Packing Arborescences in Acyclic Temporal Networks

Kamiyama, Naoyuki
Faculty of Mathematics, Kyushu University

https://hdl.handle.net/2324/26869

出版情報：Information Processing Letters. 115 (2), pp.321-325, 2015-02. Elsevier Science Publishers

MI Preprint Series

Kyushu University
The Global COE Program
Math-for-Industry Education & Research Hub

Packing Arborescences in Acyclic Temporal Networks

Naoyuki KAMIYAMA

MI 2013-9

(Received July 31, 2013)

Faculty of Mathematics
Kyushu University
Fukuoka, JAPAN

Packing Arborescences in Acyclic Temporal Networks

Naoyuki Kamiyama*

Institute of Mathematics for Industry, Kyushu University.
E-mail: kamiyama@imi.kyushu-u.ac.jp

Abstract. A temporal network is a finite directed graph in which each arc has a time label specifying the time at which its end-vertices communicate. An arborescence in a temporal network is said to be time-respecting, if the time labels on every directed path from the root in this arborescence are monotonically non-decreasing. In this paper, we consider the problem of packing time-respecting arborescences in a temporal network. Precisely speaking, we study an extension of Edmonds' arc-disjoint arborescences theorem in a temporal network. Unfortunately, it is known that a natural extension of Edmonds' arc-disjoint arborescences theorem in a temporal network does not hold. In this paper, we first show that this extension does not hold, even if an input temporal network is acyclic. Next, we prove that if an input temporal network is acyclic and pre-flow, this extension holds and we can find arc-disjoint time-respecting arborescences in polynomial time. Furthermore, we extend our main result to the problem of packing time-respecting partial arborescences.

1 Introduction

Throughout this paper, we denote by $\mathbb{N}$ the set of positive integers. For each directed graph D, we denote by $V(D)$ and $A(D)$ the sets of vertices and arcs of D, respectively. Furthermore, for each directed graph D and each vertex v of $V(D)$, let $\delta_D(v)$ and $\varrho_D(v)$ be the sets of arcs of $A(D)$ leaving and entering v, respectively. We denote by $a = (u, v)$ an arc a from u to v.

A *temporal network* N is a pair (D, τ) of a finite directed graph D and a time label function $\tau \colon A(D) \to \mathbb{N}$. For each arc a of $A(D)$, the time label $\tau(a)$ specifies the time at which its end-vertices communicate. This model is used for modeling communication in distributed networks and scheduled transportation networks (for applications of temporal networks, see [1]). If we want to communicate along a directed path P in a temporal network, then the time labels of the arcs on P must be monotonically non-decreasing. Formally speaking, a directed path P in a temporal network $N = (D, \tau)$ is said to be *time-respecting*, if

$$\tau(a_1) \leq \tau(a_2) \leq \cdots \leq \tau(a_k),$$

* This work is partly supported by KAKENHI(25240004).

where we assume that P passes through arcs $a_1, a_2, \ldots, a_k$ of $A(D)$ in this order. Time-respecting directed paths are natural and crucial structures in understanding the way in which information has disseminated through the network.

Besides directed paths, arborescences are another important structures in a directed graph from not only a theoretical point of view but also a practical point of view. Formally speaking, a subgraph T of a finite directed graph D with a specified vertex r is called an *r-arborescence* or an *arborescence rooted at r*, if

1. $V(T) = V(D)$,
2. there exists a directed path from r to every vertex v of $V(D)$ in T, and
3. $\varrho_T(r) = \emptyset$ and
$$\forall v \in V(D) \setminus \{r\}\colon\ |\varrho_T(v)| = 1.$$

It is not difficult to see that an r-arborescence is a spanning tree in D (when viewed as an undirected graph) whose arcs are directed away from r. For example, in [2], arborescences are used in the context of broadcasting.

Assume that we are given a temporal network $N = (D, \tau)$ with a specified vertex r. For each vertex v of $V(D) \setminus \{r\}$, an r-arborescence T in N is said to be *time-respecting on v*, if

$$\forall a \in \delta_T(v)\colon\ \tau(\mathsf{in}(v)) \leq \tau(a), \tag{1}$$

where $\mathsf{in}(v)$ is the unique arc of $\varrho_T(v)$. Furthermore, an r-arborescence T in N is said to be *time-respecting*, if T is time-respecting on every vertex of $V(D) \setminus \{r\}$. It is not difficult to see that an r-arborescence T in N is time-respecting if and only if for every vertex v of $V(T)$, the unique directed path from r to v in T is time-respecting.

