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Okita, Masatoshi
Graduate School of Mathematics, Kyushu University

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Masatoshi   OKITA

Faculty of Mathematics

Kyushu University
Fukuoka, JAPAN

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Masatoshi Okita

Graduate School of Mathematics,
Kyushu University,
Fukuoka 819-0395, Japan
Email: m-okita@math.kyushu-u.ac.jp

Abstract: In this paper we are concerned with the convergence rates of the global strong solution to motionless state with constant density for the compressible Navier-Stokes equations in the whole space $\mathbb{R}^n$ for $n \geq 3$. It is proved that the perturbations decay in critical spaces, if the initial perturbations of density and velocity are small in $B_{2,1}^{\frac{n}{2}}(\mathbb{R}^n) \cap \dot{B}_{1,\infty}^0(\mathbb{R}^n)$ and $B_{2,1}^{\frac{n}{2}-1}(\mathbb{R}^n) \cap \dot{B}_{1,\infty}^0(\mathbb{R}^n)$, respectively.
Key Words: compressible Navier- Stokes equations; convergence rate.

1   Introduction

This paper studies the initial value problem for the compressible Navier-Stokes equation in $\mathbb{R}^n$:

$$\begin{cases} \partial_t \rho + \nabla \cdot (\rho u) = 0, \\ \partial_t u + (u \cdot \nabla)u + \frac{\nabla P(\rho)}{\rho} = \frac{\mu}{\rho}\triangle u + \frac{\mu+\mu'}{\rho}\nabla(\nabla \cdot u), \\ (\rho, u)(0, x) = (\rho_0, u_0)(x). \end{cases} \quad (1)$$

Here $t > 0$, $x = (x_1, x_2, \cdots, x_n) \in \mathbb{R}^n$; the unknown functions $\rho = \rho(t, x) > 0$ and $u = u(t, x) = (u_1(t, x), u_2(t, x), \cdots, u_n(t, x))$ denote the density and velocity, respectively; $P = P(\rho)$ is the pressure that are assumed to be a function of the density ρ; μ and μ' are the viscosity coefficients satisfying the conditions $\mu > 0$ and $\mu' + \frac{2}{n}\mu \geq 0$; and $\nabla\cdot$, ∇ and $\triangle$ denote the usual divergence, gradient and Laplacian with respect to x, respectively.

We assume that $P(\rho)$ is smooth in a neighborhood of $\bar{\rho}$ with $P'(\bar{\rho}) > 0$, where $\bar{\rho}$ is a given positive constant.

In this paper we derive the convergence rate of solution of problem (1) to the constant stationary solution $(\bar{\rho}, 0)$ as $t \to \infty$ when the initial perturbation $(\rho_0 - \bar{\rho}, u_0)$ is sufficiently small in critical spaces and $\dot{B}_{1,\infty}^0$ for $n \geq 3$.

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Matsumura-Nishida [8] showed the global in time existence of solution of (1) for $n = 3$, provided that the initial perturbation $(\rho_0 - \bar\rho, u_0)$ is sufficiently small in $H^3(\mathbb{R}^3) \cap L^1(\mathbb{R}^3)$. Furthermore, the following decay estimates was obtained in [8];

$$\|\nabla^k(\rho - \bar\rho, u)(t)\|_{L^2} \leq C(1+t)^{-\frac{3}{4} - \frac{k}{2}} \quad k = 0, 1. \tag{2}$$

These results were proved by combining the energy method and the decay estimates of the semigroup $E(t)$ generated by the linearized operator A at the constant state $(\bar\rho, 0)$.

On the other hand, Kawashita [6] showed the global existence of solution for initial perturbations sufficiently small in $H^{s_0}(\mathbb{R}^n)$ with $s_0 = [\frac{n}{2}] + 1$, $n \geq 2$. (Note that $s_0 = 2$ for $n = 3$). Wang-Tan [11] then considered the case $n = 3$ when the initial perturbation $(\rho_0 - \bar\rho, u_0)$ is sufficiently small in $H^2(\mathbb{R}^3) \cap L^1(\mathbb{R}^3)$, and proved the decay estimates (2). Okita [10] showed that if $n \geq 2$ then the following estimates hold for the solution (ρ, u) of (1) :

$$\|\nabla^k(\rho - \bar\rho, u)(t)\|_{L^2} \leq C(1+t)^{-\frac{n}{4} - \frac{k}{2}} \quad k = 0, \cdots, s_0,$$

provided that $(\rho_0 - \bar\rho, u_0)$ is sufficiently small in $H^{s_0}(\mathbb{R}^n) \cap L^1(\mathbb{R}^n)$ with $s_0 = [\frac{n}{2}] + 1$. This result was shown by decomposition of the perturbation into low and high frequency parts. Moreover Liang Li-Zhang [7] showed the density and momentum converge at the rates $(1+t)^{\frac{3}{4} - \frac{s}{2}}$ in the L^2-norm, when initial perturbation sufficiently small in $H^l(\mathbb{R}^3) \cap \dot{B}_{1,\infty}^{-s}(\mathbb{R}^3)$ with $l \geq 4$ and $s \in [0, 1]$. Note that L^1 is included in $\dot{B}_{1,\infty}^0$.

Danchin [2] proved the global existence in critical homogeneous Besov space, which is stated as follows.

Proposition 1.1 (Danchin [2]). *Let $n \geq 2$. There are two positive constants ϵ_1 and M such that for all (ρ_0, u_0) with $(\rho_0 - \bar\rho) \in \dot{B}_{2,1}^{\frac{n}{2}} \cap \dot{B}_{2,1}^{\frac{n}{2}-1}$, $u_0 \in \dot{B}_{2,1}^{\frac{n}{2}-1}$ and*

$$\|\rho_0 - \bar\rho\|_{\dot{B}_{2,1}^{\frac{n}{2}} \cap \dot{B}_{2,1}^{\frac{n}{2}-1}} + \|u_0\|_{\dot{B}_{2,1}^{\frac{n}{2}-1}} \leq \epsilon_1, \tag{3}$$

then problem (1) has a unique global solution $(\rho, u) \in C(\mathbb{R}^+; \dot{B}_{2,1}^{\frac{n}{2}} \cap \dot{B}_{2,1}^{\frac{n}{2}-1}) \times (L^1(\mathbb{R}^+; \dot{B}_{2,1}^{\frac{n}{2}+1}) \cap C(\mathbb{R}^+; \dot{B}_{2,1}^{\frac{n}{2}-1}))$ that satisfies the estimate

$$\|\rho - \bar\rho\|_{\dot{B}_{2,1}^{\frac{n}{2}-1}} + \|u\|_{\dot{B}_{2,1}^{\frac{n}{2}-1}} + \int_0^\infty \|u\|_{\dot{B}_{2,1}^{\frac{n}{2}+1}} dt \leq M(\|\rho_0 - \bar\rho\|_{\dot{B}_{2,1}^{\frac{n}{2}} \cap \dot{B}_{2,1}^{\frac{n}{2}-1}} + \|u_0\|_{\dot{B}_{2,1}^{\frac{n}{2}-1}}).$$

For critical nonhomogeneous Besov space, Haspot [4] proved the local well-posedness.

Proposition 1.2 (Haspot [4]). *Let $n \geq 2$. Let $u_0 \in B_{2,1}^{\frac{n}{2}-1}$ and $(\rho_0 - \bar\rho) \in B_{2,1}^{\frac{n}{2}}$ with $\rho_0 - \bar\rho > 0$. Then there exist a constant $T > 0$ such that the problem (1) has a unique local solution (ρ, u) on $[0, T]$ with $\rho - \bar\rho > 0$ and:*

$$\rho - \bar\rho \in C([0, T]; B_{2,1}^{\frac{n}{2}}), \quad u \in (C([0, T]; B_{2,1}^{\frac{n}{2}-1}) \cap L^1(0, T; B_{2,1}^{\frac{n}{2}+1})).$$

We now state our main result of this paper which gives the optimal L^2 decay rate for strong solutions in critical Besov spaces.

