Role of Demand in Leontief-Sraffa System: Focusing Attention on the Duality between Quantity System and Value System

Nagata, Seiji

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Role of Demand in Leontief-Sraffa System

——Focusing Attention on the Duality between Quantity System and Value System——

Seiji Nagata

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Introduction

In his *Marx's Economics*, introducing the utility theory into Leontief-Sraffa system, Morishima advocated the doctrine stating that value = the general equilibrium price in the society with simple commodity production. In this paper it will be examined whether his advocacy is acceptable or not, paying attention to the duality between the quantity system and the value system. Especially, attentions are focused on the following two issues: (1) role of demand in Leontief-Sraffa system and (2) implications of budget constraint. Stating the conclusions beforehand, they are summerized as follows.

(1) There is no necessity of introducing the utility theory into Leontief-Sraffa system. No matter what factors determine the demand, there is the duality between the quantity system and the value sys-

tem such that if one of them has a solution then the other inevitably has. They are nothing but *Siamese twins* in the sense that they express the same condition of reproduction in terms of quantity or in terms of labour respectively. To put it strongly, in Leontief-Sraffa system even the *revealed* demand will do.

(2) So long as reproduction is maintained, the *ex ante* Walras' law implies equilibrium in labour market; there holds, as it were, Say's law in the sense that, although we are in a capitalistic society, full employment prevails. Such a consequence contradicts our experiences. Therefore, Walras' law must be abandoned; or, granting Walras' law, it should be interpreted as a mere *ex post* identity which would arose no one's interest theoretically.

§1. Mathematical Preliminaries

〔I〕 Inverse Matrices in a Decomposable System

It is well-known that the matrix $\mathbf{I}-\mathbf{A}$ induced by a productive and nonnegative matrix $\mathbf{A}$ is nonnegatively invertible[1]. Here we examine, in more detail, the composition of the inverse matrix $(\mathbf{I}-\mathbf{A})^{-1}$, assuming the decomposability of $\mathbf{A}$.

(i) $\mathbf{A}=\begin{bmatrix}\mathbf{A}_{11} & \mathbf{A}_{12}\\ \mathbf{O} & \mathbf{O}\end{bmatrix}$,

where $\mathbf{A}_{11}$ is indecomposable. In this instance

$$(\mathbf{I}-\mathbf{A})^{-1}=\begin{bmatrix}(\mathbf{I}-\mathbf{A}_{11})^{-1} & (\mathbf{I}-\mathbf{A}_{11})^{-1}\mathbf{A}_{12}\\ \mathbf{O} & \mathbf{I}\end{bmatrix}\geq\mathbf{O}, \qquad (1.\ 1)$$

where

$$(\mathbf{I}-\mathbf{A}_{11})^{-1}>\mathbf{O}^{2)}.$$

(ii) $\mathbf{A}=\begin{bmatrix}\mathbf{A}_{11} & \mathbf{A}_{12}\\ \mathbf{O} & \mathbf{A}_{22}\end{bmatrix}$,

where $\mathbf{A}_{11}$ and $\mathbf{A}_{12}$ are both indecomposable.

$$(\mathbf{I}-\mathbf{A})^{-1}=\begin{bmatrix}(\mathbf{I}-\mathbf{A}_{11})^{-1} & (\mathbf{I}-\mathbf{A}_{11})^{-1}\mathbf{A}_{12}(\mathbf{I}-\mathbf{A}_{22})^{-1}\\ \mathbf{O} & (\mathbf{I}-\mathbf{A}_{22})^{-1}\end{bmatrix}\geqq\mathbf{O};\qquad(1.\ 2)$$

$$(\mathbf{I}-\mathbf{A}_{11})^{-1},\ (\mathbf{I}-\mathbf{A}_{22})^{-1}>\mathbf{O}.$$