In this paper, we consider the problem of packing time-respecting arborescences rooted at a specified vertex in a temporal network (see Figure 1). Packing problems are one of central topics in Graph Theory and Combinatorial Optimization. Furthermore, from a practical point of view, it is natural to think that a network in which we can pack many arborescences has high robustness against troubles.

2 Problem Formulation

For defining our problem, we first consider the case where the time label of every arc is the same, i.e., we consider the problem of packing arborescences rooted at a specified vertex in a finite directed graph. In this case, the following important theorem was proved by Edmonds [3].

Theorem 1 (Edmonds [3]). *For each finite directed graph D with a specified vertex r and each positive integer k, there exist k arc-disjoint r-arborescences if and only if for every vertex v of $V(D)$, there exist k arc-disjoint directed paths from r to v.*

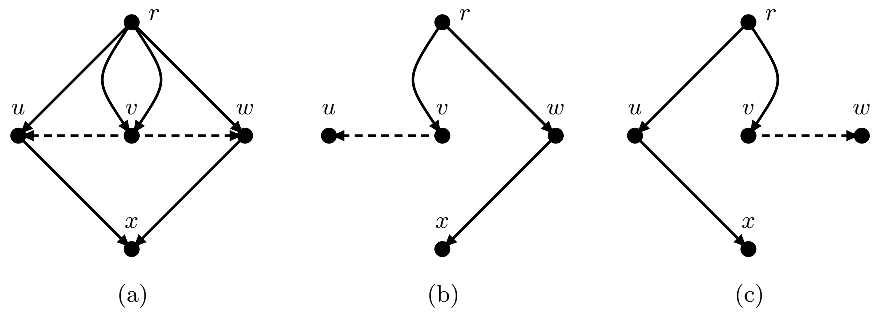

Fig. 1. (a) An example of a temporal network. A time label of each arc illustrated by a real line is equal to 1. A time label of each arc illustrated by a broken line is equal to 2. (b, c) Two arc-disjoint time-respecting r-arborescences in the temporal network illustrated in (a).

Theorem 1 is one of the most important min-max theorems in Graph Theory and Combinatorial Optimization. Furthermore, it gives us the following algorithmic implication. For checking the existence of k arc-disjoint r-arborescences, it is suffice to check whether there exist k arc-disjoint directed paths from r to every vertex. Since we can check in polynomial time whether there exist k arc-disjoint directed paths from r to every vertex (see, e.g., [4]), Theorem 1 implies that we can check in polynomial time whether there exist k arc-disjoint r-arborescences. It should be noted that Theorem 1 was extended to various settings (see, e.g., [5–7]).

In this paper, we consider the following extension of Theorem 1.

Statement A. *For each temporal network $N = (D, \tau)$ with a specified vertex r and each positive integer k, there exist k arc-disjoint time-respecting r-arborescences if and only if for every vertex v of $V(D)$, there exist k arc-disjoint time-respecting directed paths from r to v.*

Unfortunately, it is known [1] that Statement A does not hold. Although the counterexample proposed in [1] has a directed cycle, we can construct an acyclic temporal network in which Statement A does not hold by slightly modifying their counterexample (see Figure 2). Precisely speaking, a temporal network $N = (D, \tau)$ is said to be *acyclic*, if D is acyclic.

In the temporal network illustrated in Figure 2, there exist two arc-disjoint time-respecting directed paths from r to every vertex. Thus, from Theorem 1, we can see that there exist two arc-disjoint r-arborescences T_1 and T_2. However, at least one of these two r-arborescences contains at most one arc leaving r that has a time label 1. Assume that T_1 contains the arc from r to u with time label 1, and the arcs from r to v and w with time label 1 are not contained in T_1. In T_1, the unique directed path from r to z must use an arc with time label 2, i.e., this directed path is not time-respecting. This implies that there can not be two arc-disjoint time-respecting r-arborescences in this temporal network.

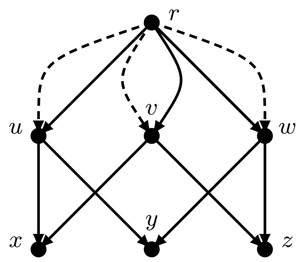

Fig. 2. A counterexample for Statement A. A time label of each arc illustrated by a real line is equal to 1. A time label of each arc illustrated by a broken line is equal to 2.

In this paper, we consider Statement A in some special case. Precisely speaking, we prove that if an input temporal network is acyclic and "pre-flow", then Statement A holds. As shown later, a temporal network illustrated in Figure 2 is not pre-flow. Thus, our result is in a sense tight for acyclic temporal networks.