Theorem 1.3. *Assume that $n \geq 3$. There exists $\epsilon > 0$ such that if $u_0 \in B_{2,1}^{\frac{n}{2}-1} \cap \dot{B}_{1,\infty}^0$, $(\rho_0 - \bar{\rho}) \in B_{2,1}^{\frac{n}{2}} \cap \dot{B}_{1,\infty}^0$ and*

$$\|\rho_0 - \bar{\rho}\|_{B_{2,1}^{\frac{n}{2}} \cap \dot{B}_{1,\infty}^0} + \|u_0\|_{B_{2,1}^{\frac{n}{2}-1} \cap \dot{B}_{1,\infty}^0} \leq \epsilon,$$

then problem (1) has a unique global solution $(\rho - \bar{\rho}, u) \in C(\mathbb{R}^+; B_{2,1}^{\frac{n}{2}}) \times \big(C(\mathbb{R}^+; B_{2,1}^{\frac{n}{2}-1}) \cap L^1(\mathbb{R}^+; \dot{B}_{2,1}^{\frac{n}{2}+1}) \big)$. Furthermore, there exists constant $C_0 > 0$, we have

$$\|(\rho - \bar{\rho}, u)\|_{B_{2,1}^{\frac{n}{2}-1}} \leq C_0 (1+t)^{-\frac{n}{4}} \tag{4}$$

for $t \geq 0$.

Remark 1.4. *If (ρ_0, u_0) satisfies the assumption of Theorem 1.3, then it also satisfies the assumption of Proposition 1.1. Therefore, we have estimate of (3).*

Remark 1.5. *We will derive the a priori estimate with time weight for $\|(\rho(t) - \bar{\rho}, u(t))\|_{B_{2,1}^{\frac{n}{2}} \times B_{2,1}^{\frac{n}{2}-1}}$ which, together with Proposition 1.2, proves the global existence in nonhomogeneous critical Besov spaces and $\dot{B}_{1,\infty}^0$.*

Remark 1.6. *The convergence rates for the problem of (1) given in (2) are optimal. By $B_{2,1}^{\frac{n}{2}-1} \subset L^2$, the convergence rate of (4) are optimal.*

To prove Theorem 1.3, as in [5], we introduce a decomposition of the perturbation $U(t) = (\rho - \bar{\rho}, u)(t)$ associated with the spectral properties of the linearized operator A. In the case of our problem, we simply decompose the perturbation $U(t)$ into low and high frequency parts. As for the low frequency part, we apply the decay estimates for the low frequency part of $E(t)$; while the high frequency part is estimated by using the energy method. One of the points of our approach is that by restricting the use of the decay estimates for $E(t)$ to its low frequency part, one can avoid the derivative loss due to the convective term of the transport equation $(1)_1$. On the other hand, the convective term of $(1)_1$ can be controlled by the energy method and commutator estimate which we apply to the high frequency part. Another point is that we have $\int_0^\infty \|u\|_{\dot{B}_{2,1}^{\frac{n}{2}+1}} dt < C\epsilon$, established in Proposition 1.1. We need this estimate when we estimate the nonlinear terms.

The paper is organized as follows. In Section 2 we introduce the notation and some properties of Besov spaces. In Section 3 we rewrite the system into the one for the perturbation and introduce auxiliary Lemmas used in this paper. In Section 4 we give the proof of Theorem 1.3.

2 Preliminaries

In this section we first introduce the notation which will be used throughout this paper. We then introduce Besov spaces and some properties of Besov spaces.

2.1 Notation

Let $L^p(1 \leq p \leq \infty)$ denote the usual L^p-Lebesgue space on $\mathbb{R}^n$. For nonnegative integer m, we denote by H^m the usual L^2-Sobolev space of order m. The inner-product of L^2 is denoted by $(\cdot, \cdot)$. If S is any nonempty set, sequence space $l^p(S)$ denote the usual l^p sequence space on S.

For any integer $l \geq 0, \nabla^l f$ denotes all of l-th derivatives of f.

For a function f, we denote its Fourier transform by $\mathfrak{F}[f] = \hat{f}$:

$$\mathfrak{F}[f](\xi) = \hat{f}(\xi) = \int_{\mathbb{R}^n} f(x)e^{-ix\cdot\xi}dx.$$

The inverse of $\mathfrak{F}$ is denoted by $\mathfrak{F}^{-1}[f] = \check{f}$,

$$\mathfrak{F}^{-1}[f](x) = \check{f}(x) = (2\pi)^{-n}\int_{\mathbb{R}^n} f(\xi)e^{i\xi\cdot x}d\xi.$$

2.2 Besov spaces

Let us now define the homogeneous and nonhomogeneous Besov spaces. First we introduce the dyadic partition of unity. We can use for instance any $(\phi, \chi) \in C^\infty$, such that ϕ is supported in $\{\xi \in \mathbb{R}^n | \frac{3}{4} \leq |\xi| \leq \frac{8}{3}\}$, χ is supported in $\{\xi \in \mathbb{R}^n | |\xi| \leq \frac{4}{3}\}$ such that

$$\forall \xi \in \mathbb{R}^n, \ \chi(\xi) + \sum_{j \geq 0} \phi(2^{-j}\xi) = 1,$$

$$\forall \xi \in \mathbb{R}^n \setminus \{0\}, \ \sum_{j \in \mathbb{Z}} \phi(2^{-j}\xi) = 1,$$

$$|j - j'| \geq 2 \Rightarrow \operatorname{Supp} \phi(2^{-j}\cdot) \cap \operatorname{Supp} \phi(2^{-j'}\cdot) = \emptyset,$$

$$j \geq 1 \Rightarrow \operatorname{Supp} \chi \cap \operatorname{Supp} \phi(2^{-j}\cdot) = \emptyset.$$

Denoting $h = \mathfrak{F}^{-1}\phi$ and $\tilde{h} = \mathfrak{F}^{-1}\chi$, we then define the dyadic blocks by

$$\triangle_{-1}u = \chi(D)u = \tilde{h} * u,$$

$$\triangle_j u = \phi(2^{-j}D)u = 2^{jn}\int_{\mathbb{R}^n} h(2^j y)u(x-y)dy \quad \text{if } j \geq 0,$$

$$\dot{\triangle}_j u = \phi(2^{-j}D)u = 2^{jn}\int_{\mathbb{R}^n} h(2^j y)u(x-y)dy \quad \text{if } j \in \mathbb{Z}.$$

The low-frequency cut-off operator is defined by

$$S_j u = \sum_{-1 \leq k \leq j-1} \triangle_k u, \quad \dot{S}_j u = \sum_{k \leq j-1} \dot{\triangle}_k u.$$

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Obviously we can write that: $Id = \sum_j \triangle_j$. The high-frequency cut-off operator $\tilde{S}_j$ is defines by

$$\tilde{S}_j = Id - S_j = \sum_{k \geq j} \triangle_k u,$$

To begin, we define Besov spaces.

Definition 1. For $s \in \mathbb{R}$ and $1 \leq p, r \leq \infty$, and $u \in \mathcal{S}'$ we set

$$\|u\|_{B^s_{p,r}} := \left\| 2^{js} \|\triangle_j u\|_{L^p} \right\|_{l^r(\{k \geq -1\})},$$

$$\|u\|_{\dot{B}^s_{p,r}} := \left\| 2^{js} \|\dot{\triangle}_j u\|_{L^p} \right\|_{l^r(\mathbb{Z})}.$$

The nonhomogeneous Besov space $B^s_{p,r}$ and the homogeneous Besov space $\dot{B}^s_{p,r}$ are set of function $u \in \mathcal{S}'$ such that $\|u\|_{B^s_{p,r}}$, $\|u\|_{\dot{B}^s_{p,r}} < \infty$ respectively.

Let us state some basic Lemmas for Besov spaces.

Lemma 2.1. *The following properties hold:*

(i) $\|\nabla \triangle_{-1} u\|_{L^2} \leq C \|\triangle_{-1} u\|_{L^2}$.

(ii) $C^{-1} 2^j \|\dot{\triangle}_j u\|_{L^2} \leq \|\nabla \dot{\triangle}_j u\|_{L^2} \leq C 2^j \|\dot{\triangle}_j u\|_{L^2}$ $(j \in \mathbb{Z})$.

