Some Aspects of the Kinked Demand Curve Hypothesis from the Sales-maximization Principle

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Makoto Hagino

CONTENTS

1. Introduction

This paper analyzes the model of duopoly with the kinked demand curve hypothesis. According to this hypothesis, a firm has two shapes of demand curve: the obtuse curve when the state of its market is stable, and the reflex curve when it goes to extremes, such as boom or depression. The first reference to these shapes of the demand curve is Sweezy[6], but he confuses the explanation of the reflex curve on two special cases: price leadership and secret price cutting[1]. This explanation is refined by Efroymson[3]. He makes it clear that the condition of the market chooses the curves. We have accepted this proposal.

In the case of the obtuse curve, the pattern of firm's behavior is subject to the sales-maximization principle, then the profit-maximization principle is satisfied with its consequence. In the case of the reflex curve, both of its pattern and consequence are consistent with the profit-maximization principle. These consequences satisfy only the profit-maximization principle, but the two patterns are derived from each principle, respectively. Shepherd[5] indicates that the consequence of the obtuse kinked demand curve is satisfied with both principles, where the sales-maximization principle consists with the profit-maximization one. This is merely a special case[2]. He never points out that in general cases the marginal revenue curve is positive.

The purpose of this paper is to demonstrate these changes in the shape of the demand curves in the kinked demand curve hypothesis. We set up our principle as the sales-maximization principle. Firms' behavioral patterns and consequences are determined only by this principle. They are restricted to one principle, whatever their market conditions may be changed. This approach would throw light on the changes of the shape. In general, a duopoly model restricts the firms' demand curves, which derive the reaction functions. These equilibrium points are the consequences of the hypothesis. The treatments of the kinked demand curve (imagined demand curve) are lacking in the model. It deals with Section 3.

2. Changes in the shape of demand curve: An analysis of the duopoly model

The phases of the changes in the shape of the kinked demand curve are taken into account on this section. Firstly, it is useful to examine the equilibrium points derived from the sales-maximization principle. These points are classified into three cases. Secondly, the effect of the condition of the market on the changes in the shape is considered. The condition causes transfers between the three cases.

These movements change the shape.

To provide simple results, let's set up a quantity-setting duopoly. Firm 1 has the quantities of the product, x_1, and the price, p_1. Firm 2 has x_2 and p_2 similarly. The utility function and the inverse demand functions used by Dixit[2] are,

$$\left.\begin{aligned} u &= x_0+\alpha_1 x_1+\alpha_2 x_2-1/2(\beta_1 x_2{}^1+2\gamma x_1 x_2+\beta_2 x_2{}^2) \\ p_1 &= \alpha_1-\beta_1 x_1-\gamma x_2 \\ p_2 &= \alpha_2-\beta_2 x_2-\gamma x_1 \end{aligned}\right\}. \qquad (1)$$

Concavity of u requires

$$\beta_1>0,\ \beta_2>0,\ \gamma^2\leqq\beta_1\beta_2.$$

The total costs are

$$c_i=A_i x_i{}^2+B_i,\quad i=1,2. \qquad (2)$$

Then marginal cost is $2A_i x_i$, and the minimum value of average cost is $2(A_iB_i)^{1/2}$ at $x_i=(B_i/A_i)^{1/2}$.

The 'conventional' reaction functions to maximize profit are

$$\left.\begin{aligned} x_1 &= (\alpha_1-\gamma x_2)/2(A_1+\beta_1) \\ x_2 &= (\alpha_2-\gamma x_1)/2(A_2+\beta_2) \end{aligned}\right\}. \qquad (3)$$

These functions yield the conventional Nash equilibrium, that is,

$$\left.\begin{aligned} N_1 &= [2\alpha_1(A_2+\beta_2)-\alpha_2\gamma]/[4(A_1+\beta_1)(A_2+\beta_2)-\gamma^2] \\ N_2 &= [2\alpha_2(A_1+\beta_1)-\alpha_1\gamma]/[4(A_1+\beta_1)(A_2+\beta_2)-\gamma^2] \end{aligned}\right\}. \qquad (4)$$

Firm 1's function meets x_1-axis at M_1, and x_2-axis at Q_1. Firm 2's one meets the x_2-axis at M_2, and x_1-axis at Q_2.