Remark. Here we give a remark about the counterexample proposed in [1]. In their example, instead of arcs with time label 2 between r and u, v, w, there exist arcs

$$(u, v), (v, w), (w, u), (v, u), (w, v), (u, w),$$

and the time labels of these arcs are 2. Our proof for the non-existence of two arc-disjoint time-respecting r-arborescences in a temporal network illustrated in Figure 2 is the same as the proof in [1] for their counterexample.

3 Main Result

For each temporal network $N = (D, \tau)$, each vertex v of $V(D)$ and each positive integer i, define $\sigma_N(v, i)$ and $\gamma_N(v, i)$ by

$$\sigma_N(v, i) := |\{a \in \varrho_D(v) \mid \tau(a) \leq i\}|,$$
$$\gamma_N(v, i) := |\{a \in \delta_D(v) \mid \tau(a) \leq i\}|,$$

respectively. A temporal network $N = (D, \tau)$ with a specified vertex r is said to be *pre-flow*, if

$$\forall v \in V(D) \setminus \{r\}, \forall i \in \mathbb{N}\colon\ \sigma_N(v, i) \geq \gamma_N(v, i). \tag{2}$$

It should be noted that the temporal network $N = (D, \tau)$ illustrated in Figure 2 is not pre-flow since

$$\sigma_N(v, 1) = 1 < 2 = \gamma_N(v, 1).$$

In this paper, we will prove that Statement A holds if an input temporal network is acyclic and pre-flow. Our main result can be described as follow.

Theorem 2. *For each acyclic and pre-flow temporal network $N = (D, \tau)$ with a specified vertex r and each positive integer k, there exist k arc-disjoint time-respecting r-arborescences if and only if for each vertex v of $V(D)$, there exist k arc-disjoint time-respecting directed paths from r to v.*

We will give the proof of Theorem 2 in the next section. Our proof of Theorem 2 is constructive. Thus, this proof gives a polynomial-time algorithm for finding arc-disjoint time-respecting arborescences.

Remark. Here we give a remark about the concept of "pre-flow". The concept of pre-flow directed graphs was introduced in [8]. A finite directed graph D is said to be *pre-flow*, if

$$\forall v \in V(D) \setminus \{r\}\colon\ |\varrho_D(v)| \geq |\delta_D(v)|. \tag{3}$$

If the time label of every arc is the same, the definition in this paper is equivalent to (3). In [8], the authors studied the problem of packing "partial" arborescences rooted at a specified vertex in a pre-flow directed graph (see Section 6).

4 Proof

In this section, we prove Theorem 2. Since the *if*-part is trivial, we prove the other direction. Throughout this section, we assume that we are given an acyclic and pre-flow temporal network $N = (D, \tau)$ with a specified vertex r and a positive integer k. Let n be the number of vertices of D. Furthermore, we assume that for each vertex v of $V(D)$, there exist k arc-disjoint time-respecting directed paths from r to v.

It is well-known (see, e.g., [4]) that since D is acyclic, there exists a function $\pi\colon V(D) \to \{1, 2, \ldots, n\}$ such that $\pi(u) > \pi(v)$ if there exists an arc of $A(D)$ from u to v. Since there exists a time-respecting directed path from r to every vertex of $V(D)$, if $\varrho_D(r) \neq \emptyset$, then there exists a directed cycle in D. So, no arc of $A(D)$ enters r, and thus we have $\pi(r) = n$.

Before proving Theorem 2, we explain about the difficulty of proving Theorem 2. Roughly speaking, we show that Theorem 2 does not seem to be able to be "straightforwardly" proved by induction on the number of vertices. Assume that we are given a temporal network $N = (D, \tau)$ with a specified vertex r illustrated in Figure 1(a). For every vertex v of $V(D)$, there exist two arc-disjoint time-respecting directed paths from r to v. It is not difficult to see that there exist two arc-disjoint time-respecting directed paths from r to every vertex, even if we remove x and arcs of $\varrho_D(x)$. Let N' be the temporal network obtained from N by removing x and arcs of $\varrho_D(x)$. By induction hypothesis, there exist two arc-disjoint time-respecting r-arborescences in N'. Assume that we have two r-arborescences in N' illustrated in Figure 3. However, we can not construct two arc-disjoint time-respecting r-arborescences in N from arborescences illustrated in Figure 3 for the following reason. For constructing an r-arborescence in N from an r-arborescence in N' illustrated in Figure 3(b), we have to add one arc of $\varrho_D(x)$. However, we can not construct a time-respecting r-arborescence in N by adding either (u, x) or (w, x) to an r-arborescence in N' illustrated in Figure 3(b). This implies that Theorem 2 does not seem to be able to be straightforwardly proved by induction on the number of vertices.