(iii) $\|\nabla S_j u\|_{L^2} \leq C 2^j \|S_j u\|_{L^2}$ $(j \geq 0)$.

(iv) $\|\tilde{S}_j u\|_{L^2} \leq C 2^j \|\nabla \tilde{S}_j u\|_{L^2}$ $(j \geq 0)$.

The assertions $(i), (ii), (iii)$ and (iv) easily follow from the Plancherel theorem.

Lemma 2.2. *The following properties hold:*

(i) $C^{-1} \|u\|_{\dot{B}^s_{p,r}} \leq \|\nabla u\|_{\dot{B}^{s-1}_{p,r}} \leq C \|u\|_{\dot{B}^s_{p,r}}$.

(ii) $\|\nabla u\|_{B^{s-1}_{p,r}} \leq C \|u\|_{B^s_{p,r}}$.

(iii) *If $s' > s$ or if $s' = s$ and $r_1 \leq r$ then $B^{s'}_{p,r_1} \subset B^s_{p,r}$.*

(iv) *If $r_1 \leq r$ then $\dot{B}^s_{p,r_1} \subset \dot{B}^s_{p,r}$.*

(v) *Let $\Lambda := \sqrt{-\triangle}$ and $t \in \mathbb{R}$. Then the operator Λ^t is an isomorphism from $\dot{B}^s_{2,1}$ to $\dot{B}^{s-t}_{2,1}$.*

See, e.g., [2] [3] and [4] for the proof.

Lemma 2.3. *The following properties hold:*

(i) $\|u\|_{L^\infty} \leq C \|u\|_{\dot{B}^{\frac{n}{2}}_{2,1}}$ $\quad \left(\dot{B}^{\frac{n}{2}}_{2,1} \subset L^\infty \right)$.

(ii) $\dot{B}^0_{1,1} \subset L^1 \subset \dot{B}^0_{1,\infty}$.

(iii) $B^s_{2,2} \approx H^s$.

(iv) $B^s_{p,r} \subset \dot{B}^s_{p,r}$ $(s > 0)$.

See, e.g., [2] [3] and [4] for the proof.

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3 Reformulation of the problem

In this section we first rewrite system (1) into the one for the perturbation. We then introduce some auxiliary lemmas which will be useful in the proof of the main result.

Let us rewrite the problem (1). We define μ_1, μ_2 and γ by

$$\mu_1 = \frac{\mu}{\bar{\rho}}, \ \mu_2 = \frac{\mu + \mu'}{\bar{\rho}}, \ \gamma = \sqrt{P'(\bar{\rho})}.$$

By using the new unknown function

$$\sigma(t, x) = \frac{\rho(t, x) - \bar{\rho}}{\bar{\rho}}, \quad w(t, x) = \frac{1}{\gamma} u(t, x),$$

the initial value problem (1) is reformulated as

$$\begin{cases} \partial_t \sigma + \gamma \nabla \cdot w = F_1(U), \\ \partial_t w - \mu_1 \triangle w - \mu_2 \nabla(\nabla \cdot w) + \gamma \nabla \sigma = F_2(U), \\ (\sigma, w)(0, x) = (\sigma_0, w_0)(x), \end{cases} \tag{5}$$

where, $U = \begin{pmatrix} \sigma \\ w \end{pmatrix}$,

$$F_1(U) = -\gamma(w \cdot \nabla \sigma + \sigma \nabla \cdot w),$$

$$\begin{aligned} F_2(U) = {} & -\gamma(w \cdot \nabla)w - \mu_1 \frac{\sigma}{\sigma + 1} \triangle w - \mu_2 \frac{\sigma}{\sigma + 1} \nabla(\nabla \cdot w) \\ & + \Big(\frac{\bar{\rho}\gamma}{\sigma + 1} - \frac{\bar{\rho}}{\gamma} \frac{\int_0^1 P''(s\bar{\rho}\sigma + \bar{\rho})ds}{\sigma + 1} \Big) \sigma \nabla \sigma. \end{aligned}$$

We set

$$A = \begin{pmatrix} 0 & -\gamma \nabla \cdot \\ -\gamma \nabla & \mu_1 \triangle + \mu_2 \nabla \nabla \cdot \end{pmatrix}.$$

By using operator A, problem (5) is written as

$$\partial_t U - AU = F(U), \quad U|_{t=0} = U_0, \tag{6}$$

where

$$F(U) = \begin{pmatrix} F_1(U) \\ F_2(U) \end{pmatrix}, \quad U_0 = \begin{pmatrix} \sigma_0 \\ w_0 \end{pmatrix}.$$

We introduce a semigroup associated with A. We set

$$E(t)u := \mathfrak{F}^{-1}[e^{\hat{A}(\xi)t}\hat{u}] \quad \text{for } u \in L^2,$$

where

$$\hat{A}(\xi) = \begin{pmatrix} 0 & -i\gamma\xi^t \\ -i\gamma\xi & -\mu_1|\xi|^2 I_n - \mu_2 \xi \xi^t \end{pmatrix}.$$

Here and in what follows the superscript $\cdot^t$ means the transposition.

We next state some basic Lemmas.

Lemma 3.1. *Let $s_1, s_2 \leq \frac{n}{2}$ such that $s_1 + s_2 > 0$, $u \in \dot{B}_{2,1}^{s_1}$ and $v \in \dot{B}_{2,1}^{s_2}$. Then $uv \in \dot{B}_{2,1}^{s_1+s_2-\frac{n}{2}}$ and*

$$\|uv\|_{\dot{B}_{2,1}^{s_1+s_2-\frac{n}{2}}} \leq C\|u\|_{\dot{B}_{2,1}^{s_1}}\|v\|_{\dot{B}_{2,1}^{s_2}}.$$

See, e.g., [1], for the proof.

Lemma 3.2. *Let $s > 0$ and $u \in \dot{B}_{2,1}^{s} \cap L^\infty$. Let $F \in W_{loc}^{[s]+2,\infty}(\mathbb{R}^n)$ such that $F(0) = 0$. Then $F(u) \in \dot{B}_{2,1}^{s}$. Moreover, there exists a function C_1 of one variable depending only on s, n and F, and such that*

$$\|F(u)\|_{\dot{B}_{2,1}^{s}} \leq C_1(\|u\|_{L^\infty})\|u\|_{\dot{B}_{2,1}^{s}}.$$

See, e.g., [2], for the proof.

Lemma 3.3. *(i) Let $a, b > 0$ satisfying $\max\{a, b\} > 1$. Then*

$$\int_0^t (1+s)^{-a}(1+t-s)^{-b}ds \leq C(1+t)^{-\min\{a,b\}}, \quad t \geq 0.$$

(ii) Let $a, b > 0$ and $f \in L^1(0, \infty)$. Then

$$\int_0^t (1+t)^{-a}(1+t-s)^{-b}f ds \leq C(1+t)^{-\min\{a,b\}} \int_0^t |f| ds, \quad t \geq 0.$$

For the proof of (i), see [9]. Proof of (ii) can be proved using Hölder inequality.

4 Proof of main result

In this section we prove Theorem 1.3. In subsections 4.1 and 4.2 we establish the necessary estimates for $\triangle_{-1}U(t)$ and $\triangle_j U(t)$ for $j \geq 0$, respectively. In subsection 4.3 we derive the a priori estimate to complete the proof of Theorem 1.3.

Proposition 4.1. *Let $T > 0$ and let (σ, w) be a solution of problem (6) on $[0, T]$ such that*

$$\sigma \in C([0,T]; B_{2,1}^{\frac{n}{2}}), w \in C([0,T]; B_{2,1}^{\frac{n}{2}}) \cap L^1(0,T; B_{2,1}^{\frac{n}{2}+1}), \tag{7}$$

Then, $\triangle_j U(t) = (\triangle_j \sigma, \triangle_j w)^t$ for $j \geq -1$ satisfy

$$\partial_t \triangle_j U - A\triangle_j U = \triangle_j F(U), \tag{8}$$

$$\triangle_j U|_{t=0} = \triangle_j U_0. \tag{9}$$

Moreover, $\triangle_j U(t)$ for $j \geq -1$ satisfy

$$\triangle_j U(t) = E(t)\triangle_j U_0 + \int_0^t E(t-s)\triangle_j F(U)(s)ds. \tag{10}$$

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Proof. Let $U(t) = (\sigma, w)^{\mathrm{t}}$ be a solution of (6) satisfying (7). Since $\triangle_j A U = A \triangle_j U$, applying $\triangle_j$ to (6), we obtain

$$\begin{cases} \partial_t \triangle_j U - A \triangle_j U = \triangle_j F(U), \\ \qquad\qquad \triangle_j U|_{t=0} = \triangle_j U_0. \end{cases} \tag{11}$$

It then follows that

$$\triangle_j U(t) = E(t) \triangle_j U_0 + \int_0^t E(t-s) \triangle_j F(U)(s) ds.$$

This completes the proof.