(iii) $\mathbf{A}=\begin{bmatrix}\mathbf{A}_{11} & \mathbf{A}_{12}\\ \mathbf{O} & \mathbf{A}_{22}\end{bmatrix}$,

where $\mathbf{A}_{11}$ is indecomposable but $\mathbf{A}_{12}$ is decomposable. Some repetitions of procedures (i) and/or (ii) give the inverse matrix in question. For instance, if $\mathbf{A}$ is a matrix such that

$$\mathbf{A}=\begin{pmatrix}\mathbf{A}_{11} & \mathbf{A}_{12} & \mathbf{A}_{13}\\ \mathbf{O} & \mathbf{A}_{22} & \mathbf{A}_{23}\\ \mathbf{O} & \mathbf{O} & \mathbf{A}_{33}\end{pmatrix},$$

where $\mathbf{A}_{11}$, $\mathbf{A}_{22}$, and $\mathbf{A}_{33}$ are indecomposable, we define $\mathbf{B}_{11}$, $\mathbf{B}_{12}$, and $\mathbf{B}_{22}$ as follows.

$$\mathbf{B}_{11}=\begin{bmatrix}\mathbf{A}_{11} & \mathbf{A}_{12}\\ \mathbf{O} & \mathbf{A}_{22}\end{bmatrix},\quad \mathbf{B}_{12}=\begin{bmatrix}\mathbf{A}_{13}\\ \mathbf{A}_{23}\end{bmatrix},\quad \mathbf{B}_{22}=\mathbf{A}_{33},$$

$$\mathbf{A}=\begin{bmatrix}\mathbf{B}_{11} & \mathbf{B}_{12}\\ \mathbf{O} & \mathbf{B}_{22}\end{bmatrix}.$$

Then we can apply the procedure (ii) to $(\mathbf{I}-\mathbf{B}_{11})^{-1}$.

$$(\mathbf{I}-\mathbf{A})^{-1}=\begin{bmatrix}(\mathbf{I}-\mathbf{B}_{11})^{-1} & (\mathbf{I}-\mathbf{B}_{11})^{-1}\mathbf{B}_{12}(\mathbf{I}-\mathbf{B}_{22})^{-1}\\ \mathbf{O} & (\mathbf{I}-\mathbf{B}_{22})^{-1}\end{bmatrix}\geqq\mathbf{O},$$

where

$$(\mathbf{I}-\mathbf{B}_{11})^{-1}=\begin{bmatrix}(\mathbf{I}-\mathbf{A}_{11})^{-1} & (\mathbf{I}-\mathbf{A}_{11})^{-1}\mathbf{A}_{12}(\mathbf{I}-\mathbf{A}_{22})^{-1}\\ \mathbf{O} & (\mathbf{I}-\mathbf{A}_{22})^{-1}\end{bmatrix},$$

$$\mathbf{B}_{12}=\begin{bmatrix}\mathbf{A}_{13}\\ \mathbf{A}_{23}\end{bmatrix},\quad (\mathbf{I}-\mathbf{B}_{22})^{-1}=(\mathbf{I}-\mathbf{A}_{33})^{-1}.$$

〔II〕 Duality

Let ρ be a real number. Since the principal minors of the original system

$$(\rho\mathbf{I}-\mathbf{A})\mathbf{x}=\mathbf{c},\quad \mathbf{c}\geqq\mathbf{O},\quad \mathbf{x}\geqq\mathbf{O}\qquad(1.\ 3)$$

and those of the dual system

$$(\rho\mathbf{I}-\mathbf{A}')\mathbf{p}=\mathbf{v},\quad \mathbf{v}\geqq\mathbf{O},\quad \mathbf{p}\geqq\mathbf{O}\qquad(1.\ 4)$$

are equal in numerical value each other, it follows that if one of these systems satisfies the Hawkins-Simon condition, then the other satisfies the same condition. Consequently, if one of them has a set of nonnegative solutions, the other also has[3]. Moreover, there holds the relation between any solution $\mathbf{x}$ for the original system (1. 3) and any solution $\mathbf{p}$ for the dual system (1. 4) such that

$$\mathbf{p}'\mathbf{c}=\mathbf{v}'\mathbf{x}. \qquad (1.\ 5)$$

Indeed

$$\mathbf{p}'\mathbf{c}=\mathbf{p}'(\rho\mathbf{I}-\mathbf{A})\mathbf{x}=\mathbf{v}'\mathbf{x}.$$

The latter half of this proposition, *i. e.* (1. 5), holds independently of whether the first half is met or not.