Fig. 3. Two partial time-respecting r-arborescences that can not be extended to two arc-disjoint time-respecting r-arborescences.

In the rest of this section, for proving Theorem 1, we first propose an algorithm for finding k arc-disjoint time-respecting r-arborescences. After that, we prove its correctness.

4.1 Algorithm

For proposing our algorithm, we first introduce an auxiliary bipartite graph. For each vertex v of $V(D) \setminus \{r\}$, we construct a bipartite graph $G_v = (P_v, Q_v; E_v)$ as follows. The vertex set P_v (resp., Q_v) contains a vertex $p(a)$ (resp., $q(a)$) for each arc a of $\varrho_D(v)$ (resp., $\delta_D(v)$). Furthermore, the edge set E_v contains an edge between a vertex $p(a)$ of P_v and a vertex $q(b)$ of Q_v, if $\tau(a) \le \tau(b)$. These are all the arcs of E_v.

The following lemma plays an important role in our algorithm.

Lemma 1. *For each vertex v of $V(D)$, there exists a matching M_v in G_v such that it covers all the vertices of Q_v, i.e., for every vertex q of Q_v, there exists an edge of M_v that is incident to q.*

Proof. It is known [9] that there exists a matching M_v in G_v covering all the vertices of Q_v if and only if

$$\forall X \subseteq Q_v \colon\ |\Gamma(X)| \ge |X|, \tag{4}$$

where $\Gamma(X)$ is the set of vertices of P_v that is adjacent to a vertex of X. Let us fix a subset X of Q_v. Define

$$t := \max\{\tau(a) \mid q(a) \in X\}.$$

Furthermore, let a^* be an arc of $\delta_D(v)$ such that $\tau(a^*) = t$. Since $\tau(a) \le t$ for every vertex $q(a)$ of X, we have

$$\gamma_N(v, t) \ge |X|. \tag{5}$$

From the definition of the edge set E_v, we can see that there exists an edge between $p(a)$ and $q(a^*)$ for every arc a of $\varrho_D(a)$ such that $\tau(a) \leq t$. So, we have

$$|\Gamma(X)| \geq \sigma_N(v,t). \tag{6}$$

From (2), (5) and (6), we can see that (4) holds. □

For each vertex v of $V(D) \setminus \{r\}$, we fix a matching M_v in G_v covering all the vertices of Q_v. For each arc $a = (v, w)$ of $A(D) \setminus \delta_D(r)$, we denote by $\mu(a)$ the arc b of $\varrho_D(v)$ such that there exists an edge between $p(b)$ and $q(a)$ in M_v.

We are now ready to propose our algorithm.

Algorithm 1

Step 1: For each $i = 1, 2, \ldots, k$, set $A_i^0 := \emptyset$. Furthermore, set $t := 1$.
Step 2: If $t = n$, then halt and output $A_1^{n-1}, A_2^{n-1}, \ldots, A_k^{n-1}$.
Step 3: Set v be the vertex of $V(D)$ such that $\pi(v) = t$, and do the following.

(3-a) Partition $\{1, 2, \ldots, k\}$ into I^+ and I^- so that

$$I^+ := \{i = 1, 2, \ldots, k \mid \delta_D(v) \cap A_i^{t-1} \neq \emptyset\},$$
$$I^- := \{i = 1, 2, \ldots, k \mid \delta_D(v) \cap A_i^{t-1} = \emptyset\}.$$

(3-b) For each positive integer i of I^+, find an arc a^* of $\delta_D(v) \cap A_i^{t-1}$ such that

$$\tau(a^*) = \min\{\tau(a) \mid a \in \delta_D(v) \cap A_i^{t-1}\}, \tag{7}$$

and then set $a_i^t := \mu(a^*)$.

(3-c) For each positive integer i of I^-, choose an arbitrary arc a_i^t of $\varrho_D(v)$ so that

$$\forall i, j \in I^- \text{ s.t. } i \neq j\colon\ a_i^t \neq a_j^t, \qquad \forall i \in I^-, \forall j \in I^+\colon\ a_i^t \neq a_j^t. \tag{8}$$

(3-d) For each $i = 1, 2, \ldots, k$, set $A_i^t := A_i^{t-1} \cup \{a_i^t\}$.
(3-e) Update $t := t + 1$, and then go to Step 2.