$$M_1=\alpha_1/(2(A_1+\beta_1)),\qquad M_2=\alpha_2/(2(A_2+\beta_2)),$$

$$Q_1=\alpha_1/\gamma,\qquad Q_2=\alpha_2/\gamma.$$

Now we introduce the sales-maximization principle into this model. Firms behave to maximize their revenues where their marginal revenues equal to zero. The new reaction functions maximizing their sales are

$$\left.\begin{aligned} x_1 &= (\alpha_1-\gamma x_2)/(2\beta_1) \\ x_2 &= (\alpha_2-\gamma x_1)/(2\beta_2) \end{aligned}\right\}. \qquad (5)$$

Firm 1′s function meets x_1-axis at S_1 and x_2-axis at T_1. Firm 2′s one meets x_2-axis at S_2 and x_1-axis at T_2.

$$S_1=\alpha_1/(2\beta_1), \qquad S_2=\alpha_2/(2\beta_2),$$
$$T_1=\alpha_1/\gamma\ (=Q_1), \qquad T_2=\alpha_2/\gamma\ (=Q_2).$$

New Nash equilibrium point, R, is

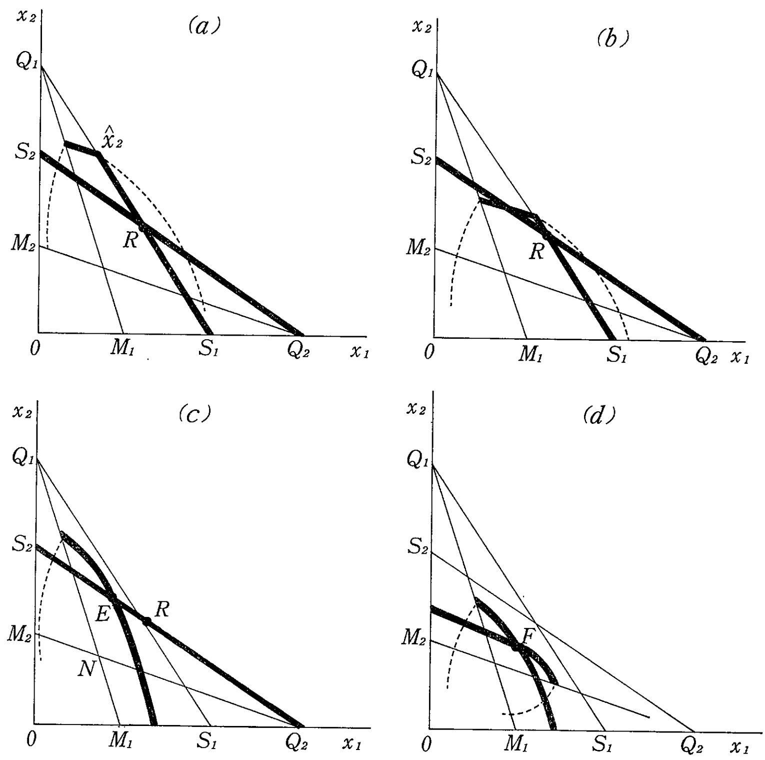
$$\left.\begin{array}{l} R_1=(2\alpha_1\beta_2-\alpha_2\gamma)/(4\beta_1\beta_2-\gamma^2) \\ R_2=(2\alpha_2\beta_1-\alpha_1\gamma)/(4\beta_1\beta_2-\gamma^2) \end{array}\right\}. \qquad (6)$$

On this point, firms' behavioral patterns and their consequences are subject to the sales-maximization priciple. It is the first classification of equilibrium.

A behavior maximizing the sales is confronted with a restraint; that is, the minimum profit[3]. This restraint gives rise to the appearances of other equilibrium points. A firm is not able to stay on R if it yields less profit than the minimum profit. The minimum profit erases a part of reaction function(5). Its disappearance is compensated by iso-profit curve yielding the minimum profit. Then sales-maximizing reaction function consists of (5) and iso-profit curve. It is shown in Figure 1.[4]

Figure 1(d) is illustrated according to the minimum profit of firm 1 and firm 2. Figure 1(b) and (c) are according only to firm 1′s minimum profit. Figure 1(a) has equilibrium point at R without restraint. Figure 1(b) has two intersection points. If equilibrium is realized, it is only at R. It is acceptable to classify (a) and (b) into the same case. Figure 1(c) has the equilibrium point on firm 2′s reaction curve. An intersection point of firms' iso-profit curves is represented in Figure 1(d). There are much more combinations. We can sort them out to three cases, which are applicable to (a) or (b); and (c) and (d). These cases demonstrate three equilibrium points: the first is R; the second is an intersection point between reaction function and iso-profit curve,

FIGURE 1 REACTION FUNCTION CURVES AND ISO-PROFIT CURVES



E; the third is a intersection point between iso-profit curves, F.