$\square$

Set

$$M_1(t) := \sup_{0 \le \tau \le t} (1+\tau)^{\frac{n}{4}} \| \triangle_{-1} U(\tau) \|_{L^2},$$

$$M_\infty(t) := \sup_{0 \le \tau \le t} (1+\tau)^{\frac{n}{4}} \sum_{j=0}^\infty 2^{(\frac{n}{2}-1)j} \big\{ \| \triangle_j U(\tau) \|_{L^2} + 2^j \| \triangle_j \sigma \|_{L^2} \big\},$$

$$M(t) := M_1(t) + M_\infty(t).$$

4.1 Estimate of low frequency parts

In this subsection we derive the estimate of $\triangle_{-1} U(t)$, in other words, we estimate $M_1(t)$.

Lemma 4.2 (Matsumura-Nishida [9]). *(i) The set of all eigenvalues of $\hat{A}(\xi)$ consists of $\lambda_i(\xi)$ $(i = 1, 2, 3)$, where*

$$\begin{cases} \lambda_1(\xi) = \frac{-(\mu_1+\mu_2)|\xi|^2 + i|\xi|\sqrt{4\gamma^2 - (\mu_1+\mu_2)|\xi|^2}}{2}, \\ \lambda_2(\xi) = \overline{\lambda_1(\xi)}, \\ \lambda_3(\xi) = -\mu_1|\xi|^2, \end{cases}$$

for all $\xi \in \mathbb{R}^n$ Here $\overline{\lambda_1(\xi)}$ denotes the complex conjugate of $\lambda_1(\xi)$.

(ii) $e^{t\hat{A}(\xi)}$ has the spectral resolution

$$e^{t\hat{A}(\xi)} = \sum_{j=1}^3 e^{t\lambda_j(\xi)} P_j(\xi),$$

for all $|\xi|$ except at most points of $|\xi| > 0$, where $P_j(\xi)$ is the eigenprojection for $\lambda_j(\xi)$ and $P_j(\xi)$ satisfies

$$\| P_j(\xi) \| \le C \qquad \big(|\xi| \le r\big).$$

where $r = \frac{\gamma}{\sqrt{\mu_1+\mu_2}}$.
Moreover it has the estimate

$$\| e^{t\hat{A}(\xi)} \| \le C e^{-\beta t},$$

for all $|\xi| \ge r$ and a positive constant β.

Remark 4.3. *For each $M > 0$ there exist $C_2 = C_2(M) > 0$ and $\beta_2 = \beta_2(M) > 0$ such that the estimate*

$$\|e^{t\hat{A}(\xi)}\| \le C_2 e^{-\beta_2|\xi|^2 t}$$

holds for $|\xi| \le M$ and $t > 0$.

Lemma 4.4. *$E(t)$ satisfies the estimate,*

$$\|E(t)\triangle_{-1}U_0\|_{L^2} \le C(1+t)^{-\frac{n}{4}}\|U_0\|_{\dot{B}^0_{1,\infty}}$$

for $t \ge 0$.

Proof. By Plancherel's theorem and Lemma 4.2 (ii), we have

$$
\begin{aligned}
\|E(t)\triangle_{-1}U_0(t)\|_{L^2} &\le C\Big(\int_{|\xi|\le 2}\big|e^{\hat{A}(\xi)t}\hat{U}_0\big|^2 d\xi\Big)^{\frac{1}{2}}\\
&\le C\Big(\int_{0<|\xi|\le 2}e^{-\beta|\xi|^2 t}|\hat{U}_0(\xi)|^2 d\xi\Big)^{\frac{1}{2}}\\
&\le C\sum_{j\le 2}\big(\|\triangle_j U_0\|_{L^1}\|\tilde{\triangle}_j U_0\|_{L^1}\int_{2^{j-1}<|\xi|\le 2^{j+1}}e^{-\beta|\xi|^2 t}d\xi\big)^{\frac{1}{2}}\\
&\le Ct^{-\frac{n}{4}}\|U_0\|_{\dot{B}^0_{1,\infty}}, \qquad\qquad\qquad (12)
\end{aligned}
$$

where $\tilde{\triangle}_j := \triangle_{j-1} + \triangle_j + \triangle_{j+1}$.

We also find that

$$
\begin{aligned}
\|E(t)\triangle_{-1}U_0\|_{L^2} &\le C\sum_{j\le 2}\big(\|\triangle_j U_0\|_{L^1}\|\tilde{\triangle}_j U_0\|_{L^1}\int_{2^{j-1}<|\xi|<2^{j+1}}e^{-\beta|\xi|^2 t}d\xi\big)^{\frac{1}{2}}\\
&\le C\|U_0\|_{\dot{B}^0_{1,\infty}}. \qquad\qquad\qquad (13)
\end{aligned}
$$

The estimate of Lemma 4.4 follows from (12) and (13). $\qquad\square$

As for $M_1(t)$, we show the following estimate.

Proposition 4.5. *Let $n \ge 3$. There exists a $\epsilon > 0$ such that if*

$$\|w_0\|_{B^{\frac{n}{2}-1}_{2,1}} + \|\sigma_0\|_{B^{\frac{n}{2}}_{2,1}} \le \epsilon,$$

then there exists a constant $C > 0$ independent of T such that

$$M_1(t) \le C\|U_0\|_{\dot{B}^0_{1,\infty}} + C\epsilon M(t) + CM^2(t)$$

for $t \in [0, T]$

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To prove Proposition 4.5, we will use the following estimates on $F(U)$.

Lemma 4.6. *There exists a $\epsilon > 0$ such that if*

$$\|w_0\|_{B_{2,1}^{\frac{n}{2}-1}} + \|\sigma_0\|_{B_{2,1}^{\frac{n}{2}}} \leq \epsilon,$$

then there exists a constant $C > 0$ independent of T such that

$$\|F(U)\|_{\dot{B}_{1,\infty}^0} \leq C(1+t)^{-\frac{n}{4}} M(t) f(t) + C(1+t)^{-\frac{n}{2}} M^2(t)$$

for $t \in [0,T]$, where $0 \leq f \leq \|w\|_{\dot{B}_{2,1}^{\frac{n}{2}+1}} \in L^1(0,\infty)$.

We will prove Lemma 4.6 later. Now we prove Proposition 4.5.