End of Algorithm 1

4.2 Correctness

Here we prove the correctness of Algorithm 1. We first prove that Algorithm 1 is well-defined. For this, it suffices to prove that in Step (3-c), we can find an arc a_i^t satisfying (8) for each positive integer i of I^-. This can be proved as follows. Since for each vertex v of $V(D)$, there exist k arc-disjoint time-respecting directed paths from r to v, we have

$$\forall v \in V(D) \setminus \{r\}\colon\ |\varrho_D(v)| \geq k. \tag{9}$$

This implies that we can find an arc a_i^t satisfying (8) for each positive integer i of I^-.

Assume that Algorithm 1 outputs subsets $A_1^{n-1}, A_2^{n-1}, \dots, A_k^{n-1}$ of A. For each $i = 1, 2, \dots, k$, let T_i be a subgraph of D such that $V(T_i) = V(D)$ and $A(T_i) = A_i^{n-1}$. Since $\mu(a) \neq \mu(b)$ for every distinct arcs a, b of $A(D) \setminus \delta_D(r)$, subgraphs $T_1, T_2, \dots, T_k$ are clearly arc-disjoint. Thus, Theorem 2 immediately follows from the following lemma.

Lemma 2. *For $i = 1, 2, \dots, k$, T_i is a time-respecting r-arborescence.*

Proof. Let us fix a positive integer i such that $i \leq k$. It is not difficult to see that

$$\forall v \in V(D) \setminus \{r\} \colon \; |\varrho_{T_i}(v)| = 1. \tag{10}$$

Although it is well-known that (10) and the fact that D is acyclic imply that T_i is r-arborescence, we give its proof for completeness. It sufficient to prove that there exists a directed path from r to every vertex of $V(D)$ in T_i. Let us fix a vertex v of $V(D) \setminus \{r\}$. Let u be the tail of the unique arc of $\varrho_{T_i}(v)$. From the definition of the function π, we can see that $\pi(u) > \pi(v)$. Furthermore, for the tail of the unique arc of $\varrho_{T_i}(u)$, we have $\pi(w) > \pi(u)$. So, by repeating this, we can find a directed path from r to v in T_i.

Now we prove that T_i is time-respecting. Let us fix a vertex v of $V(D) \setminus \{r\}$. Assume that $\pi(v) = t$. Let $\mathsf{in}(v)$ be the unique arc of $\varrho_{T_i}(v)$. Moreover, let a^* be the unique arc of $\delta_{T_i}(v)$ such that $\mathsf{in}(v) = \mu(a^*)$. Since D is acyclic,

$$\forall j = t, t+1, \dots, n-1 \colon \; \delta_D(v) \cap A_i^j = \delta_D(v) \cap A_i^{t-1}.$$

From this and (7), we can see that

$$\begin{aligned} \tau(a^*) &= \min\{\tau(a) \mid a \in \delta_D(v) \cap A_i^{t-1}\} \\ &= \min\{\tau(a) \mid a \in \delta_D(v) \cap A_i^{n-1} \, (= \delta_{T_i}(v))\}. \end{aligned}$$

This and the definition of $\mu(\cdot)$ imply

$$\tau(\mathsf{in}(v)) = \tau(\mu(a^*)) \leq \tau(a^*) = \min\{\tau(a) \mid a \in \delta_{T_i}(v)\} \leq \tau(b)$$

for every arc b of $\delta_{T_i}(v)$. This completes the proof. $\square$

5 Time complexity

In this section, we analyze the time required to check whether there exist k arc-disjoint time-respecting r-arborescences in an acyclic and pre-flow temporal network $N = (D, \tau)$ with a specified vertex r and a positive integer k, and find them if they exist. Define $m := |A(D)|$. We assume that D is weakly connected, which implies $|V(D)| = O(m)$. By Theorem 2, for checking whether there exist k arc-disjoint time-respecting r-arborescences in N, it suffice to check where there exist k arc-disjoint time-respecting directed paths from r to every vertex of $V(D)$. From the proof of Theorem 2, we can see that there exist k arc-disjoint time-respecting directed paths from r to every vertex of $V(D)$ if and only if (9) holds. Thus, we can check this in $O(m)$ time. Next, we analyze the time required to find k arc-disjoint time-respecting r-arborescences, i.e., the time complexity of Algorithm 1. It is not difficult to see that if we know $\mu(a)$ for every arc a of $A(D)\setminus\delta_D(r)$, the rest of Algorithm 1 can be done in $O(m)$ time. What remains is to analyze the time required to compute M_v for all the vertices v of M_v. Although we can compute M_v for each vertex v of $V(D)\setminus\{r\}$ in polynomial time by using any polynomial-time algorithm for the maximum matching problem (see, e.g., [10]), we can compute it faster as follows. Let us fix a vertex v of $V(D)\setminus\{r\}$. Assume that $\varrho_D(v) = \{a_1, a_2, \ldots, a_l\}$ and