On these equilibrium points, firm's demand curve should be the obtuse shape because it is derived from reaction function maximizing sales. The reflex curves emerge as a peculiar process where an equilibrium point replaces other. The condition of the market brings about these process.

A condition of market; that is,shift of demand, has an effect on α_1 and α_2, proportionally. Its effect on β_1 and β_2 is neglected. The relation between α_1 and α_2 is, as k is fixed,

$$\alpha_2 = k\alpha_1. \tag{7}$$

Then R_1 and R_2 are rewritten

$$\left.\begin{aligned} R_1&=\alpha_1(2\beta_2-k\gamma)/(4\beta_1\beta_2-\gamma^2) \\ R_2&=\alpha_1(2k\beta_1-\gamma)/(4\beta_1\beta_2-\gamma^2) \end{aligned}\right\}. \tag{8}$$

As R_1 and R_2 are positive, conditions are

$$2\beta_2-k\gamma>0, \qquad 2k\beta_1-\gamma>0.$$

Firm 1′s iso-profit curve at a profit, π_0, denoted by $\hat{x}_2$, is

$$\hat{x}_2=[\alpha_1-(A_1+\beta_1)x_1-(B_1+\pi_0)x_1^{-1}]/\gamma. \tag{9}$$

Firm 2′s reaction function (5) is

$$\hat{x}_2=(\alpha_1k-\gamma x_1)/2\beta_2. \tag{10}$$

Then we find

$$\partial\hat{x}_2/\partial\alpha_1=1/\gamma, \qquad \partial\hat{\hat{x}}_2/\partial\alpha_1=k/2\beta_2,$$

then $\quad \partial\hat{x}_2/\partial\alpha_1> \partial\hat{\hat{x}}_2/\partial\alpha_1.$

When the increase of demand rises up α_1, the iso-profit curve shifts up further than the reaction curve. These results are explained by Figure 1. An increase of α_1 illustrates from F to E, and to an extreme, R. Inversely, along with the decrease of α_1, the equilibrium point changes from R to E, and to an extreme, F. Firm 2 has the same effect.

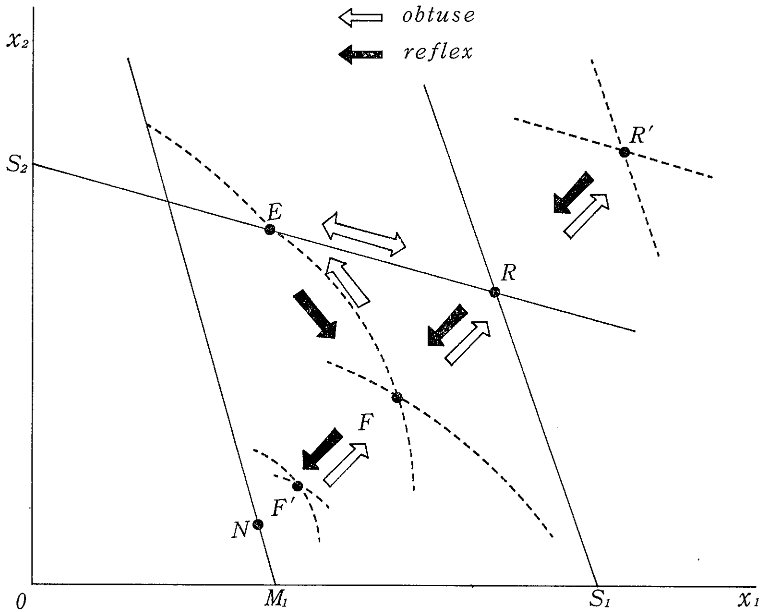
A condition of the market also alters the firm i′s profit at R, $\pi_i(R)$, that is,

$$\partial\pi_i(R)/\partial\alpha_1=2R_i^2(\beta_i-A_i)/\alpha_1. \tag{11}$$

If $\beta_i>A_i$, then $\partial\pi_i(R)/\partial\alpha_1>0$, and if $\beta_i<A_i$, then $\partial\pi_i(R)/\partial\alpha_1<0$. This implies that the profits at R vary in β_i and A_i, that is, the relation between the elasticity of the demand and the shape of the cost curve. On the condition supporting R and increasing the demand, if $\beta_i>A_i$, firm i increases its profit, and if $\beta_i<A_i$, then firm i decreases it.