Notes

1) See Nikaido [8], [9], or [10], for example.
2) Since $\mathbf{A}_{11}$ is an indecomposable and nonnegative matrix, according to Frobenius' theorem, this inequality holds.
3) See Nikaido [10] pp. 93–95.

§ 2. Utility Theory and Labour Value

In his *Marx's Economics* Morishima proposed an interpretation of labour value such that

> 'values are the general equilibrium prices in the society with simple commodity production[1)]';
>
> 'the values are the general equilibrium prices prevailing in the society with simple commodity production where people behave in the Walrasian manner[2)]',

where the society with simple commodity production stands for a *imaginary* society in which

> 'the labourers themselves are in possession of their respective means of production and exchange their commodities with one another[3)]'.

And he regarded the society with simple commodity production[4] as a *normative* or *ideal* one in the sense that it is comparable with capitalistic society.

Summing up his arguments, the following six items are listed.

i) Demand

Let the utility function and the budget constraint of the jth individual ($j=1, 2, \cdots, N^s$) be

$$u^j = u^j(\mathbf{d}^j), \tag{2. 1}$$

$$\mathbf{p}'\mathbf{d}^j = w\sigma \tag{2. 2}$$

respectively, where

$\mathbf{d}^j = [d_i^j]$,

d_i^j : the jth individual's demand for the ith commodity,

$\mathbf{p} = [p_i]$,

p_i : the price of the ith commodity,

w : the income rate[5],

σ : the standard working day.

And we assume that

$$\frac{\partial u^j}{\partial \mathbf{d}^j}(\mathbf{d}^j) = \left[\frac{\partial u^j}{\partial d_i^j}(\mathbf{d}^j)\right] > \mathbf{O}. \tag{2. 3}$$

Solving the problem of utility maximization under budget constraint

$$\max_{\mathbf{d}^j} \quad u^j(\mathbf{d}^j) \quad \text{subject to} \quad \frac{1}{w}\mathbf{p}'\mathbf{d}^j = \sigma,$$

we get well-known Lagrange conditions.

$$\frac{\partial u^j}{\partial \mathbf{d}^j}(\mathbf{d}^j) - \lambda\frac{\mathbf{p}}{w} = \mathbf{O}, \tag{2. 4}$$

$$\sigma - \frac{1}{w}\mathbf{p}'\mathbf{d}^j = 0, \tag{2. 5}$$

where λ denotes the Lagrange multiplier. According to these conditions, the following demand function is obtained[6].

$$\mathbf{d}^j=\mathbf{d}^j(\mathbf{p}/w), \tag{2. 6}$$

$$\frac{1}{w}\mathbf{p}'\mathbf{d}^j=\sigma. \tag{2. 7}$$

The equation (2. 7) stands for the budget constraint and nothing other. Summing up demand functions over all individuals, we get the demand function of this society

$$\mathbf{d}=\mathbf{d}(\mathbf{p}/w), \tag{2. 8}$$

$$\mathbf{d}=\sum_{j=1}^{N^S}\mathbf{d}^j$$

and society's budget constraint or Walras' law[7)]

$$\frac{1}{w}\mathbf{p}'\mathbf{d}=\sigma N^S. \tag{2. 9}$$

ii) Production

Let **A**, **x**, and **f** be Leontief matrix, gross product, and net product respectively. There holds the input-output relation between them such that

$$\mathbf{Ax}+\mathbf{f}=\mathbf{x}. \tag{2. 10}$$

Suppose that **A** is a productive and nonnegative matrix, then there exists the inverse matrix $(\mathbf{I}-\mathbf{A})^{-1}\geqq\mathbf{O}$. Therefore

$$(\mathbf{I}-\mathbf{A})^{-1}\mathbf{f}=\mathbf{x}, \tag{2. 11}$$

and, according to $\mathbf{f}\geqq\mathbf{O}$, nonnegative **x** is uniquely determined.

iii) Employment[8)]

Let $\mathbf{a}_0$ be the labour input coefficient, that is

$\mathbf{a}_0=[a_{0j}]$,

a_{0j} : the quantity of labour required for the production of one unit of the jth commodity,

then, in accordance with the output **x** determined in ii), the required labour L^D is determined by the following equation.

