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Napier's Constant, Imaginary Unit, Circular Constant and Some Natural Phenomena

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This study investigated relationships between some mathematical constants [Napier's constant (e), imaginary unit ($\boldsymbol{i}$), circular constant (π)] and some natural phenomena. The results obtained were as follows. The equality between exponential function based on e and Bondi K–factor suggested hypotheses that both expressions required each other and exponential function was associated with the space–time continuum. The mathematical application of $v \to \infty$ to Bondi K–factor led to $\boldsymbol{i}$, where $\boldsymbol{i}$ was interpreted as something other than v due to the definition of v as $0 \leq v < c$ (c=speed of light in vacuum). This suggested a hypothesis that the role of $\boldsymbol{i}$, though the phenomenon related to $\boldsymbol{i}$ looked like the phenomenon related to the infinite velocity, was to prohibit the state of $c \leq v < \infty$ by taking the form of wave. Euler's identity was related with Bondi K–factor to the power of $\boldsymbol{i}$ where v=$c(\exp(2\pi)-1)/(\exp(2\pi)+1) \approx 0.996c$ (≈ muon's velocity). Applying $v \to \infty$ to Bondi K–factor to the power of $\boldsymbol{i}$ gave '$\boldsymbol{i}$ to the power of $\boldsymbol{i}$', a many–valued function that suggested numeral approximations to some physical constants. Some natural phenomena including metallic number and muon's velocity were related to π. It was suggested that e, $\boldsymbol{i}$ and π were the basso continuo being played in some natural phenomena.

Key words: Bondi K–factor, circular constant, imaginary unit, Napier's constant, natural phenomena

INTRODUCTION

Exponential function based on Napier's constant is frequently used for the simulation of various natural phenomena including the growth of plants and animals. Shimojo and Nakano (2013) investigated relationships between exponential functions and some natural phenomena. Circular constant and imaginary unit are also important components of many simulation models.

The present study was designed to investigate relationships between some mathematical constants (Napier's constant, imaginary unit, circular constant) and some natural phenomena by giving additional explanations and information to the previous report (Shimojo and Nakano, 2013).

THREE MATHEMATICAL CONSTANTS AND SOME NATURAL PHENOMENA

Exponential function, Bondi K–factor and related problems

As shown in reports (Bondi, 1964; Shimojo, 2011b, 2011c; Shimojo and Nakano, 2012, 2013), solving simultaneous equations in hyperbolic function and Lorentz factor gives expressions that connect exponential function to Bondi K–factor (1) or to Lorentz factor (2),

$$\exp(\pm\theta) = \sqrt{\frac{1 \pm v/c}{1 \mp v/c}}, \tag{1}$$

$$= \frac{1}{\sqrt{1-(v/c)^2}} \cdot (1 \pm v/c), \tag{2}$$

where $0 \leq \theta < \infty$, v=velocity of matter, c=speed of light in vacuum, $0 \leq v < c$, double–sign corresponds.
Expression (1) seems to suggest hypotheses that exponential function and Bondi K–factor require each other and exponential function is associated with the space–time continuum.

As shown in a report (Shimojo and Nakano, 2012), replacing $\pm\theta$ with rt gives

$$\exp(rt) = \sqrt{\frac{1+v/c}{1-v/c}}, \tag{3}$$

where r=relative growth rate, t=time.
One of the possible interpretations of expression (3) seems to suggest a hypothesis that when the weight or space shows an exponential increase ($r>0$, $t>0$), there is a causal energy ($v>0$). Expression $\exp(rt)$ is used to simulate the growth of an individual plant (Blackman, 1919) or animal (Brody, 1945).

By the way, the power series of $\exp(rt)$ is given by

$$\exp(rt) = \sum_{k=0}^{\infty} \frac{(rt)^k}{k!}. \tag{4}$$

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The definite integral of power series of exp(rt) with respect to time is given by

$$\int_0^t \left(\sum_{k=0}^{\infty} \frac{(rt)^k}{k!} \right) dt = \frac{1}{r} \left[\sum_{k=1}^{\infty} \frac{(rt)^k}{k!} \right]_0^t . \quad (5)$$

If based on the power series, then the conservation of the form of exp(rt) under its definite integral seems to suggest a mathematical field that causes the pair appearance of '1' and '–1' (Shimojo *et al*., 2004),

$$\frac{1}{r} \left[\sum_{k=1}^{\infty} \frac{(rt)^k}{k!} \right]_0^t = \frac{1}{r} \left[\left(1 + \sum_{k=1}^{\infty} \frac{(rt)^k}{k!} \right) - 1 \right]_0^t , \quad (6)$$

$$= \frac{1}{r} \left[\sum_{k=0}^{\infty} \frac{(rt)^k}{k!} - 1 \right]_0^t . \quad (7)$$