The phases of the change in the shape of the curve should be explained from the process. In depression, the decrease of α_1 will result in two processes. One is from R to E, the other is from R or E to F. On the former process, the firm 1 has kept the obtuse curve, because firm 2 behaves along with its reaction curve. On the latter process, the firm 1 and firm 2 have the reflex curves, because both firms decrease their quantities and increase their prices. The process provides firms with the obtuse curve if the equilibrium point moves along reaction curve, and with the reflex curve if it moves to the intersection point of iso-profit curves (Figure 2).

FIGURE 2 CHANGES IN THE SHAPE OF THE DEMAND CURVES



In the increase of demand, however, there is not the reflex curve process that departs from F. In that case, firms increase their quantities similarly. At first, we should describe the profit at R. If β_i is less than A_i, then the profit at R becomes smaller along with the increase of α_1. The more demand increases, the less the profit at R yields. The same process as depression occurs in boom as soon as the profit is less than the minimum profit. Then, the movement to F explains the reflex curve. If β_i is more than A_i, there is just the obtuse case. In addition, the movement from R to another R, and from F to another F should be explained. In the increase of demand, both movements result in the obtuse process. In the decrease of demand, both results in reflex process.

Efroymson[3] points out the existence of the reflex curve at boom and depression. In depression case, his interpretation are valid, but in boom case, it needs the restriant, $\beta_i < A_i$. It means that firm i's market share is smaller, and its marginal cost increases greater. This appearance applies to the smaller firm in the industry, or a firm that belongs to an industry which has a steep shape of marginal cost curve, or that is confronted with a lot of rivals.

It proves that the reflex processes emerge as the movements from R or E to F in any conditions, and from R to another R or from F to another F when demand decreases. In other cases, they are the obtuse process.

3. A stability of the equilibrium: the length of discontinuous part of marginal revenue curves

Now we introduce the changes of cost into our model. It is assumed that a condition of the market is invariable. The position of the equilibrium point is also altered only by cost because of shifts of iso-profit curve. Firm 1's iso-profit curve shifts according to the change of coefficients in the cost function, A_1 and B_1, that is,[5]

$$\partial \hat{x}_2/\partial A_1 = -x_1/\gamma < 0, \quad \partial \hat{x}_2/\partial B_1 = -x_1^{-1}/\gamma < 0. \tag{12}$$

Firm 1's iso-profit curves shift down if its cost increases. This induces the movements of the equilibrium point such as in Section 2. It is certain that these movements change the shape of demand curve.

These processes are applied to the movements from R to E, or from R or E to F in Figure 1. The former yields the obtuse curve, and the latter yields the reflex curve. But the reflex curve is not emerged from the cost reduction which decreases A_i and B_i. The change of cost has no effect on the sales-maximizing reaction function, (5).

The kinked demand curve hypothesis has the reputation of explicitly representing the mechanism of the price rigidity, which is interpreted as the situation that the marginal cost curve cuts across the discontinuous part of marginal revenue curves. On this situation, a firm with the obtuse curve judges that it can also maximizes its profit[6]. If the situation occurs, it attains to maximizing both of profit and sales. It is the case of Shepherd[5]. We, however, are not able to analyze the rigidity of price precisely. It is not necessary for the marginal cost curve to cut across the discontinuous part in our equilibrium points in Section 2. Then it is possible to denote the length of discontinuous part of marginal revenue curves. This length determines the probability of a price rigidity. The sales-maximized equilibrium becomes more stable if the situation of the rigidity is satisfied. To put it concretely, an enterprenuer who is worried about his decision rests confident in himself due to the attainment of both principles.

Firstly, we set up the firm's behavior according to the kinked demand hypothesis. If firm i increases its price on the condition that firm j keeps its price, $\bar{p}_j$, at the starting point, $Y(y_i,y_j)$, then firm j has reaction function, that is,

$$x_j = y_j + \gamma(y_i - x_i)/\beta_j \tag{13}$$

$$(\because \bar{p}_j(=\alpha_j - \beta_j y_j - \gamma y_i) = \alpha_j - \beta_j x_j - \gamma x_i).$$

This reaction function restricts firm i's estimate. Then, firm i's value of marginal revenue, $MRd(x_i)$, is

$$MRd(x_i) = (\alpha_i - \gamma y_j - (\gamma^2 y_i)/\beta_j) - 2x_i(\beta_i\beta_j - \gamma^2)/\beta_j \tag{14}$$

$$MRd(y_i)=(\alpha_i-\gamma y_j)-y_i(2\beta_i\beta_j-\gamma^2)/\beta_j. \tag{14}$$