$$\mathbf{a}_0{}'\mathbf{x}=L^D. \tag{2. 12}$$

Since the standard working day is σ, there holds

$$\mathbf{a}_0{}'\mathbf{x}=\sigma N^D, \tag{2. 13}$$

where N^D denotes the required workers.

iv) Price

In S. S. C. P., prices satisfying conditions of reproduction expressed as (2.10) meet the equation (2.14).

$$\mathbf{A}'\mathbf{p}+w\mathbf{a}_0=\mathbf{p}. \tag{2. 14}$$

Accordingly, the price vector in terms of the wage unit $\mathbf{p}/w$ is uniquely determined by (2.15) below.

$$\frac{\mathbf{p}}{w}=(\mathbf{I}-\mathbf{A}')^{-1}\mathbf{a}_0. \tag{2. 15}$$

v) Value

Let us define the value of the jth commodity as labour required directly or indirectly for the production of one unit of that commodity. Denoting the value vector by $\mathbf{v}$, therefore, there holds

$$\mathbf{A}'\mathbf{v}+\mathbf{a}_0=\mathbf{v}. \tag{2. 16}$$

So

$$\mathbf{v}=(\mathbf{I}-\mathbf{A}')^{-1}\mathbf{a}_0. \tag{2. 17}$$

vi) Market Equilibrium

The equilibrium condition between the demand determined by (2. 8) and the net product expressed as (2.10) is as follows;

$$\mathbf{d}=\mathbf{f}. \tag{2. 18}$$

On the other hand, the equilibirum condition in labour market is expressed as

$$N^D=N^S. \tag{2. 19}$$

Determination of variables in the systen ⅰ)-ⅵ) is as follows. First, according to equations (2.15) and (2.17), both $\mathbf{p}/w$ and $\mathbf{v}$ are determined exclusively by Leontief matrix and the labour input coefficient. In accordance with $\mathbf{p}/w$ obtained in this way, the demand $\mathbf{d}$ is determined by (2. 8). Next, equations (2.18) and (2.10) determine the output $\mathbf{x}$. Finally, as is shown in (2.13), the demand for labour N^D is determined. Besides, satisfactorily enough, it is equal to the supply of labour N^S appeared in (2. 9). In other words, the equilibrum condition in labour market is spontaneously met. Indeed

$$N^D=\frac{1}{\sigma}\mathbf{a}_0'\mathbf{x}=\frac{1}{\sigma}\mathbf{a}_0'(\mathbf{I}-\mathbf{A})^{-1}\mathbf{f}=\frac{1}{\sigma}\frac{1}{w}\mathbf{p}'\mathbf{f}=N^S.$$

This process is illustrated with the Figure 1 below.

$$\left.\begin{matrix}\mathbf{A}\\ \mathbf{a}_0\end{matrix}\right\}\begin{matrix}\nearrow \dfrac{\mathbf{p}}{w}\longrightarrow\mathbf{d}\longrightarrow\mathbf{f}\longrightarrow\mathbf{x}\longrightarrow N^D(=N^S)\\ \searrow \mathbf{v}\left(=\dfrac{\mathbf{p}}{w}\right)\end{matrix}$$

Figure 1. Determination of variables in S. S. C. P.

Comparing (2.15) with (2.17), we quite obviously see that

$$\frac{\mathbf{p}}{w}=\mathbf{v}. \tag{2. 20}$$

Here exists the rationale for Morishima's advocacy of the doctrine stating that value = the general equilibirum price in S. S. C. P..

So far we regarded the matrix $\mathbf{A}$ as a productive and nonnegative matrix generally. Now, following Morishima, let us assume $\mathbf{A}$ to be a matrix expressed in §1 (ⅰ). It will be noted, nevertheless, that the above arguments are still adequate irrespective of physical classification between capital goods and consumer goods. Anyhow, rewriting the above system into Morishima's expression, we get the following;

$$\mathbf{d}=\mathbf{d}(\mathbf{p}_{\text{II}}/w), \tag{2. 8'}$$

$$\frac{1}{w}[\mathbf{p}_{\text{I}}'\ \mathbf{p}_{\text{II}}']\begin{bmatrix}\mathbf{O}\\ \mathbf{d}\end{bmatrix}=\sigma N^S, \tag{2. 9'}$$