The numeral '1' seems to suggest a hypothetic condensation of many sets of complex number and its complex conjugate (Shimojo *et al*., 2004),

$$1 = \prod_{k=1}^{m} \exp(\boldsymbol{i}\theta_k) \cdot \exp(-\boldsymbol{i}\theta_k), \quad (8)$$

where $\boldsymbol{i}$ = imaginary unit.
The natural logarithm of expression (8) gives

$$0 = \sum_{k=1}^{m} ((\boldsymbol{i}\theta_k) + (-\boldsymbol{i}\theta_k)), \quad (9)$$

$$= \sum_{k=1}^{m} \left(\exp\left(\boldsymbol{i} \left(\frac{1}{2} + 2n \right) \pi \right) \theta_k + \exp\left(-\boldsymbol{i} \left(\frac{1}{2} + 2n \right) \pi \right) \theta_k \right), \quad (10)$$

where n=set of all integers, π=circular constant.
This seems to suggest a collapse of the hypothetic condensation (8), leading to the pair appearance of $\boldsymbol{i}\theta_k$ and $-\boldsymbol{i}\theta_k$ and the immediate pair disappearance of them. This is based on the principal value (n=0). Does the many–valued property of complex functions [ln(1)=$2n\pi\boldsymbol{i}$, $\pm\boldsymbol{i}(\theta_k+2n\pi)$, $n\neq0$] seem to suggest a hypothesis that $2n\pi\boldsymbol{i}$ plays a background role in this phenomenon?

If there are replacements (11) and (12) in expression (8), then does this seem to suggest a hypothetic condensation of matter and antimatter?

$$\exp(\boldsymbol{i}\theta) \rightarrow A \exp\left(\boldsymbol{i} \frac{2\pi}{h} (px - Et) \right), \quad (11)$$

$$\exp(-\boldsymbol{i}\theta) \rightarrow A \exp\left(-\boldsymbol{i} \frac{2\pi}{h} (px - Et) \right), \quad (12)$$

where A=amplitude, p=momentum, x=position, E=energy, h=Planck constant.
Those hypotheses suggested in this section will be severely criticized or disregarded.

Limitation of the velocity of matter

The velocity (v) at which the matter moves is limited by $0\leq v<c$, where c is speed of light in vacuum. Since the state that is equal to or greater than c does not exist, the phenomenon that looks like the phenomenon associated with $c\leq v<\infty$ is considered something other than v. This hypothesis will be severely criticized or disregarded.

Mathematical application of $v \rightarrow \infty$ to Bondi K–factor and Lorentz transformation

The mathematical application of $v \rightarrow \infty$ to Bondi K–factor and Lorentz transformation breaks them and leads to expressions (13) ~ (20) (Shimojo, 2011b, 2011c; Shimojo and Nakano, 2013),

$$\lim_{v\to\infty} \left(\sqrt{\frac{1 \pm v/c}{1 \mp v/c}} \right) = \lim_{v\to\infty} \left(\sqrt{\frac{c/v \pm 1}{c/v \mp 1}} \right)$$

$$= \sqrt{-1}$$

$$= \boldsymbol{i}, \quad (13)$$

$$= \exp\left(\boldsymbol{i} \left(\frac{1}{2} + 2n \right) \pi \right), \quad (14)$$

$$m' = \frac{m}{\sqrt{1-(v/c)^2}} = \frac{mc/v}{\sqrt{(c/v)^2 - 1}} \rightarrow 0, \quad (15)$$

$$E = \frac{mc^2}{\sqrt{1-(v/c)^2}} = \frac{mc^3/v}{\sqrt{(c/v)^2 - 1}} \rightarrow 0, \quad (16)$$

$$p = \frac{mv}{\sqrt{1-(v/c)^2}} = \frac{mc}{\sqrt{(c/v)^2 - 1}} \rightarrow -\boldsymbol{i}mc, \quad (17)$$

$$E^2 = m^2c^4 + p^2c^2 \rightarrow m^2c^4 + (-\boldsymbol{i}mc)^2c^2 = 0, \quad (18)$$

$$x' = \frac{x - vt}{\sqrt{1-(v/c)^2}} = \frac{cx/v - ct}{\sqrt{(c/v)^2 - 1}} \rightarrow \boldsymbol{i}ct, \quad (19)$$