If firm i decreases p_i, firm j decreases p_j as soon as possible. We assume that firm i increases x_i in proportion to the rate of market share, $\varepsilon=y_j/y_i$, and has another value of price and marginal revenue, $MRD(x_i)$,

$$\left.\begin{aligned} &p_i=\alpha_i-\beta_i x_i-\gamma\varepsilon x_i \\ &MRD(x_i)=\alpha_i-2(\beta_i+\gamma\varepsilon)x_i \\ &MRD(y_i)=\alpha_i-2\beta_i y_i-2\gamma y_j \end{aligned}\right\}. \tag{15}$$

Firm i's length of discontinuous part of marginal revenue curves at Y, $MU(y_i)$, (11)-(12), is

$$MU(y_i)=y_i(\gamma^2/\beta_j)+y_j\cdot\gamma. \tag{16}$$

Now we must describe the case which is illustrated with E in Figure 1. The equilibrium point stands on the reaction curve (5). Firm 2's reaction function (5) eliminates x_2 from (14), (15) and (16).

$$\left.\begin{aligned} &MRd(y_1)=[(2\alpha_1\beta_2-\alpha_2\gamma)+(3\gamma^2-4\beta_1\beta_2)y_1]/2\beta_2 \\ &MRD(y_1)=[(2\alpha_1\beta_2-2\alpha_2\gamma)+(2\gamma^2-4\beta_1\beta_2)y_1]/2\beta_2 \\ &MU(y_1)=(\alpha_2\gamma/2\beta_2)+(\gamma^2/2\beta_2)y_1 \end{aligned}\right\}. \tag{17}$$

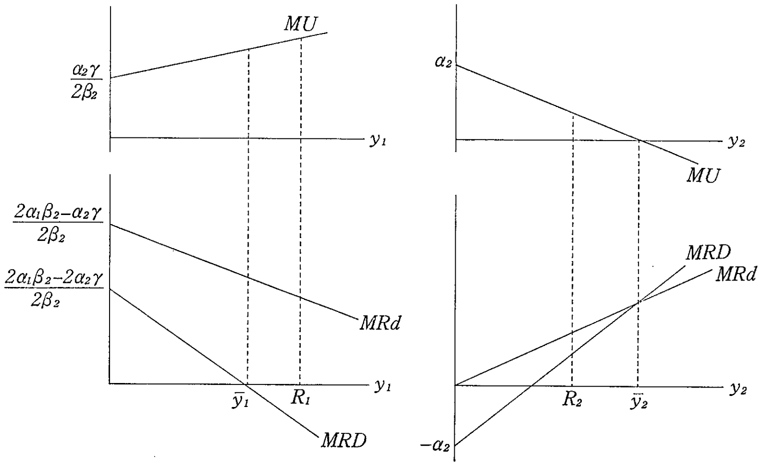
Firm 2 has similarly

$$\left.\begin{aligned} &MRd(y_2)=(\gamma^2/\beta_1)y_2 \\ &MRD(y_2)=(-\alpha_2)+2\beta_2 y_2 \\ &MU(y_2)=\alpha_2+[(\gamma^2-2\beta_1\beta_2)/\beta_1]y_2 \\ &(\because y_1=(\alpha_2-2\beta_2 y_2)/\gamma) \end{aligned}\right\}. \tag{18}$$

These value are depicted in Figure 3[7]. If firm 1 increases its cost, then iso-profit curves shift down. The equilibrium point moves to the left along with firm 2's reaction curve (5). Firm 1 reduces x_1 and firm 2 raises up x_2. On this situation, the length of both of $MU(y_1)$ and $MU(y_2)$ are shortened. If firm 1 decreases its cost, then iso-profit curves shi ft up. The equilibrium point moves to the right. Firm 1 increases x_1 and firm 2 reduces x_2. Both of $MU(y_1)$ and $MU(y_2)$ are

FIGURE 3 THE LENGTH OF DISCONTINUOUS PART OF MARGINAL CURVE

(Firm 1) (Firm 2)



lengthened.

The equilibrium point, $F(=Y)$, induces the relation, that is, as $K>0$,

$$y_2=[(\alpha_2-\gamma y_1)/(2\beta_2)]-K. \qquad (19)$$

Then values of (14), (15) and (16) are derived from (19).

