is written again as

$$x_{it}=K_{i1}+K_{i2}x_{it-1}+K_{i3}x_{it-2}+K_{i4}\frac{1}{\lambda_t p_{it}}+K_{i5}\frac{1}{\lambda_{t-1}p_{it-1}} \tag{3-15}$$

where

$$\begin{cases} K_{i1}=\dfrac{2\delta_i\theta_i}{\delta_i+2} & (3\text{-}16) \\ K_{i2}=\dfrac{\alpha_i-\delta_i+\eta_i\delta_i+2\eta_i+2}{\delta_i+2} & (3\text{-}17) \\ K_{i3}=\dfrac{\eta_i\delta_i-2\eta_i+\alpha_i}{\delta_i+2} & (3\text{-}18) \end{cases}$$

$$\begin{cases} K_{i4}=\beta_i & (3\text{-}19) \\ K_{i5}=\dfrac{\beta_i(\delta_i-2)}{\delta_i+2} & (3\text{-}20) \end{cases}$$

On solving this set of regression coefficients in terms of the five structure coefficients, we hold

$$\begin{cases} \delta_i=\dfrac{2(K_{i4}+K_{i5})}{K_{i4}-K_{i5}} & (3\text{-}21) \\ \eta_i=\dfrac{(K_{i2}-K_{i3})K_{i4}+K_{i5}}{K_{i4}+K_{i5}} & (3\text{-}22) \\ \alpha_i=\dfrac{K_{i3}K_{i4}-K_{i2}K_{i5}}{4K_{i4}}-\dfrac{K_{i5}^2}{4K_{i4}^2} & (3\text{-}23) \\ \theta_i=\dfrac{2K_{i1}(K_{i4}+K_{i5})}{K_{i4}} & (3\text{-}24) \\ \beta_i=K_{i4} & (3\text{-}25) \end{cases}$$

We discuss the effect of long run and short run next. In the long run equilibrium point, consumption quantity for each goods is x_i^* and income is y^*. From the consumer demand equation and budget constraint we have the following system.

$$\begin{cases} x_i^*=\theta_i+\alpha_i s_i+\eta_i x_i^*+\dfrac{\beta_i}{\lambda p_i} \\ \sum_i p_i x_i^*=y^*. \end{cases} \qquad (3\text{-}26)$$

At the equilibrium point the stock adjustment has finished so that stock s_i should be constant, *e.g* $\dot{s}_i=0$. In the case of the long run equililibrium, $x_i^*-\delta_i s_i=0$. Thus we get

$$x_i^*=\theta_i\alpha_i\frac{x_i^*}{\delta_i}+\eta_i x_i^*+\frac{\beta_i}{\lambda p_i}\ . \qquad (3\text{-}27)$$

The equation (3-27) is transformed as

$$\left(1-\frac{\alpha_i}{\delta_i}-\eta_i\right)x_i^*=\theta_i+\frac{\beta_i}{\lambda p_i}\ . \qquad (3\text{-}28)$$

Finally

$$p_i x_i^*=p_i\gamma_i^*+\beta_i^*(y^*-\sum_j p_j\gamma_j^*) \qquad (3\text{-}29)$$

where

$$\beta_i^* = \frac{\delta_i \beta_i}{\sigma(\delta_i - \alpha_i - \delta_i \eta_i)} \tag{3-30}$$

$$\sigma = \sum_i^n \frac{\delta_i \beta_i}{\delta_i - \alpha_i - \delta_i \eta_i} \tag{3-31}$$

$$\gamma_i^* = \frac{\delta_i \theta_i}{\delta_i - \alpha_i - \delta_i \eta_i} . \tag{3-32}$$

The long run demand function is of the same form as static linear expenditure system so that we can use formulae (2-2) to compute the long run income and price elasticity with β_i and γ_i replaced by β_i^* and γ_i^* .