$$t' = \frac{t - vx/c^2}{\sqrt{1-(v/c)^2}} = \frac{ct/v - x/c}{\sqrt{(c/v)^2 - 1}} \rightarrow \frac{\boldsymbol{i}x}{c}, \quad (20)$$

where m and m'=mass of matter, $\boldsymbol{i}$=imaginary unit.
These expressions seem to suggest the phenomenon in the world of imaginary numbers, where $\boldsymbol{i}$ is interpreted as something other than v because $\boldsymbol{i}$ is given by applying $v \rightarrow \infty$ to Bondi K–factor. The mutually exclusive relationship between $\boldsymbol{i}$ and v seems to suggest the mutually

exclusive relationship between wave and matter. Does this seem to suggest a hypothesis that the role of $\boldsymbol{i}$, though the phenomenon related to $\boldsymbol{i}$ looks like the phenomenon related to the infinite velocity, is to prohibit the state of $c \leq v < \infty$ by taking the form of wave? If so, does $\boldsymbol{i}$ seem to suggest a hypothetic association with the simultaneous existence of different states or the nonlocal correlation between states that exist apart? Those hypotheses suggested in this section will be severely criticized or disregarded.

Euler's formula and Bondi K–factor to the power of *i*

As shown in the previous report (Shimojo and Nakano, 2013), extending expression (1) into complex numbers (21) gives expressions that connect Euler's formula with Bondi K–factor to the power of $\boldsymbol{i}$ (22) or with Lorentz factor to the power of $\boldsymbol{i}$ (23),

$$\exp(\pm\boldsymbol{i}\theta) = \cos(\pm\theta) + \boldsymbol{i}\sin(\pm\theta), \tag{21}$$

$$= \left(\sqrt{\frac{1 \pm v/c}{1 \mp v/c}}\right)^{\boldsymbol{i}}, \tag{22}$$

$$= \left(\frac{1}{\sqrt{1-(v/c)^2}} \cdot (1 \pm v/c)\right)^{\boldsymbol{i}}. \tag{23}$$

Expression (22) seems to suggest a hypothetic relationship between wave and Bondi K–factor in the world of complex numbers. Shimojo *et al.* (2003a) applied the symmetric property of Euler's formula under its differentiation to the description of matter cycling in field–forage–ruminant relationships. Shimojo *et al.* (2003b) applied the stereographic representation of Euler's formula to the spiral form of micro–structures observed in plants and animals.

If expression (22) is related to replacements (11) and (12), then does expression (24) seem to suggest a hypothetic relationship between Bondi K–factor to the power of $\boldsymbol{i}$ and wave function? In addition, do inequalities (27) and (28) seem to suggest a hypothetic difference between matter and antimatter?

$$\left(\sqrt{\frac{1 \pm v_{\boldsymbol{d}}/c}{1 \mp v_{\boldsymbol{d}}/c}}\right)^{\boldsymbol{i}} = \exp\left(\pm\boldsymbol{i}\,\frac{2\pi}{h}\,(px - Et)\right), \tag{24}$$

$$\ln\sqrt{\frac{1 \pm v_{\boldsymbol{d}}/c}{1 \mp v_{\boldsymbol{d}}/c}} = \pm\frac{2\pi}{h}(px - Et), \tag{25}$$

$$= \pm\frac{2\pi}{h}\left(\frac{h}{\lambda}x - h\nu t\right), \tag{26}$$

$$-\frac{2\pi}{h}(px - Et) < 0 < \frac{2\pi}{h}(px - Et), \tag{27}$$

$$-\frac{2\pi}{h}\left(\frac{h}{\lambda}x - h\nu t\right) < 0 < \frac{2\pi}{h}\left(\frac{h}{\lambda}x - h\nu t\right), \tag{28}$$

where $v_{\boldsymbol{d}}=\pm c(\exp(\pm d)-1)/(\exp(\pm d)+1)$, $d=4\pi(px-Et)/h$, $d=4\pi(hx/\lambda-h\nu t)/h$, ln=natural logarithm, λ= wavelength, ν=frequency, double–sign corresponds.

As for Euler's identity that is a special case of Euler's formula,

$$\exp(\pm\boldsymbol{i}\pi) = -1, \tag{29}$$

$$= \left(\sqrt{\frac{1 \pm v_\pi/c}{1 \mp v_\pi/c}}\right)^{\boldsymbol{i}}, \tag{30}$$

$$= \left(\frac{1}{\sqrt{1-(v_\pi/c)^2}} \cdot (1 \pm v_\pi/c)\right)^{\boldsymbol{i}}, \tag{31}$$

where $v_\pi=c(\exp(2\pi)-1)/(\exp(2\pi)+1) \approx 0.996c \approx$ muon's velocity.