$\partial MRd(y_1)/\partial K=\gamma>0,$ $\quad\partial MRD(y_1)/\partial K=2\gamma>0,$

$\partial MU(y_1)/\partial K=-\gamma<0,$

and

$\partial MRd(y_2)/\partial K=2\beta_2>0,$ $\quad\partial MRD(y_2)/\partial K=4\beta_2>0,$

$\partial MU(y_2)/\partial K=-2\beta_2<0,$

$(\because y_1=[\alpha_2-2\beta_2(y_2+K)]/\gamma).$

If either of firm increases its cost, it result in an increase of K. Then either MU is shortened. This case is applicable only to the movement between F.

Consequently, it appears that a shift of cost also brings about the change in the shape of demand curve. This explanation seems similar to Section 2. The reflex curve is found out only if the cost shifts up.

It is different from the above-mentioned point in the preceding section. The probability of the price rigidity is shown by the length of the discontinuous part of the marginal revenue curves. If an equilibrium point puts on the reaction curve, the probability falls off as the point goes away from R. If the equilibrium point is displayed by F in Figure 1, the probability falls off as any increment of either firm's cost. The length of the discontinuous part at E is imcomparable with that at F in our analysis. It is possible to denote that the further the equilibrium point stands from R, the shorter the length becomes.

4. Concluding comments

In this paper, we interpreted the kinked demand curve hypothesis as a kind of sales-maximization principle, and then focused our analysis on the shapes of demand curves. A change of the shape is emerged from *the process* of moving to a new equilibrium point because of the property of our model. It means that there is a time lag between the change of the condition of the market and the decision of the enterprenuers. The enterprenuer decides price and output after the condition has changed.

There are many incomplete points. For example, it is necessary to denote the equilibrium point such as E and F precisely. Our model has limitations. Duopoly model should be expanded into the oligopoly model because the kinked demand curve hypothesis was developed in order to analyze the oligopoly market.

If our analyses are acceptable, we suggest that the following conclusions may be drawn:

(i) The hypothesis requires that firms should be confronted with the reflex curve in boom and depression. This requirement is satisfied only if $\beta_1 < A_1$ and $\beta_2 < A_2$. In all the other conditions, firms are confronted with the reflex curve only in depression.

(ii) An increase of cost induces either the obtuse process or the reflex process, but a decrease of cost induces only the obtuse process.

(iii) The equilibrium point is most stable at R. The further the point goes away from R, the less the stability of the equilibrium becomes.

Notes

1. Sweezy mentions a condition of the demand, but he explains it incompletely. See Sweezy[6], pp. 570-571 and Reid[4], pp.31-32.
2. See Reid[4], pp. 19-20 and Shepherd[5], pp. 422.
3. See Baumol[1], Chapter 7.
4. This reaction curve departs from the point on another reaction curve (3). Firm 1 selects right side one along with the sales-maximization priciple if it has two output against x_2.
5. An increase of A_i brings about an increase of the marginal cost, $2A_ix_1$, and moves up the point of the minimum average cost to the left. An increase of B_i moves up the minimum point to the right.
6. See Reid[4], pp. 25-27.
7. As $R_1 > 0$ and $R_2 > 0$, then $2\alpha_1\beta_2 - \alpha_2\gamma > 0$ and $2\alpha_2\beta_1 - \alpha_1\gamma > 0$.

A sign of $(2\alpha_1\beta_2 - 2\alpha_2\gamma)$ is indeterminate. We assumed that the sign is positive in Figure 3.

The position between y_i and R_i is as follows,

$$\bar{y}_1 = (\alpha_1\beta_2 - \alpha_2\gamma)/(2\beta_1\beta_2 - \gamma^2),$$

$$\bar{y}_2 = (\alpha_2\beta_1)/(2\beta_1\beta_2 - \gamma^2),$$

$$R_1 - \bar{y}_1 = \beta_2\gamma(2\alpha_2\beta_1 - \alpha_1\gamma)/[(4\beta_1\beta_2 - \gamma^2)(2\beta_1\beta_2 - \gamma^2)] > 0,$$

$$R_2 - \bar{y}_2 = -[\alpha_1\gamma(2\beta_1\beta_2 - \gamma^2) + \alpha_2\beta_1\gamma^2]/[(4\beta_1\beta_2 - \gamma^2)(2\beta_1\beta_2 - \gamma^?)] < 0,$$

$$\therefore R_1 > y_1, \quad R_2 < y_2.$$

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