For the short run price effect, on the assumption

$$\begin{cases} \dfrac{\partial s_{i\,t-1}}{\partial p_{i\,t}} = 0 \\ \dfrac{\partial x_{i\,t-1}}{\partial p_{i\,t}} = 0 \end{cases} \tag{3-33}$$

we differentiate

$$x_{i\,t} = \theta_i + \alpha_i s_{i\,t-1} + \eta_i x_{i\,t-1} + \frac{\beta_i}{\lambda_t p_{i\,t}} \tag{3-34}$$

with respect to $p_{i\,t}$ and get

$$\frac{\partial x_{i\,t}}{\partial p_{i\,t}} = -\frac{\beta_i}{\lambda_t^2 p_{i\,t}} \frac{\partial \lambda_t}{\partial p_{i\,t}} - \frac{\beta_i}{\lambda_t p_{i\,t}^2} . \tag{3-35}$$

After multiplying $p_{i\,t}$,

$$p_{i\,t} \frac{\partial x_{i\,t}}{\partial p_{i\,t}} = -\frac{\beta_i}{\lambda_t^2} \frac{\partial \lambda_t}{\partial p_{i\,t}} - \frac{\beta_i}{\lambda_t p_{i\,t}} . \tag{3-36}$$

Let us recall budget constraint again

$$\sum_j^n p_{j\,t} x_{j\,t} = y_t \tag{3-37}$$

and differentaite it with respect to $p_{i\,t}$

$$x_{i\,t} + \sum_j^n p_{j\,t} \frac{\partial x_{j\,t}}{\partial p_{i\,t}} = 0. \tag{3-38}$$

From equation (3-33) we get again

$$\frac{\partial x_{jt}}{\partial p_{it}}=-\frac{\beta_j}{\lambda_t^2 p_{jt}}\frac{\partial \lambda_t}{\partial p_{it}} \tag{3-39}$$

which can be written as

$$p_{jt}\frac{\partial x_{jt}}{\partial p_{it}}=-\frac{\beta_j}{\lambda_t^2}\frac{\partial \lambda_t}{\partial p_{it}}\ . \tag{3-40}$$

We sum up equation (3-40) with j

$$\sum_j^n p_{jt}\frac{\partial x_{jt}}{\partial p_{it}}=-\frac{1}{\lambda_t^2}\left(\sum_j^n \beta_j\right)\frac{\partial \lambda_t}{\partial p_{it}}-\frac{\beta_i}{\lambda_t p_{it}} \tag{3-41}$$

and make use of equation (3-38)

$$-\frac{1}{\lambda_t^2}\frac{\partial \lambda_t}{\partial p_{it}}=\frac{\frac{\beta_i}{\lambda_t p_{it}}-x_{it}}{\sum_j^n \beta_j}\ . \tag{3-42}$$

Substituting equation (3-42) into equation (3-35)

$$\begin{aligned}\frac{\partial x_{it}}{\partial p_{it}}&=\frac{\beta_i}{p_{it}\sum_j^n \beta_j}\left(\frac{\beta_i}{\lambda_t p_{it}}-x_{it}\right)-\frac{\beta_i}{\lambda_t p_{it}^2}\\&=\frac{\beta_i^2}{\lambda_t p_{it}^2\sum_j^n \beta_j}-\frac{\beta_i x_{it}}{p_{it}\sum_j^n \beta_j}-\frac{\beta_i}{\lambda_t p_{it}^2}\ .\end{aligned} \tag{3-43}$$

Also we can derive

$$\frac{\partial x_{it}}{\partial p_{jt}}=\frac{\beta_i\beta_j}{\lambda_t p_{it} p_{jt}\sum_j^n \beta_j}-\frac{\beta_i x_{jt}}{p_{jt}\sum_j^n \beta_j}\ . \tag{3-44}$$

For the short run income effect, we differentiate equation (3-33) with respect to y on taking assumption as follows.

$$\begin{cases}\dfrac{\partial \alpha_i}{\partial y_t}=0\\[2mm]\dfrac{\partial x_{i\,t-1}}{\partial y_t}=0\ .\\[2mm]\dfrac{\partial s_{i\,t-1}}{\partial y_t}=0\end{cases} \tag{3-45}$$

Then

$$\frac{\partial x_{it}}{\partial y_t}=-\frac{1}{\lambda_t^2}\frac{\partial \lambda_t}{\partial y_t}\frac{\beta_i}{p_{it}}\ . \tag{3-46}$$