$$\tau(a_1) \leq \tau(a_2) \leq \cdots \leq \tau(a_l).$$

Moreover, assume that $\delta_D(v) = \{b_1, b_2, \ldots, b_h\}$ and

$$\tau(b_1) \leq \tau(b_2) \leq \cdots \leq \tau(b_h).$$

It follows from (2) that $l \geq h$ and $\tau(a_i) \leq \tau(b_i)$ for every $i = 1, 2, \ldots, h$. So, we can set $a_i = \mu(b_i)$ for every $i = 1, 2, \ldots, h$, and thus we can compute M_v for all the vertices of v of $V(D)\setminus\{r\}$ in $O(m \log m)$ time. This implies that the time complexity of Algorithm 1 is $O(m \log m)$.

6 Generalization

In this section, we consider an extension of Theorem 2. Let D be a finite directed graph with a specified vertex r. For each vertex v of $V(D)$, we denote by $\lambda_D(v)$ the maximum number of arc-disjoint directed paths from r to v in D. A subgraph T of D is called a *partial r-arborescence*, if $r \in V(T)$ and T is an r-arborescence in the subgraph of D induced by $V(T)$. Notice that $V(T)$ is not necessarily equal to $V(D)$.

The following extension of Theorem 1 in a pre-flow directed graph is known. It should be noted that if we set

$$k := \min\{\lambda_D(v) \mid v \in V(D)\},$$

then Theorem 3 corresponds to Theorem 1 in a pre-flow directed graph.

Theorem 3 (Bang-Jensen, Frank and Jackson [8]). *For each pre-flow finite directed graph D with a specified vertex r and each positive integer k such that*

$$k \leq \max\{\lambda_D(v) \mid v \in V(D)\setminus\{r\}\},$$

there exist k arc-disjoint partial r-arborescences such that each vertex v of $V(D)$ is contained in exactly $\min\{k, \lambda_D(v)\}$ arborescences.

In the rest of this paper, we prove that Theorem 3 can be extended in a pre-flow and acyclic temporal network. Let $N = (D, \tau)$ be a temporal network with a specified vertex r. For each vertex v of $V(D)$, we denote by $\lambda_N(v)$ the maximum number of arc-disjoint time-respecting directed paths from r to v in N. We call a partial r-arborescence in N *time-respecting*, if (1) holds for every vertex v of $V(D) \setminus \{r\}$ such that $\varrho_T(v) \neq \emptyset$.

An extension of Theorem 3 in an acyclic and pre-flow network can be described as follows.

Theorem 4. *For each acyclic and pre-flow temporal network $N = (D, \tau)$ with a specified vertex r and each positive integer k such that*

$$k \leq \max\{\lambda_N(v) \mid v \in V(D) \setminus \{r\}\},$$

there exist k arc-disjoint time-respecting partial r-arborescences $T_1, T_2, \ldots, T_k$ such that each vertex v of $V(D)$ is contained in exactly $\min\{k, \lambda_N(v)\}$ arborescences of $T_1, T_2, \ldots, T_k$.

We will give the proof of Theorem 4 in the next subsection. The following lemma plays an important role in the proof of Theorem 4.

Lemma 3. *For each acyclic and pre-flow temporal network $N = (D, \tau)$ with a specified vertex r and each vertex v of $V(D)$, we have $|\varrho_D(v)| = \lambda_N(v)$.*

Proof. Let us fix a vertex v of $V(D)$. Define $d := \lambda_N(v)$. From the definition of $\lambda_N(v)$, we can see that $|\varrho_D(v)| \geq d$. Assuming that $|\varrho_D(v)| > d$, we prove this lemma by contradiction. For each arc a of $A(D) \setminus \delta_D(r)$, define $\mu(a)$ in the same way as in Section 4.1. Since D is acyclic, for each arc a of $\varrho_D(v)$ we have a directed path P_a from r to v that passes arcs

$$a, \mu(a), \mu(\mu(a)), \mu(\mu(\mu(a))), \ldots,$$

in the reverse order. For the definition of $\mu(\cdot)$, the directed paths P_a and P_b are clearly arc-disjoint for every distinct arcs a, b of $\varrho_D(v)$. This implies that there exist more than d arc-disjoint time-respecting directed paths from r to v in N, which contradicts $\lambda_N(v) = d$. □

6.1 Proof

Here we prove Theorem 4. Assume that we are given an acyclic and pre-flow temporal network $N = (D, \tau)$ with a specified vertex r and a positive integer k such that

$$k \leq \max\{\lambda_N(v) \mid v \in V(D) \setminus \{r\}\}.$$

Assume that $|V(D)| = n$, and define a function π in the same way as in Section 4. We propose an algorithm arc-disjoint time-respecting partial r-arborescences by modifying Algorithm 1. Notice that the difference between Algorithm 1 and Algorithm 2 is only Step (3-a).