$$\begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{O} & \mathbf{O} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{\mathrm{I}} \\ \mathbf{x}_{\mathrm{II}} \end{bmatrix} + \begin{bmatrix} \mathbf{f}_{\mathrm{I}} \\ \mathbf{f}_{\mathrm{II}} \end{bmatrix} = \begin{bmatrix} \mathbf{x}_{\mathrm{I}} \\ \mathbf{x}_{\mathrm{II}} \end{bmatrix}, \tag{2. 10'}$$

$$[\mathbf{a}_{0\mathrm{I}}' \quad \mathbf{a}_{0\mathrm{II}}'] \begin{bmatrix} \mathbf{x}_{\mathrm{I}} \\ \mathbf{x}_{\mathrm{II}} \end{bmatrix} = \sigma N^D, \tag{2. 13'}$$

$$\begin{bmatrix} \mathbf{A}_{11}' & \mathbf{O} \\ \mathbf{A}_{12}' & \mathbf{O} \end{bmatrix} \begin{bmatrix} \mathbf{p}_{\mathrm{I}} \\ \mathbf{p}_{\mathrm{II}} \end{bmatrix} + w \begin{bmatrix} \mathbf{a}_{0\mathrm{I}} \\ \mathbf{a}_{0\mathrm{II}} \end{bmatrix} = \begin{bmatrix} \mathbf{p}_{\mathrm{I}} \\ \mathbf{p}_{\mathrm{II}} \end{bmatrix}, \tag{2. 14'}$$

$$\begin{bmatrix} \mathbf{A}_{11}' & \mathbf{O} \\ \mathbf{A}_{12}' & \mathbf{O} \end{bmatrix} \begin{bmatrix} \mathbf{v}_{\mathrm{I}} \\ \mathbf{v}_{\mathrm{II}} \end{bmatrix} + \begin{bmatrix} \mathbf{a}_{0\mathrm{I}} \\ \mathbf{a}_{0\mathrm{II}} \end{bmatrix} = \begin{bmatrix} \mathbf{v}_{\mathrm{I}} \\ \mathbf{v}_{\mathrm{II}} \end{bmatrix}, \tag{2. 16'}$$

$$\begin{bmatrix} \mathbf{O} \\ \mathbf{d} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_{\mathrm{I}} \\ \mathbf{f}_{\mathrm{II}} \end{bmatrix}, \tag{2. 18'}$$

$$N^D = N^S, \tag{2. 19}$$

where it is assumed that departments I and II produce capital goods and consumer goods respectively. In this instance we ascertain that

$$N^D = \frac{1}{\sigma} \frac{1}{w} \mathbf{p}_{\mathrm{II}}' \mathbf{d} = N^S,$$

$$\begin{bmatrix} \mathbf{p}_{\mathrm{I}}/w \\ \mathbf{p}_{\mathrm{II}}/w \end{bmatrix} = \begin{bmatrix} \mathbf{v}_{\mathrm{I}} \\ \mathbf{v}_{\mathrm{II}} \end{bmatrix} = \begin{bmatrix} (\mathbf{I} - \mathbf{A}_{11}')^{-1} & \mathbf{O} \\ \mathbf{A}_{12}' (\mathbf{I} - \mathbf{A}_{11}')^{-1} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{a}_{0\mathrm{I}} \\ \mathbf{a}_{0\mathrm{II}} \end{bmatrix}.$$

Notes

1) See Morishima [5] p. 44.
2) See Morishima [5] p. 45.
3) See Morishima [5] p. 36.
4) From now on this word will be abbreviated to S. S. C. P..
5) By definition there exist no workers in S. S. C. P., so that income never turns into the wage form.
6) For a strict proof the implicit function theorem should be applied to those conditions. Having nothing to do with the following arguments, it is omitted.
7) See Negishi [7] p. 66 or Lancaster [4] chapter 9.
8) Strictly speaking, the term employment is a specific concept incidental to the capitalistic relations of production. Since an apt term could not be found, this term is applied to the relation between labour input and physical output in S. S. C. P. here.