Proof of Proposition 4.5. By Lemma 4.4 and (10), we see that

$$\|\triangle_{-1}U(\tau)\|_{L^2} \leq \|E(\tau)\triangle_{-1}U_0\|_{L^2} + \int_0^\tau \|E(\tau-s)\triangle_{-1}F(U(s))\|_{L^2}ds$$

$$\leq C(1+\tau)^{-\frac{n}{4}}\|U_0\|_{\dot{B}_{1,\infty}^0} + \int_0^t (1+\tau-s)^{-\frac{n}{4}}\|F(U(s))\|_{\dot{B}_{1,\infty}^0}ds. \tag{14}$$

Using Lemma 4.6, we have

$$\int_0^\tau (1+\tau-s)^{-\frac{n}{4}}\|F(U(s))\|_{\dot{B}_{1,\infty}^0}ds$$

$$\leq C\int_0^\tau (1+\tau-s)^{-\frac{n}{4}}\{(1+s)^{-\frac{n}{4}}f(s)M(t) + (1+s)^{-\frac{n}{2}}M^2(t)\}ds$$

$$\leq CM(t)\int_0^\tau (1+\tau-s)^{-\frac{n}{4}}(1+s)^{-\frac{n}{4}}f(s)ds$$

$$\quad + CM^2(t)\int_0^\tau (1+\tau-s)^{-\frac{n}{4}}(1+s)^{-\frac{n}{2}}ds$$

$$\leq C(1+\tau)^{-\frac{n}{4}}\epsilon M(t) + C(1+\tau)^{-\frac{n}{4}}M^2(t). \tag{15}$$

Here we used Lemma 3.3, in other words, we used $\frac{n}{2} > 1$ for $n \geq 3$ and $\int_0^\tau f(s)ds \leq M\epsilon$. By (14) and (15), we obtain

$$\|\triangle_{-1}U(\tau)\|_{L^2} \leq C(1+\tau)^{-\frac{n}{4}}\|U_0\|_{\dot{B}_{1,\infty}^0} + C\epsilon(1+\tau)^{-\frac{n}{4}}M(t) + C(1+\tau)^{-\frac{n}{4}}M^2(t),$$

and hence,

$$(1+\tau)^{\frac{n}{4}}\|\triangle_{-1}U(\tau)\|_2 \leq C\|U_0\|_{\dot{B}_{1,\infty}^0} + C\epsilon M(t) + CM^2(t).$$

Taking the supremum in $\tau \in [0,t]$, we obtain the desired estimate.

$\square$

It remains to prove Lemma 4.6.

Proof of Lemma 4.6. Since $L^1 \subset \dot{B}^0_{1,\infty}$, it suffices to estimate $\|F(U)\|_{L^1}$. By the Hölder inequality, we have

$$
\begin{aligned}
\|F(U)\|_{L^1} \leq\ & C\big\{\|w\|_{L^2}\|\nabla\sigma\|_{L^2} + \|\sigma\|_{L^2}\|\nabla w\|_{L^2} + \|w\|_{L^2}\|\nabla w\|_{L^2} \\
& +\|\sigma\|_{L^2}\|\nabla^2 w\|_{L^2} + \|\sigma\|_{L^2}\|\nabla\sigma\|_{L^2}\big\},
\end{aligned}
$$

We see from $B^s_{2,1} \subset L^2 \quad (s \geq 0)$ that

$$
\|w\|_{L^2}\|\nabla\sigma\|_{L^2} \leq C\|w\|_{B^{\frac{n}{2}-1}_{2,1}}\|\nabla\sigma\|_{B^{\frac{n}{2}-1}_{2,1}} \leq C(1+s)^{-\frac{n}{2}}M^2(t),
$$

$$
\begin{aligned}
\|\sigma\|_{L^2}\|\nabla w\|_{L^2} \leq\ & C\|\sigma\|_{B^{\frac{n}{2}-1}_{2,1}}\big\{\|\triangle_{-1}w\|_{L^2} + \|\tilde{S}_0 w\|_{B^{\frac{n}{2}}_{2,1}}\big\} \\
\leq\ & C\|\sigma\|_{B^{\frac{n}{2}-1}_{2,1}}\big\{\|w\|_{B^{\frac{n}{2}-1}_{2,1}} + \|w\|_{\dot{B}^{\frac{n}{2}+1}_{2,1}}\big\} \\
\leq\ & C(1+s)^{-\frac{n}{2}}M^2(t) + C(1+s)^{-\frac{n}{4}}M(t)f(t),
\end{aligned}
$$

where $0 \leq f(t) \leq \|w\|_{\dot{B}^{\frac{n}{2}+1}_{2,1}}$. As for $\|\nabla^2 w\|_{L^2}$, we get

$$
\begin{aligned}
\|\nabla^2 w\|_{L^2} \leq\ & C\big\{\|\triangle_{-1}w\|_{L^2} + \|\tilde{S}_0 w\|_{\dot{B}^{\frac{n}{2}+1}_{2,1}}\big\} \\
\leq\ & C\big\{\|w\|_{B^{\frac{n}{2}-1}_{2,1}} + \|w\|_{\dot{B}^{\frac{n}{2}+1}_{2,1}}\big\}.
\end{aligned}
$$

The other terms are estimated similarly. Hence we have

$$
\|F(U)\|_{\dot{B}^0_{1,\infty}} \leq C(1+s)^{-\frac{n}{2}}M^2(t) + C(1+s)^{-\frac{n}{4}}M(t)f(t).
$$

This completes the proof.

$\square$

4.2 Estimate of high frequency parts

We next derive estimates for M_∞. The system (8) is written as

$$
\begin{cases}
\partial_t \triangle_j \sigma + \gamma\nabla\cdot\triangle_j w = \triangle_j F_1(U), \\
\partial_t \triangle_j w - \mu_1\triangle\triangle_j w - \mu_2\nabla\cdot(\nabla\triangle_j w) + \gamma\nabla\triangle_j\sigma = \triangle_j F_2(U).
\end{cases} \tag{16}
$$

Proposition 4.7. *Let $j \geq 0$. There holds*

$$
\begin{aligned}
& \frac{1}{2}\frac{d}{dt}\|\triangle_j U(t)\|^2_{L^2} + \mu_1\|\nabla\triangle_j w(t)\|^2_{L^2} + \mu_2\|\nabla\cdot\triangle_j w(t)\|^2_{L^2} \\
& = (\triangle_j F_1(U), \triangle_j\sigma) + (\triangle_j F_2(U), \triangle_j w)
\end{aligned} \tag{17}
$$

for a.e. $t \in [0, T]$.

Proof. We take the inner product of $(16)_1$ and $(16)_2$ with $\triangle_j\sigma$ and $\triangle_j w$ respectively, integrating by parts and then adding them together, we obtain our proposition. $\square$

For $s \in \mathbb{R}$, we denote $\Lambda^s z := \mathfrak{F}^{-1}[|\xi|^s \hat{z}]$. Let $d = \Lambda^{-1}\nabla \cdot w$ be the "compressible part" of the velocity. Applying $\Lambda^{-1}\nabla\cdot$ to $(16)_2$, system (16) writes

$$\begin{cases} \partial_t \triangle_j\sigma + \gamma\Lambda\triangle_j d = \triangle_j F_1(U), \\ \partial_t \triangle_j d - \nu\triangle\triangle_j d - \gamma\Lambda\triangle_j\sigma = \Lambda^{-1}\nabla \cdot \triangle_j F_2(U), \end{cases} \tag{18}$$

where we denote $\nu = \mu_1 + \mu_2$.

Proposition 4.8. *Let $j \geq 0$. There holds*

$$\frac{1}{2}\frac{\nu}{\gamma}\frac{d}{dt}\|\Lambda\triangle_j\sigma\|_{L^2}^2 - \frac{d}{dt}(\Lambda\triangle_j\sigma, \triangle_j d) + \|\Lambda\triangle_j\sigma\|_{L^2}^2 = \gamma\|\Lambda\triangle_j d\|_{L^2}^2$$

$$-(\Lambda\triangle_j F_1(U), \triangle_j d) - (\Lambda^{-1}\nabla \cdot \triangle_j F_2(U), \Lambda\triangle_j\sigma) + \frac{\nu}{\gamma}(\Lambda\triangle_j F_1(U), \Lambda\triangle_j\sigma) \tag{19}$$

for a.e. $t \in [0, T]$.

Proof. We apply Λ to the equation $(18)_1$ and then take L^2 inner product with $\triangle_j d$. We take L^2 inner product of $(18)_2$ with $\Lambda\triangle_j\sigma$. We also apply Λ to the equation $(18)_1$ and take L^2 inner product with $\frac{\nu}{\gamma}\triangle_j\sigma$. By a suitable linear combination of them, we obtain the desired identity of the proposition. $\square$

We introduce a lemma for estimates of the right hande side.