Algorithm 2

Step 1: For each $i = 1, 2, \ldots, k$, set $A_i^0 := \emptyset$. Furthermore, set $t := 1$.
Step 2: If $t = n$, then halt and output $A_1^{n-1}, A_2^{n-1}, \ldots, A_k^{n-1}$.

Step 3: Set v be the vertex of $V(D)$ such that $\pi(v) = t$, and do the following.

(3-a) Define

$$I^+ := \{i = 1, 2, \ldots, k \mid \delta_D(v) \cap A_i^{t-1} \neq \emptyset\},$$

and let I^- be an arbitrary subset of positive integer of $\{1, 2, \ldots, k\} \setminus I^+$ such that

$$|I^-| = \min\{k, \lambda_N(v)\} - |I^+|.$$

(3-b) For each positive integer i of I^+, find an arc a^* of $\delta_D(v) \cap A_i^{t-1}$ such that

$$\tau(a^*) = \min\{\tau(a) \mid a \in \delta_D(v) \cap A_i^{t-1}\},$$

and then set $a_i^t := \mu(a^*)$.

(3-c) For each positive integer i of I^-, choose an arbitrary arc a_i^t of $\varrho_D(v)$ so that

$$\forall i, j \in I^- \text{ s.t. } i \neq j\colon\ a_i^t \neq a_j^t, \qquad \forall i \in I^-, \forall j \in I^+\colon\ a_i^t \neq a_j^t.$$

(3-d) For each $i = 1, 2, \ldots, k$, set $A_i^t := A_i^{t-1} \cup \{a_i^t\}$.

(3-e) Update $t := t + 1$, and then go to Step 2.

End of Algorithm 2

We first prove that Algorithm 2 is well-defined. For this, it suffices to prove that $|I^+| \leq \lambda_N(v)$ in Step (3-a). Since $A_1^{t-1}, A_2^{t-1}, \ldots, A_k^{t-1}$ are arc-disjoint, it follows from (2) and Lemma 3 that

$$|I^+| \leq |\delta_D(v)| \leq |\varrho_D(v)| = \lambda_N(v).$$

Assume that Algorithm 2 outputs subsets $A_1^{n-1}, A_2^{n-1}, \ldots, A_k^{n-1}$ of A. For each $i = 1, 2, \ldots, k$, let T_i be a subgraph of D satisfying

$$\begin{aligned} V(T_i) &:= \{r\} \cup \{v \in V(D) \mid \varrho_D(v) \cap A_i^{n-1} \neq \emptyset\} \\ A(T_i) &:= A_i^{n-1}. \end{aligned}$$

Notice that we can prove that for each $i = 1, 2, \ldots, k$, every end-vertices of an arc of $A(T_i)$ is contained in $V(T_i)$ as follows. From the definition of Step (3-a), we can see that for every vertex v of $V(D) \setminus \{r\}$, if $\delta_{T_i}(v) \neq \emptyset$, then $\varrho_{T_i}(v) \neq \emptyset$. This implies that the head of the unique arc of $\varrho_{T_i}(v)$ is contained in $V(T_i)$ for each vertex v of $V(D)$ such that $\varrho_{T_i}(v) \neq \emptyset$. Moreover, from the definition of I^+ and I^-, we can see that each vertex v of $V(D)$ is contained in exactly $\min\{k, \lambda_N(v)\}$ subgraphs of $T_1, T_2, \ldots, T_k$. So, Theorem 4 immediately follows from the following lemma.

Lemma 4. *For $i = 1, 2, \ldots, k$, T_i is a time-respecting partial r-arborescence.*

Proof. Let us fix a positive integer i such that $i \leq k$. Since $\varrho_{T_i}(v) \neq \emptyset$ for every vertex v of $V(D) \setminus \{r\}$ such that $\delta_{T_i}(v) \neq \emptyset$, there exists a directed path from r to every vertex of v of $V(D)$ such that $\varrho_{T_i}(v) \neq \emptyset$ in T_i. This implies that T_i is a partial r-arborescence. Furthermore, we can prove that T_i is time-respecting in the same way as in the proof of Lemma 2. □

7 Conclusion

In this paper, we proved that Edmonds' arc-disjoint arborescences theorem can be naturally extended in an acyclic and pre-flow temporal network. Furthermore, we generalized our main theorem to the packing problem of partial arborescences. An apparent next step is to reveal whether Edmonds' arc-disjoint arborescences theorem can be extended in a general pre-flow temporal network.