Making use of the relation

$$\sum_i^n p_{it}\frac{\partial x_{it}}{\partial y_t}=1 \tag{3-47}$$

we have

$$-\frac{1}{\lambda_t^2}\frac{\partial \lambda_t}{\partial y_t}=\frac{1}{\sum_i^n \beta_i}\ . \tag{3-48}$$

Finally

$$\frac{\partial x_{it}}{\partial y_t}=\frac{\beta_i}{p_{it}\sum_i^n \beta_i}\ . \tag{3-49}$$

Having discussed above we can compute the prices and income elasticity of short run and long run.

4. Some Empirical Result

For quarterly data we utilized Composition of Final Consumption Expenditure of Households published in the Annual Report on National Accounts for the periods 1980-I to 1986-I. Quantities purchased (x_{it}) are measured by expenditure in constant prices of the year 1980. Prices are obtained from Price Indexes Monthly. Income is defined as the sum of the consumption expenditure at current prices. All expenditures are per capita.

We use iterative method to estimate our model because there is a nonlinear relation between the parameters of our model. From following equations (4-1) and (4-2)

$$x_{it}=K_{i1}+K_{i2}x_{it-1}+K_{i3}x_{it-2}+K_{i4}\frac{1}{\lambda_t p_{it}}+K_{i5}\frac{1}{\lambda_{t-1}p_{it-1}} \tag{4-1}$$

$$\sum_i^n p_{it}x_{it}=y_t \tag{4-2}$$

we deduce

$$\lambda_t = \frac{\sum_i^n K_{i4}}{y_t - \sum_i^n p_{it} K_{i1} - \sum_i^n p_{it} K_{i2} x_{it-1} - \sum_i^n p_{it} K_{i3} x_{it-2} - \frac{1}{\lambda_{t-1}} \sum_i^n \frac{p_{it} K_{i4}}{p_{it-1}}} \tag{4-3}$$

The estimating procedure is as follows:

(1) Set λ_t equal to some initial value and estimate system equation (4-1)

(2) Use the estimates for K_{ij} to compute new set λ_t by equation (4-3) and estimate K_{ij} again on using new set λ_t

(3) Repeat this process until the estimated expenditures add up to observed total expenditure for each period. As this condition is satisfied, the estimating equations have become demand equations.

The system of equations has to be estimated by an approriate regression technique at each step of iterative estimation. Given the condition that observed total expenditure is equal to estimated total expenditure, the error terms between the equations are dependent. We should use Zellner's method for seemingly unrelated regression to estimate our model. However it seems impossible for us to estimate demand system from n-1 equations with Zellner's method here as we have no simple constraint on our regression coefficients over all commidities. For convenience we use OLS to estimate demand system alternatively and do give up to consider dependent relation between equations in our model· The estimated result with standard errors between brackets as usual is as follows:

	K_{i1}	K_{i2}	K_{i3}	K_{i4}	K_{i5}
durable $R^2=0.942$	−4378.02 (4590.98)	0.6543 (0.2183)	0.1914 (0.2093)	0.0570 (0.0464)	−0.0331 (0.03414)
non-durable $R^2=0.845$	23118.76 (19390.86)	0.2989 (0.2006)	0.3195 (0.1953)	0.1312 (0.0539)	−0.0841 (0.0371)
semi-durable $R^2=0.876$	6114.15 (6563.63)	0.6358 (0.1285)	−0.1919 (0.1484)	0.1496 (0.00215)	−0.1032 (0.0147)
services $R^2=0.431$	58682.96 (41420.21)	0.4193 (0.2081)	0.1362 (0.2152)	0.1110 (0.0518)	−0.0696 (0.0320)

From K_{ij} coefficients we calculate structure coefficients of our model:

commodity	α_i	β_i	δ_i	η_i	θ_i
durable	0.143	0.057	0.533	−0.074	−1841.32
non-durable	0.128	0.131	0.437	−0.403	8294.70
semi-durable	0.062	0.150	0.368	0.082	1898.97
services	0.100	0.111	0.458	−0.212	21863.31