Expressions (29) and (30) seem to suggest a hypothesis that Euler's identity is related with Bondi K–factor to the power of $\boldsymbol{i}$ where $v \approx$ muon's velocity. Those hypotheses suggested in this section will be severely criticized or disregarded.

Mathematical application of $v \rightarrow \infty$ to Bondi K–factor to the power of *i*

The mathematical application of $v \rightarrow \infty$ to expression (22) gives expressions (32) ~ (38),

$$\lim_{v\to\infty}\left(\sqrt{\frac{1 \pm v/c}{1 \mp v/c}}\right)^{\boldsymbol{i}} = \lim_{v\to\infty}\left(\sqrt{\frac{c/v \pm 1}{c/v \mp 1}}\right)^{\boldsymbol{i}}$$

$$= \left(\sqrt{-1}\right)^{\boldsymbol{i}} = \boldsymbol{i}^{\boldsymbol{i}}, \tag{32}$$

$$\pm\theta \quad \rightarrow \quad \ln\boldsymbol{i} = \boldsymbol{i}\left(\frac{1}{2} + 2n\right)\pi, \tag{33}$$

$$\exp(\pm\boldsymbol{i}\theta) \quad \rightarrow \quad \boldsymbol{i}^{\boldsymbol{i}}, \tag{34}$$

$$\boldsymbol{i}^{\boldsymbol{i}} = \cos(\ln\boldsymbol{i}) + \boldsymbol{i}\sin(\ln\boldsymbol{i}), \tag{35}$$

$$= \cos\left(\boldsymbol{i}\left(\frac{1}{2} + 2n\right)\pi\right) + \boldsymbol{i}\sin\left(\boldsymbol{i}\left(\frac{1}{2} + 2n\right)\pi\right), \tag{36}$$

$$= \cosh\left(\left(\frac{1}{2} + 2n\right)\pi\right) - \sinh\left(\left(\frac{1}{2} + 2n\right)\pi\right), \tag{37}$$

$$= \exp\left(-\left(\frac{1}{2} + 2n\right)\pi\right), \tag{38}$$

where cosh = hyperbolic cosine, sinh = hyperbolic sine. The mathematical application of $v \to \infty$ to Bondi K–factor to the power of $\boldsymbol{i}$ gives '$\boldsymbol{i}$ to the power of $\boldsymbol{i}$' that takes real numbers. It takes one real number in response to one integer that n takes. Does many–valued function (38) seem to suggest a hypothesis that '$\boldsymbol{i}$ to the power of $\boldsymbol{i}$' takes two or more real numbers simultaneously unless n takes one of the integers? Those hypotheses suggested in this section will be severely criticized or disregarded.

Mathematical application of '$\boldsymbol{i}$ to the power of $\boldsymbol{i}$' to some physical constants

In spite of just playing with numbers based on the many–valued property of '$\boldsymbol{i}$ to the power of $\boldsymbol{i}$', function (38) gives numeral approximations to some physical constants, apart from their units. Those are Newtonian constant of gravitation (39), speed of light in vacuum (40), Planck constant (41), Boltzmann constant (42), electron mass (43), proton mass (44), neutron mass (45) and Avogadro constant (46),

$$26.39823 \cdot \exp(-(1/2+2\cdot 4)\pi) = 26.39823 \cdot \boldsymbol{i}^{\boldsymbol{i}} \approx 6.67384 \cdot 10^{-11} = G\ (\mathrm{m}^3 \cdot \mathrm{kg}^{-1} \cdot \mathrm{s}^{-2}), \tag{39}$$

$$9.39184155 \cdot \exp(-(1/2+2\cdot(-3))\pi) = 9.39184155 \cdot \boldsymbol{i}^{\boldsymbol{i}} \approx 299{,}792{,}458 = c\ (\mathrm{m} \cdot \mathrm{s}^{-1}), \tag{40}$$

$$1.77205281 \cdot \exp(-(1/2+2\cdot 12)\pi) = 1.77205281 \cdot \boldsymbol{i}^{\boldsymbol{i}} \approx 6.62606957 \cdot 10^{-34} = h\ (\mathrm{J} \cdot \mathrm{s}), \tag{41}$$

$$0.4490483 \cdot \exp(-(1/2+2\cdot 8)\pi) = 0.4490483 \cdot \boldsymbol{i}^{\boldsymbol{i}} \approx 1.3806488 \cdot 10^{-23} = k\ (\mathrm{J} \cdot \mathrm{K}^{-1}), \tag{42}$$