§ 3. Budget Constraint and Say's Law

It is not a mere chance that there holds the equilibirum in labour market as (2.19) shows. As a matter of fact, whenever the equilibrium condition (2.18) is met in commodity market, so long as the budget constraint or Walras' law (2. 9) holds, labour market is in equilibrium in S. S. C. P.. In the consequence, in this instance, the budget constraint plays a role of Say's law in the sense that whenever commodity market is in equilibrium also labour market is. Some embracers of utility theory may be discontented with this consequence. It may be objected that the equality in the budget constraint is extremely strict so that nowadays inequality in (2. 2) is generally admitted. In other words, now the equation (2. 2) must be replaced by

$$\mathbf{p}'\mathbf{d}^j \leqq w\sigma, \tag{3. 1}$$

and, correspondingly, the problem of utility maximization is nowadays rewritten as (3. 2) below.

$$\max_{\mathbf{d}^j} \quad u^j(\mathbf{d}^j) \quad \text{subject to} \quad \frac{1}{w}\mathbf{p}'\mathbf{d}^j \leqq \sigma. \tag{3. 2}$$

It is regrettable, nevertheless, that the equation (3. 1) always turns out to be equality and that Say's law prevails; for, according to Kuhn=Tucker conditions[1], there holds the following.

$$\frac{\partial u^j}{\partial \mathbf{d}^j}(\mathbf{d}^j) - \lambda\frac{\mathbf{p}}{w} \leqq \mathbf{O}^{2)}, \tag{3. 3}$$

$$\left(\frac{\partial u^j}{\partial \mathbf{d}^j}(\mathbf{d}^j) - \lambda\frac{\mathbf{p}}{w}\right)'\mathbf{d}^j = 0, \tag{3. 4}$$

$$\mathbf{d}^j \geqq \mathbf{O}, \tag{3. 5}$$

$$\sigma - \frac{1}{w}\mathbf{p}'\mathbf{d}^j \geqq 0, \tag{3. 6}$$

$$\lambda\left(\sigma - \frac{1}{w}\mathbf{p}'\mathbf{d}^j\right) = 0, \tag{3. 7}$$

$$\lambda \geqq 0. \tag{3. 8}$$

Suppose that λ were equal to zero. Then the inequality (3. 3) contradicts the assumption (2. 3), so that λ is positive. Taking the equation (3. 7) into consideration, the positiveness of λ implies that (3. 6) turns out to be equality[3]. After all, *harmonie préétablir* comes true also in this instance.

Viewing this subject from another angle, the odds are still against Morishima. As a matter of fact, there is no necessity of introducing the utility theory into the theory taken up in §2. Indeed, no matter what factor determines the demand **d**, so far as Walras' law (2. 9) is assumed, labour market is always in equilibrium. It is not surprising at all that, in spite of such poor information about the demand function, Say's law still prevails; for replacement of the equation (2. 8) by

$$\mathbf{d}=\bar{\mathbf{d}}\,(\text{const.}) \tag{3. 9}$$

makes no serious alteration hereafter, so that always $N^D=N^S$ holds. In this case the Figure 1 turns out to be the Figure 2 below.

$$\mathbf{d} \longrightarrow \mathbf{f} \longrightarrow \mathbf{x} \longrightarrow N^D(=N^S)$$

$$\left.\begin{matrix}\mathbf{A}\\ \mathbf{a}_0\end{matrix}\right\} \begin{matrix}\nearrow \dfrac{\mathbf{p}}{w}\\ \searrow \mathbf{v}\left(=\dfrac{\mathbf{p}}{w}\right)\end{matrix}$$

Figure 2. Determination of variables with exogenous demand

Notes

1) As to Kuhn-Tucker conditions, see Intriligator [1] chapter 4 or [2].

2) Following Morishima, in the department I this turns out to be strict inequality, so that, according to (3. 4) and (3. 5), $\mathbf{d}_{\mathrm{I}}^{\mathrm{I}}=\mathbf{O}$. In other words, in the department I utilities of goods are not so great that consumption demand would not be positive; but they are indispensable, as the means of production, for the production of the goods with positive consumption demand, *i. e.* the goods in the department II. Therefore, in spite of their lacking consumption demand, every goods in the department I has positive output and posi-

tive value. Thus, economic theories have to be investigated from the standpoint of reproduction, otherwise there is the possibility of lapsing into the $v+m$ dogma and feeling embarrassment in face of the fact that whereas $\mathbf{d}_{\mathrm{I}}^{i}=\mathbf{O}$, $\mathbf{p}>\mathbf{O}$. Incidentally, in a capitalistic economy the demand for capital goods should not be explained in terms of utility. Capitalist is by no means interested in any material element of capital goods; the only matter of concern he interested in is profitability of investment or, in terms of labour, production of surplus value.