Lemma 4.9. *The following inequalities hold*

$$\left|\left(\Lambda\triangle_j(w \cdot \nabla\sigma), \Lambda\triangle_j\sigma\right)\right| \leq C\alpha_j 2^{-(\frac{n}{2}-1)j}\|w\|_{\dot{B}_{2,1}^{\frac{n}{2}+1}}\|\sigma\|_{\dot{B}_{2,1}^{\frac{n}{2}}}\|\Lambda\triangle_j\sigma\|_{L^2},$$

$$\left|\left(\Lambda\triangle_j(w \cdot \nabla\sigma), \Lambda\triangle_j d\right) + \left(\triangle_j(w \cdot \nabla d), \Lambda\triangle_j\sigma\right)\right|$$
$$\leq C\alpha_j 2^{-(\frac{n}{2}-1)j}\|w\|_{\dot{B}_{2,1}^{\frac{n}{2}+1}}\left(\|\Lambda\triangle_j\sigma\|_{L^2}\|d\|_{\dot{B}_{2,1}^{\frac{n}{2}-1}} + \|\sigma\|_{\dot{B}_{2,1}^{\frac{n}{2}}}\|\triangle_j d\|_{L^2}\right),$$

where C independs on j and $\|\{\alpha_j\}\|_{l^1} \leq 1$.

See, e.g., [2], for the proof.

Proposition 4.10. *There exists a $\epsilon > 0$ such that if*

$$\|w_0\|_{B_{2,1}^{\frac{n}{2}-1}} + \|\sigma_0\|_{B_{2,1}^{\frac{n}{2}}} \leq \epsilon$$

then there holds

$$\frac{d}{dt}E_j(t) + c_0 E_j(t) \leq C\{\alpha_j(1+t)^{-\frac{n}{4}}M(t)f(t)$$
$$+2^{(\frac{n}{2}-1)j}\|\Lambda\triangle_j(\sigma\nabla\cdot w)\|_{L^2} + 2^{(\frac{n}{2}-1)j}\|\triangle_j(w\cdot\nabla d)\|_{L^2}$$
$$+2^{(\frac{n}{2}-1)j}\|\triangle_j F_1(U)\|_{L^2} + 2^{(\frac{n}{2}-1)j}\|\triangle_j F_2(U)\|_{L^2}\}, \tag{20}$$

for $t \in [0, T]$ and $j \geq 1$, where $\sum_{j=\mathbb{Z}}\alpha_j \leq 1$, $\int_0^\infty f(t)dt \leq \int_0^\infty \|w\|_{\dot{B}_{2,1}^{\frac{n}{2}+1}} < C\epsilon$ and c_0 is not depend on j. Here, $E_j(t)$ is equivalent to $2^{(\frac{n}{2}-1)j}\|\triangle_j U(t)\|_{L^2} + 2^{\frac{n}{2}j}\|\triangle_j\sigma\|_{L^2}$. That is, there exists a D_1 such that

$$\frac{1}{D_1}\left(2^{(\frac{n}{2}-1)j}\|\triangle_j U(t)\|_{L^2} + 2^{\frac{n}{2}j}\|\triangle_j\sigma\|_{L^2}\right) \leq E_j \leq D_1\left(2^{(\frac{n}{2}-1)j}\|\triangle_j U(t)\|_{L^2} + 2^{\frac{n}{2}j}\|\triangle_j\sigma\|_{L^2}\right).$$

Proof. We add (17) to $\kappa \times$ (19) with a constant $\kappa > 0$ to be determined later. Then, we obtain

$$\frac{d}{dt}\left\{\frac{1}{2}\|\triangle_j U\|_{L^2}^2 + \frac{\kappa}{2}\frac{\nu}{\gamma}\|\Lambda\triangle_j\sigma\|_{L^2}^2 - \kappa(\Lambda\triangle_j\sigma, \triangle_j d)\right\}$$
$$+\mu_1\|\nabla\triangle_j w\|_{L^2}^2 + \mu_2\|\nabla\cdot\triangle_j w\|_{L^2}^2 + \kappa\|\Lambda\triangle_j\sigma\|_{L^2}^2$$
$$= \gamma\kappa\|\Lambda\triangle_j w\|_{L^2}^2 + \left(\triangle_j F_1(U), \triangle_j\sigma\right) + \left(\triangle_j F_1(U), \triangle_j w\right) + \kappa\frac{\nu}{\gamma}\left(\Lambda\triangle_j F_1(U), \Lambda\triangle_j\sigma\right)$$
$$-\kappa\left(\Lambda\triangle_j F_1(U), \triangle_j d\right) - \kappa\left(\Lambda^{-1}\nabla\cdot\triangle_j F_2(U), \Lambda\triangle_j\sigma\right). \tag{21}$$

We set

$$E_j^2(t) = 2^{2(\frac{n}{2}-1)j}\left\{\frac{1}{2}\|\triangle_j U\|_{L^2}^2 + \frac{\kappa}{2}\frac{\nu}{\gamma}\|\Lambda\triangle_j\sigma\|_{L^2}^2 - \kappa(\Lambda\triangle_j\sigma, \triangle_j d)\right\}.$$

For each $\kappa \leq 1$, there exists a $D_1 > 3$ such that

$$E_j^2 \leq D_1^2\left(2^{(\frac{n}{2}-1)j}\|\triangle_j U(t)\|_{L^2} + 2^{\frac{n}{2}j}\|\triangle_j\sigma\|_{L^2}\right)^2.$$

By Cauchy's inequality with δ, we have

$$\left(2^{(\frac{n}{2}-1)j}\|\triangle_j U(t)\|_{L^2} + 2^{\frac{n}{2}j}\|\triangle_j\sigma\|_{L^2}\right)^2 + D_1^2\kappa 2^{2(\frac{n}{2}-1)j}(\triangle_j d, \Lambda\triangle_j\sigma)$$
$$\leq 2\left\{\left(2^{(\frac{n}{2}-1)j}\|\triangle_j U(t)\|_{L^2}\right)^2 + \left(2^{\frac{n}{2}j}\|\triangle_j\sigma\|_{L^2}\right)^2\right\}$$
$$+D_1^2\kappa\delta\left(2^{(\frac{n}{2}-1)j}\|\Lambda\triangle_j\sigma\|_{L^2}\right)^2 + D_1^2\kappa\frac{1}{4\delta}\left(2^{(\frac{n}{2}-1)j}\|\triangle_j w\|_{L^2}\right)^2.$$

We select $\delta = \frac{\nu}{4\gamma D_1}$ and κ is fixed in such a way that $\kappa \leq \min\{\delta, \frac{\mu_1}{4\gamma}, 1\}$. We then obtain

$$\frac{1}{D_1^2}\left(2^{(\frac{n}{2}-1)j}\|\triangle_j U(t)\|_{L^2} + 2^{\frac{n}{2}j}\|\triangle_j\sigma\|_{L^2}\right)^2$$
$$\leq 2^{2(\frac{n}{2}-1)j}\left\{\frac{1}{2}\|\triangle_j U\|_{L^2}^2 + \frac{\kappa}{2}\frac{\nu}{\gamma}\|\Lambda\triangle_j\sigma\|_{L^2}^2 - \kappa(\Lambda\triangle_j\sigma, \triangle_j d)\right\} = E_j^2.$$

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For $j \geq 0$, by Lemma 2.1, there exists a $c_0 > 0$ such that

$$2c_0 E_j^2 \leq 2^{2(\frac{n}{2}-1)j}\big\{\mu_1\|\nabla\triangle_j w\|_{L^2}^2 + \mu_1\|\nabla\cdot\triangle_j w\|_{L^2}^2 + \kappa\|\Lambda\triangle_j\sigma\|_{L^2}^2 - \gamma\kappa\|\Lambda\triangle_j w\|_{L^2}^2\big\}.$$

Let us next estimate the right hande side of $2^{2(\frac{n}{2}-1)j}\times(21)$. By Hölder's inequality, we obtain

$$2^{2(\frac{n}{2}-1)j}\big(\triangle_j F_1(U),\triangle_j\sigma\big) \leq 2^{(\frac{n}{2}-1)j}\|\triangle_j F_1(U)\|_{L^2}2^{(\frac{n}{2}-1)j}\|\triangle_j\sigma\|_{L^2},$$

$$2^{2(\frac{n}{2}-1)j}\big(\triangle_j F_2(U),\triangle_j\sigma\big) \leq 2^{(\frac{n}{2}-1)j}\|\triangle_j F_2(U)\|_{L^2}2^{(\frac{n}{2}-1)j}\|\triangle_j w\|_{L^2},$$

$$2^{2(\frac{n}{2}-1)j}\big(\Lambda^{-1}\nabla\cdot\triangle_j F_2(U),\triangle_j\sigma\big) \leq 2^{(\frac{n}{2}-1)j}\|\triangle_j F_2(U)\|_{L^2}2^{(\frac{n}{2}-1)j}\|\triangle_j\sigma\|_{L^2}.$$