The positive α_i's indicate habit-formation while the negative α_i's characterize stock effect of durable goods. From the structure coefficients we have following conclusion that all goods hold the charcteristic of habit-formation. We guess that habit-formation plays an important role in the durable goods consumption. Subtracting α_i from δ_i, we get an estimate of adjustment coefficient for each goods. They are lower than unity and follow the partial adjustment assumption so that consumer adjusts his consumer behavior slowly. From η_i's sign, we know the latest period's consumption exerts a minus influence on present consumption in most goods. The more is consumption in the latest period, the less consumption in the present period. The negative η_i seems rational for durable goods while it is an open question for the other goods. Moreover, marginal budget shares β_i's are positive as expected and the smallest is durable goods.

We compute price and income elasticity for short run. The short run uncompensated price elasticity is

	durable	non-durable	semi-durable	services
durable	−0.9244	−0.3845	0.0724	−0.8430
non-durable	−0.0048	−0.6353	0.0337	−0.3920
semi-durable	−0.0144	−0.5318	−1.3123	−1.1660
services	−0.0027	−0.0982	0.0185	−0.4827

and the short run compensated prices elasticity is

	durable	non-durable	semi-durable	services
durable	−0.7980	0.2863	0.3411	0.2584
non-durable	0.0540	−0.3233	0.1587	0.1202
semi-durable	0.1605	0.3960	−0.9405	0.3574
services	0.0297	0.0732	0.0872	−0.2012

Next we discuss the long run effect of demand equations. We get the estimates of long run demand equation from corresponding short run demand equation.

commodity	β_i^*	θ_i^*
durable	0.142	−2284.72
non-durable	0.236	7465.50
semi-durable	0.399	2529.50
services	0.223	22004.00

For long run, the largest marginal budget share is semi-durable goods as same as short run case. From β_i^* and θ_i^* we find the long run uncompensated price elasticity is

	durable	non-durable	semi-durable	services
durable	−1.0955	0.0153	0.0150	0.0148
non-durable	−0.0174	−0.9457	−0.0164	−0.0162
semi-durable	−0.0258	−0.0249	−0.9632	−0.0242
services	−0.0314	−0.0303	−0.0297	−0.8977

and the long run compensated price elasticity is

	durable	non-durable	semi-durable	services
durable	−0.9647	0.6871	0.2853	1.1182
non-durable	0.0415	−0.6994	0.1067	0.4938
semi-durable	0.1492	0.9012	−0.7000	1.4959
services	0.0011	0.1391	0.0357	−0.7409

All direct price elasticity has the appropriate sign and all direct compensated price elasticity is smaller than the corresponding uncompensated price elasticity. But all short run price elasticity is not smaller than the corresponding long run price elasticity.

Finally we get the short run and long run income elasticity as follows

	short run	long run
durable	1.94	2.17
non-durable	0.90	0.73
semi-durable	2.68	3.20
services	0.50	0.45

The income elasticity for semi-durable goods is the largest not only for long run but for short run. Goods with durability has larger income elasticity. The interesting phenomenon is that income elasticity for services goods is the smallest of four kinds of goods for both short run and long run. This is probably related to goods classification.

5. Summary

In this paper we developed a new dynamic model based on linear expenditure system and applied it to Japan consumption analysis. From empirical result, we suggest the difference between short run and long run is obvious for some goods. It seems necessary for us to be aware of the distinction between short run and long run in economic analysis. From our empirical analysis we know following conclusion:

(1) The short run price elasticity is different from the long run one whether uncompensated or compensated.

(2) The short run direct price elasticity is smaller than the long run one except semi-durables.

(3) All direct compensated price elasticity is smaller than the corresponding uncompensated price elasticity for both short run and long run.

(4) For semi-durables and durables, the long run income elasticity is larger than the short run one.

(5) For non-durables, the long run income elasticity is smaller than the short run one.

There are some problems to be discussed further such as the elasticity for service goods with respect to price and income, the negative η_i's sign which exerts a minus effect on minimum required quantity, and so on. There is an estimation method problem in our model and it is necessary for us to develop a new method to estimate our model on thinking over the dependent disturbance terms of system equation.

6. References

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