3) See Intriligator 〔1〕 p. 150 or 〔2〕 pp. 80-81.

§ 4. Reproduction and Duality

In the preceding section irrationality of utility theoretic interpretation of valus as the general equilibrium price in S. S. C. P. was shown. Whatever factors determine the demand, there holds the duality, as we have seen in §1, between output $\mathbf{x}$ and labour value $\mathbf{v}$ so long as they satisfy the conditions of reproduction (2.10) and (2.16) respectively. In other words, there holds the relation between equations

$$(\mathbf{I}-\mathbf{A})\mathbf{x}=\mathbf{f}, \quad \mathbf{f}\geqq\mathbf{O}, \quad \mathbf{x}\geqq\mathbf{O} \tag{4. 1}$$

and

$$(\mathbf{I}-\mathbf{A}')\mathbf{v}=\mathbf{a}_0, \quad \mathbf{a}_0\geqq\mathbf{O}, \quad \mathbf{v}\geqq\mathbf{O} \tag{4. 2}$$

such that if one of them has a solution then the other inevitably has; moreover that always

$$\mathbf{v}'\mathbf{f}=\mathbf{a}_0{}'\mathbf{x} \tag{4. 3}$$

holds, so that the value of net product is identically equal to the required labour. That is all we can say. There leaves any room neither for the utility function as a subjective factor nor for the budget constraint expressing *harmonie préétablir*. Thus the quantity system and the value system, expressed as (4. 1) and (4. 2) respectively, are nothing but *Siamese twins* in the sense that they express the same condition of reproduction in terms of quantity or that of labour respectively. Consequently, so long as reproduction is maintained, the duality mentioned above would be found in any society, ——whether in a capitalistic society or in a socialistic society, besides in a feudalistic one——; whereas it does not imply any occurrence in such a *imaginary* society at all as the society with simple commodity production. In other words, throughout the ages, the duality holds for the production in a society stamping its hallmark with history.

Incidentally, since

$$(\mathbf{I}-\mathbf{A}')\mathbf{p}/w=\mathbf{a}_0, \quad \mathbf{a}_0 \geqq \mathbf{O}, \quad \mathbf{p}/w \geqq \mathbf{O} \tag{4. 4}$$

holds in a price system satisfying the equation (2.14), there holds the duality proposition between equations (4. 4) and (4. 1), and between equations (4. 2) and (4. 1), so that these three equations are trinitarian. In the consequence there holds the following;

$$\mathbf{p}'\mathbf{f}=w\mathbf{a}_0'\mathbf{x}, \tag{4. 5}$$

$$\frac{\mathbf{p}}{w}=\mathbf{v}, \tag{4. 6}$$

that is, national income produced equals national income distributed, and price in terms of the wage unit is equal to value.

Now, suppose that a capitalistic price system (4. 7) prevails. Is there a duality between prices and quantities on earth ?

$$(1+r)(\mathbf{A}'\mathbf{p}+w\mathbf{a}_0)=\mathbf{p}, \tag{4. 7}$$

where r denotes the rate of profit and w the wage rate. This reduces to

$$(\mathbf{I}-\mathbf{A}')\mathbf{p}=r\mathbf{A}'\mathbf{p}+(1+r)w\mathbf{a}_0. \tag{4. 8}$$

If $r=0$, this equals the equation (4. 4), so that the value, the price, and the quantity systems are trinitarian. On the other hand, once a positive rate of profit is recognized, the first half of the duality proposition is unavailable. Even if there is a solution satisfying (4. 1), it does not necessarily follow that there exists a solution satisfying (4. 8)[1]. Economically speaking, production would not be performed in a capitalistic society unless any profit were yielded. On the other hand, whenever a solution for the latter exists, also a solution for the former does; in other words, on sufficiently profitable prices there comes about an incentive for capitalist to perform production.