By Lemma 4.9 we have

$$2^{2(\frac{n}{2}-1)j}\big(\Lambda\triangle_j F_1(U),\Lambda\triangle_j\sigma\big)$$
$$= 2^{2(\frac{n}{2}-1)j}\big(\Lambda\triangle_j(w\cdot\nabla\sigma),\Lambda\triangle_j\sigma\big) + 2^{2(\frac{n}{2}-1)j}\big(\Lambda\triangle_j(\sigma\nabla\cdot w),\Lambda\triangle_j\sigma\big)$$
$$\leq C\alpha_j\|w\|_{\dot{B}_{2,1}^{\frac{n}{2}+1}}\|\sigma\|_{\dot{B}_{2,1}^{\frac{n}{2}}}2^{(\frac{n}{2}-1)j}\|\Lambda\triangle_j\sigma\|_{L^2} + 2^{2(\frac{n}{2}-1)j}\|\Lambda\triangle_j(\sigma\nabla\cdot w)\|_{L^2}\|\Lambda\triangle_j\sigma\|_{L^2},$$

and

$$2^{2(\frac{n}{2}-1)j}\big(\Lambda\triangle_j F_1(U),\triangle_j d\big)$$
$$= 2^{2(\frac{n}{2}-1)j}\big(\Lambda\triangle_j(w\cdot\nabla\sigma),\triangle_j d\big) + 2^{2(\frac{n}{2}-1)j}\big(\Lambda\triangle_j(\sigma\nabla\cdot w),\triangle_j d\big)$$
$$\qquad + 2^{2(\frac{n}{2}-1)j}\big(\triangle_j(w\cdot\nabla d),\Lambda\triangle_j\sigma\big) - 2^{2(\frac{n}{2}-1)j}\big(\triangle_j(w\cdot\nabla d),\Lambda\triangle_j\sigma\big)$$
$$\leq C\alpha_j\|w\|_{\dot{B}_{2,1}^{\frac{n}{2}+1}}\big(\|d\|_{\dot{B}_{2,1}^{\frac{n}{2}-1}}2^{(\frac{n}{2}-1)j}\|\Lambda\triangle_j\sigma\|_{L^2} + \|\sigma\|_{\dot{B}_{2,1}^{\frac{n}{2}}}2^{(\frac{n}{2}-1)j}\|\triangle_j d\|_{L^2}\big)$$
$$\qquad + 2^{2(\frac{n}{2}-1)j}\|\triangle_j(w\nabla\cdot d)\|_{L^2}\|\Lambda\triangle_j\sigma\|_{L^2} + 2^{2(\frac{n}{2}-1)j}\|\Lambda\triangle_j(\sigma\nabla\cdot w)\|_{L^2}\|\triangle_j d\|_{L^2},$$

where $\sum_{j\in\mathbb{Z}}\alpha_j \leq 1$. Hence we obtain

$$\frac{d}{dt}E_j^2 + 2c_0 E_j^2 \leq CE_j\big\{\alpha_j(1+t)^{-\frac{n}{4}}M(t)f(t)$$
$$+ 2^{(\frac{n}{2}-1)j}\|\Lambda\triangle_j(\sigma\nabla\cdot w)\|_{L^2} + 2^{(\frac{n}{2}-1)j}\|\triangle_j(w\cdot\nabla d)\|_{L^2}$$
$$+ 2^{(\frac{n}{2}-1)j}\|\triangle_j F_1(U)\|_{L^2} + 2^{(\frac{n}{2}-1)j}\|\triangle_j F_2(U)\|_{L^2}\big\}. \qquad (22)$$

Let $\delta_1 > 0$ be a small parameter (which will tend to 0) and denote $H_j^2 = E_j^2 + \delta_1^2$. From (22) and dividing by H_j, we gather

$$\frac{d}{dt}H_j + c_0 H_j \leq C\big\{\alpha_j(1+t)^{-\frac{n}{4}}M(t)f(t)$$
$$+ 2^{(\frac{n}{2}-1)j}\|\Lambda\triangle_j(\sigma\nabla\cdot w)\|_{L^2} + 2^{(\frac{n}{2}-1)j}\|\triangle_j(w\cdot\nabla d)\|_{L^2}$$
$$+ 2^{(\frac{n}{2}-1)j}\|\triangle_j F_1(U)\|_{L^2} + 2^{(\frac{n}{2}-1)j}\|\triangle_j F_2(U)\|_{L^2}\big\} + c_0\delta_1^2.$$

Having δ_1 tend to 0, we get the desired result. $\qquad\square$

4.3 Proof of Theorem 1.3.

Proposition 4.11. *There exists a constant $\epsilon_2 > 0$ such that if*

$$\|U_0\|_{B_{2,1}^{\frac{n}{2}-1}\cap \dot{B}_{1,\infty}^0} + \|\sigma_0\|_{B_{2,1}^{\frac{n}{2}}} \leq \epsilon_2,$$

then there holds

$$M(t) \leq C\big\{\|U_0\|_{B_{2,1}^{\frac{n}{2}-1}\cap \dot{B}_{1,\infty}^0} + \|\sigma_0\|_{B_{2,1}^{\frac{n}{2}}}\big\}$$

for $0 \leq t \leq T$, where the constant C does not depend on T.

Proof. By (20) we have

$$
\begin{aligned}
E_j(t) \;\leq\; & e^{-c_0 t} E_j(0) \\
& + C \int_0^t e^{-c_0(t-\tau)}\big\{\alpha_j(1+t)^{-\frac{n}{4}} M(t) f(t) + \alpha_j(1+t)^{-\frac{n}{2}} M^2(t) \\
& \quad + 2^{(\frac{n}{2}-1)j}\|\Lambda\triangle_j(\sigma\nabla\cdot w)\|_{L^2} + 2^{(\frac{n}{2}-1)j}\|\triangle_j(w\cdot\nabla d)\|_{L^2} \\
& \quad + 2^{(\frac{n}{2}-1)j}\|\triangle_j F_1(U)\|_{L^2} + 2^{(\frac{n}{2}-1)j}\|\triangle_j F_2(U)\|_{L^2}\big\}d\tau, \hspace{1cm} (23)
\end{aligned}
$$

where $\sum_{j=0}^{\infty}\alpha_j \leq 1$ and $\int_0^\infty f(t)dt \leq \int_0^\infty \|w\|_{\dot{B}_{2,1}^{\frac{n}{2}+1}}dt < C\epsilon_2$. Hence summing up on $j \geq 0$, by the monotone convergence theorem, we obtain

$$
\begin{aligned}
\sum_{j=0}^{\infty} E_j(t) \;\leq\; & e^{-c_0 t}\sum_{j=0}^{\infty} E_j(0) \\
& + C\int_0^t e^{-c_0(t-\tau)}\big\{\alpha_j(1+t)^{-\frac{n}{4}}M(t)f(t) + (1+t)^{-\frac{n}{2}}M^2(t) \\
& \quad + \|\sigma\nabla\cdot w\|_{\dot{B}_{2,1}^{\frac{n}{2}}} + \|w\cdot\nabla d\|_{\dot{B}_{2,1}^{\frac{n}{2}-1}} + \|F_1(U)\|_{\dot{B}_{2,1}^{\frac{n}{2}-1}} + \|F_2(U)\|_{\dot{B}_{2,1}^{\frac{n}{2}-1}}\big\}d\tau.
\end{aligned}
$$