$$4.54942929 \cdot \exp(-(1/2+2\cdot 11)\pi) = 4.54942929 \cdot \boldsymbol{i}^{\boldsymbol{i}} \approx 9.10938291 \cdot 10^{-31} = e\ (\mathrm{kg}), \tag{43}$$

$$15.599583411 \cdot \exp(-(1/2+2\cdot 10)\pi) = 15.599583411 \cdot \boldsymbol{i}^{\boldsymbol{i}} \approx 1.672621777 \cdot 10^{-27} = p\ (\mathrm{kg}), \tag{44}$$

$$15.621086177 \cdot \exp(-(1/2+2\cdot 10)\pi) = 15.621086177 \cdot \boldsymbol{i}^{\boldsymbol{i}} \approx 1.674927351 \cdot 10^{-27} = n\ (\mathrm{kg}), \tag{45}$$

$$0.80013773 \cdot \exp(-(1/2+2\cdot(-9))\pi) = 0.80013773 \cdot \boldsymbol{i}^{\boldsymbol{i}} \approx 6.02214129 \cdot 10^{23} = N_{\mathbf{A}}\ (\mathrm{mol}^{-1}). \tag{46}$$

Do these calculated approximations seem to suggest a hypothetic existence of '$\boldsymbol{i}$ to the power of $\boldsymbol{i}$' at the back of those physical constants? There is a hypothesis that the weight of an individual plant or animal comes mainly from masses of proton and neutron. Those hypotheses suggested in this section will be severely criticized or disregarded.

Some natural phenomena related to circular constant

In the previous report (Shimojo and Nakano, 2013) a special relationship between Euler's formula, Bondi K–factor and zeta function was shown through $\pi^2/6$. This section adds some more natural phenomena to the previous report (Shimojo and Nakano, 2013) in order to show the relationship to π. Those are an example of Euler product (47), an example of zeta function (48), Euler's identity (49), metallic number multiplied by its conjugate (50), '$\boldsymbol{i}$ to the power of $\boldsymbol{i}$' (51), an example of Bondi K–factor (52) where $v=v_\pi$ ($\approx$ muon's velocity) and uncertainty principle [(53), (54)],

$$\pi = \sqrt{6\prod_p \frac{p^2}{p^2-1}}, \tag{47}$$

$$= \sqrt{6\sum_{k=1}^{\infty} \frac{1}{k^2}}, \tag{48}$$

$$= \frac{1}{1+2n} \cdot \ln(-1)^{-i}, \tag{49}$$

$$= \frac{1}{1+2n} \cdot \ln\left(\frac{m+\sqrt{m^2+4}}{2} \cdot \frac{m-\sqrt{m^2+4}}{2}\right)^{-i}, \tag{50}$$

$$= \frac{-1}{1/2+2n} \cdot \ln(\boldsymbol{i}^{\boldsymbol{i}}), \tag{51}$$

$$= \ln\sqrt{\frac{1+v_\pi/c}{1-v_\pi/c}}, \tag{52}$$

$$\pi \geq \frac{h}{4 \cdot \Delta x \cdot \Delta p}, \tag{53}$$

$$\pi \geq \frac{h}{4 \cdot \Delta E \cdot \Delta t}, \tag{54}$$

where p=prime numbers, $(m+(m^2+4)^{0.5})/2$=metallic number, $v_\pi=c(\exp(2\pi)-1)/(\exp(2\pi)+1)\approx 0.996c\approx$ muon's velocity.

Metallic number (50) gives golden number when m=1, silver number when m=2, and bronze number when m=3, respectively. Shimojo *et al.* (2011a) suggested, in a simple model of plant canopy, a hypothetic relationship between the golden number and the product of light extinction coefficient and leaf area index.

In addition, do expressions (55) and (56) seem to suggest a hypothetic relationship between π and the product of wave components and the natural logarithm of Bondi K–factor?

$$\pi = \frac{h}{2(px - Et)} \cdot \ln \sqrt{\frac{1 + v_d/c}{1 - v_d/c}} \qquad (55)$$

$$= \frac{h}{2(hx/\lambda - h\nu t)} \cdot \ln \sqrt{\frac{1 + v_d/c}{1 - v_d/c}}, \qquad (56)$$

where $v_d = c(\exp(d) - 1)/(\exp(d) + 1)$, $d = 4\pi(px - Et)/h$, $d = 4\pi(hx/\lambda - h\nu t)/h$.

Those hypotheses suggested in this section will be severely criticized or disregarded.

Conclusions

This study suggests that Napier's constant, imaginary unit and circular constant are the basso continuo being played in some natural phenomena.

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