Incidentally, according to the latter half of the duality proposition, there identically holds the relation between solutions for (4. 1) and for (4. 7) such that

$$\mathbf{p}'\mathbf{f} = r(\mathbf{p}'\mathbf{A}\mathbf{x} + w\mathbf{a}_0{}'\mathbf{x}) + w\mathbf{a}_0{}'\mathbf{x}, \qquad (4.\ 9)$$

that is,

national income produced = profit + wages

(= national income distributed).

Since $\mathbf{f}=\mathbf{d}$, $\mathbf{p}'\mathbf{d}=\mathbf{p}'\mathbf{f}$, so that

national income spended = national income produced

= national income distributed.

These equations imply so-called *equivalent of three aspects*. Though these equations seem to imply Walras' law, this is not true; for $\mathbf{a}_0{}'\mathbf{x}$ is stubbornly labour demanded or labour employed, but is not labour supply at all. Once we accept Walras' law, so long as commodity market is in equilibrium, there would be no unvoluntary unemployment at all even though we are in a capitalistic society. Indeed, since Walras' law implies that

$$\mathbf{p}'\mathbf{d} = r(\mathbf{p}'\mathbf{A}\mathbf{y} + w\sigma N^S) + w\sigma N^S$$

where

$$\mathbf{y} = \mathbf{A}\mathbf{y} + \mathbf{d},$$

there holds the following proposition;

$$\mathbf{d} = \mathbf{f} \Longrightarrow \mathbf{y} = \mathbf{x}$$

$$\Longrightarrow N^D=\frac{1}{\sigma}\mathbf{a}_0{}'\mathbf{x}=\frac{1}{\sigma}\frac{1}{w}\frac{1}{1+r}(\mathbf{p}'\mathbf{f}-r\mathbf{p}'\mathbf{A}\mathbf{x})=N^S.$$

In a capitalistic economy there does not exist *the Providence of God* celebrating *harmonie préétablir*. We must explode the superstition of the *classical*[2] *Euclidean* world, and direct our steps towards a *Keynesian non-Euclidean* world[3].

Notes

1) The condition of the existence of solutions satisfying (4. 8) is as follows.

$$R \geqq r,$$

where R denotes the maximum rate of profit defined as

$$R=\frac{1}{\lambda(\mathbf{A})}-1,$$

$\lambda(\mathbf{A})$: the Frobenius root of $\mathbf{A}$.

See Nagata [6], also Nikaido [8], [9], or [10].

2) Here usage of the term *classical* follows Keynes'. See Keynes [3] chapters 1 and 2. Ironically enough, Leontief-Sraffa system succeeds to the Ricardian tradition as the original classical economics.

3) It was Keynes who exquisitely pointed out this in 1936. See Keynes [3] pp. 16–17. But his warning seems to have been forgotten as so-called neo-classicals had cut a conspicuous figure.

Concluding Remarks

As has already been seen, Morishima's utility theoretic interpretation of value is fallacious. First, value is not the equilibrium price in such a *imaginary* society as S. S. C. P., but another version of physical reproduction in terms of labour. Embodying in itself the general condition of reproduction which must be dealt with in any society, value has no connection with any specific form, whether a capitalistic price system or, if any, a price system in S. S. C. P.. In the second place, Walras' law is not acceptable at all, except when it is regarded as a mere *ex post* identity, on the ground that the law implies full employment so long as reproduction is maintained. In an economy where Walras' law prevails, there leaves no room for the specific character of labour-power commodity to be introduced, although this commodity is specific in the sense that, for subsistence, workers are obliged to sell their own labour=power as a commodity and that it is the very commodity that creates value.

As Okishio had set out[1], reproduction must be dealt with synthetically from two aspects: the general aspect and the specific. Morishima committed a double error. On the one hand he took the general concept of value for such a specific concept as the general equilibrium price in S. S. C. P.; on the other hand he introduced the general concept of utility into the analysis of labour market where the specific character of labour-power commodity, especially its condition of reproduction, would be essential. These errors seem to originate from Morishima's bias in favour for the general equilibrium theory. Having made a compromise between consecutive production and inconsecutive demand, he became a eclectic and adhered to the Hicksian principle of living only for the moment. Was he too Hicksian to analize a capitalistic economy?

Note

1) See Okishio 〔11〕 pp. 3–8.

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