We next estimate the right hand side. From Lemma 3.1 and Lemma 3.2, we have

$$\|\sigma\nabla\cdot w\|_{\dot{B}_{2,1}^{\frac{n}{2}}} \leq C\|\sigma\|_{\dot{B}_{2,1}^{\frac{n}{2}}}\|\nabla\cdot w\|_{\dot{B}_{2,1}^{\frac{n}{2}}} \leq C(1+\tau)^{-\frac{n}{4}}M(\tau)f(\tau),$$

$$\|w\cdot\nabla d\|_{\dot{B}_{2,1}^{\frac{n}{2}-1}} \leq C\|w\|_{\dot{B}_{2,1}^{\frac{n}{2}-1}}\|\nabla d\|_{\dot{B}_{2,1}^{\frac{n}{2}}} \leq C(1+\tau)^{-\frac{n}{4}}M(\tau)f(\tau).$$

Let us next consider $\|F_1(U)\|_{\dot{B}_{2,1}^{\frac{n}{2}-1}}$, $\|F_2(U)\|_{\dot{B}_{2,1}^{\frac{n}{2}-1}}$:

$$
\begin{aligned}
\|w\cdot\nabla\sigma\|_{\dot{B}_{2,1}^{\frac{n}{2}-1}} \;\leq\; & C\|w\|_{\dot{B}_{2,1}^{\frac{n}{2}}}\|\nabla\sigma\|_{\dot{B}_{2,1}^{\frac{n}{2}-1}} \\
\leq\; & C\big(\|\triangle_{-1}w\|_{\dot{B}_{2,1}^{\frac{n}{2}}} + \|\tilde{S}_0 w\|_{\dot{B}_{2,1}^{\frac{n}{2}}}\big)\|\sigma\|_{\dot{B}_{2,1}^{\frac{n}{2}}} \\
\leq\; & C(1+\tau)^{-\frac{n}{2}}M^2(\tau) + C(1+\tau)^{-\frac{n}{4}}M(\tau)f(\tau),
\end{aligned}
$$

$$\|\sigma\nabla\cdot w\|_{\dot{B}_{2,1}^{\frac{n}{2}-1}} \;\leq\; C\|\sigma\|_{\dot{B}_{2,1}^{\frac{n}{2}-1}}\|\nabla w\|_{\dot{B}_{2,1}^{\frac{n}{2}}} \leq C(1+\tau)^{-\frac{n}{4}}M(\tau)f(\tau).$$

Hence, we obtain the estimate of $\|F_1(U)\|_{\dot{B}_{2,1}^{\frac{n}{2}-1}}$. By using Lemma 3.1 and Lemma 3.2, $\|F_2(U)\|_{\dot{B}_{2,1}^{\frac{n}{2}-1}}$ is estimated as

$$
\begin{aligned}
\|\frac{\sigma}{\sigma+1}\triangle w\|_{\dot{B}_{2,1}^{\frac{n}{2}-1}} &\leq C\|\frac{\sigma}{\sigma+1}\|_{\dot{B}_{2,1}^{\frac{n}{2}}}\|\triangle w\|_{\dot{B}_{2,1}^{\frac{n}{2}-1}} \\
&\leq C\|\sigma\|_{\dot{B}_{2,1}^{\frac{n}{2}}}\|w\|_{\dot{B}_{2,1}^{\frac{n}{2}+1}} \leq C(1+\tau)^{-\frac{n}{4}}M(\tau)f(\tau),
\end{aligned}
$$

$$
\|\frac{\bar{\rho}\gamma}{\sigma+1}\sigma\nabla\sigma\|_{\dot{B}_{2,1}^{\frac{n}{2}-1}} \leq C\|\frac{\bar{\rho}\gamma}{\sigma+1}\sigma\|_{\dot{B}_{2,1}^{\frac{n}{2}}}\|\nabla\sigma\|_{\dot{B}_{2,1}^{\frac{n}{2}-1}} \leq C(1+\tau)^{-\frac{n}{2}}M^2(\tau),
$$

$$
\|(w\cdot\nabla)w\|_{\dot{B}_{2,1}^{\frac{n}{2}-1}} \leq C\|w\|_{\dot{B}_{2,1}^{\frac{n}{2}-1}}\|\nabla w\|_{\dot{B}_{2,1}^{\frac{n}{2}-1}} \leq C(1+\tau)^{-\frac{n}{4}}M(\tau)f(\tau).
$$

In the same way as we can obtain estimates of other terms on $\|F_2(U)\|_{\dot{B}_{2,1}^{\frac{n}{2}-1}}$. Hence, by using Lemma 3.3, the integral of the right hand side of (23) is estimated as

$$
\int_0^t e^{-c_0(t-\tau)}\big\{(1+\tau)^{-\frac{n}{2}}M^2(\tau)+(1+\tau)^{-\frac{n}{4}}M(\tau)f(\tau)\big\}d\tau
$$

$$
\leq M(t)\int_0^t e^{-c_0(t-\tau)}(1+\tau)^{-\frac{n}{4}}f(\tau)d\tau + M^2(t)\int_0^t e^{-c_0(t-\tau)}(1+\tau)^{-\frac{n}{2}}d\tau
$$

$$
\leq C(1+t)^{-\frac{n}{4}}\epsilon_2 M(t)+C(1+t)^{-\frac{n}{2}}M^2(t).
$$

Hence, we obtain

$$
M_\infty(t) \leq C\big(\|U_0\|_{\dot{B}_{2,1}^{\frac{n}{2}-1}}+\|\sigma_0\|_{\dot{B}_{2,1}^{\frac{n}{2}}}\big)+C\epsilon_2 M(t)+CM^2(t). \tag{24}
$$

By Proposition 4.5 and (24), we have

$$
M(t) \leq C\big(\|U_0\|_{\dot{B}_{2,1}^{\frac{n}{2}-1}\cap\dot{B}_{1,\infty}^0}+\|\sigma_0\|_{\dot{B}_{2,1}^{\frac{n}{2}}}\big)+C\epsilon_2 M(t)+CM^2(t).
$$

By taking $\epsilon_2 > 0$ suitable small, we obtain

$$
M(t) \leq C\big(\|U_0\|_{B_{2,1}^{\frac{n}{2}-1}\cap\dot{B}_{1,\infty}^0}+\|\sigma_0\|_{B_{2,1}^{\frac{n}{2}}}\big)
$$

for all $0 \leq t \leq T$. $\qquad\square$

It follows from Proposition 1.2 and Proposition 4.11 that

$$
M(t) \leq C_3 \qquad \text{for all } t.
$$

Hence we obtain the desired decay estimate in Theorem 1.3.

References

[1] H. Bahouri, J. Y. Chemin, R. Danchin *Fourier analysis and nonlinear partial differential equation*, Springer, Heidelberg, 2011

[2] R. Danchin, Global existence in critical spaces for compressible Navier-Stokes equations, Invent. Math. **141**, (2000) 579-614

[3] R. Danchin, On the uniqueness in critical spaces for compressible Navier-Stokes equations, Nonlinear Differ. Equ. Appl **12** (2005) 111-128

[4] B. Haspot, Well-posedness in critical spaces for the system of compressible Navier-Stokes in larger spaces, J. Differential Equations **251** (2011) 2262-2295

[5] Y. Kagei, Asymptotic behavior of solutions to the compressible Navier-Stokes equation around a parallel flow, Arch. Rational Mech. Anal. **205** (2012), 585-650.

[6] M. Kawashita, On global solution of Cauchy problems for compressible Navier-Stokes equation , Nonlinear Analysis **48** (2002), 1087-1105.

[7] H-Liang. Li, T. Zhang, Large time behavior of isentropic compressible Navier-Stokes system in $\mathbb{R}^3$, Math. Meth Appl. Sci **34** (2011), 670-682.

[8] A. Matsumura, T. Nishida, The initial value problem for the equation of motion of compressible viscous and heat-conductive fluids, Proc. Japan Acad. Ser. A **55** (1979), 337-342.

[9] A. Matsumura, T. Nishida, The initial value problems for the equation of motion of compressible viscous and heat-conductive gases, J. Math. Kyoto Univ. **20** (1980), 67-104.

[10] M. Okita, On the convergence rates for the compressible Navier- Stokes equations with potential force, to appear in Kyushu J. Math.

[11] Y. Wang, Z. Tan, Global existence and optimal decay rate for the strong solution in H^2 to the compressible Navier-Stokes equation, Applied Mathematics Letters **24** (2011), 1778-1784.

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