References

1. Kempe, D., Kleinberg, J., Kumar, A.: Connectivity and inference problems for temporal networks. Journal of Computer and System Sciences **64**(4) (2002) 820–842
2. Li, Y., Thai, M.T., Wang, F., Du, D.Z.: On the construction of a strongly connected broadcast arborescence with bounded transmission delay. IEEE Transactions on mobile computing **5**(10) (2006) 1460–1470
3. Edmonds, J.: Edge-disjoint branchings. In Rustin, R., ed.: Combinatorial Algorithms. Academic Press (1973) 91–96
4. Schrijver, A.: Combinatorial Optimization - Polyhedra and Efficiency. Springer (2003)
5. Huck, A.: Independent branchings in acyclic digraphs. Discrete Mathematics **199** (1999) 245–249
6. Kamiyama, N., Katoh, N., Takizawa, A.: Arc-disjoint in-trees in directed graphs. Combinatorica **29**(2) (2009) 197–214
7. Fujishige, S.: A note on disjoint arborescences. Combinatorica **30**(2) (2010) 247–252
8. Bang-Jensen, J., Frank, A., Jackson, B.: Preserving and increasing local edge-connectivity in mixed graphs. SIAM Journal on Discrete Mathematics **8**(2) (1995) 155–178
9. Hall, P.: On representatives of subsets. The Journal of the London Mathematical Society **10** (1935) 26–30
10. Hopcroft, J.E., Karp, R.M.: An $n^{5/2}$ algorithm for maximum matchings in bipartite graphs. SIAM Journal on Computing **2**(4) (1973) 225–231

List of MI Preprint Series, Kyushu University

The Global COE Program
Math-for-Industry Education & Research Hub

MI

MI2008-1 Takahiro ITO, Shuichi INOKUCHI & Yoshihiro MIZOGUCHI
Abstract collision systems simulated by cellular automata

MI2008-2 Eiji ONODERA
The intial value problem for a third-order dispersive flow into compact almost Hermitian manifolds

MI2008-3 Hiroaki KIDO
On isosceles sets in the 4-dimensional Euclidean space

MI2008-4 Hirofumi NOTSU
Numerical computations of cavity flow problems by a pressure stabilized characteristic-curve finite element scheme

MI2008-5 Yoshiyasu OZEKI
Torsion points of abelian varieties with values in nfinite extensions over a p-adic field

MI2008-6 Yoshiyuki TOMIYAMA
Lifting Galois representations over arbitrary number fields

MI2008-7 Takehiro HIROTSU & Setsuo TANIGUCHI
The random walk model revisited

MI2008-8 Silvia GANDY, Masaaki KANNO, Hirokazu ANAI & Kazuhiro YOKOYAMA
Optimizing a particular real root of a polynomial by a special cylindrical algebraic decomposition

MI2008-9 Kazufumi KIMOTO, Sho MATSUMOTO & Masato WAKAYAMA
Alpha-determinant cyclic modules and Jacobi polynomials

MI2008-10 Sangyeol LEE & Hiroki MASUDA
Jarque-Bera Normality Test for the Driving Lévy Process of a Discretely Observed Univariate SDE

MI2008-11 Hiroyuki CHIHARA & Eiji ONODERA
A third order dispersive flow for closed curves into almost Hermitian manifolds

MI2008-12 Takehiko KINOSHITA, Kouji HASHIMOTO and Mitsuhiro T. NAKAO
On the L^2 a priori error estimates to the finite element solution of elliptic problems with singular adjoint operator

MI2008-13 Jacques FARAUT and Masato WAKAYAMA
Hermitian symmetric spaces of tube type and multivariate Meixner-Pollaczek polynomials

MI2008-14 Takashi NAKAMURA
Riemann zeta-values, Euler polynomials and the best constant of Sobolev inequality

MI2008-15 Takashi NAKAMURA
Some topics related to Hurwitz-Lerch zeta functions

MI2009-1 Yasuhide FUKUMOTO
Global time evolution of viscous vortex rings

MI2009-2 Hidetoshi MATSUI & Sadanori KONISHI
Regularized functional regression modeling for functional response and predictors

MI2009-3 Hidetoshi MATSUI & Sadanori KONISHI
Variable selection for functional regression model via the L_1 regularization

MI2009-4 Shuichi KAWANO & Sadanori KONISHI
Nonlinear logistic discrimination via regularized Gaussian basis expansions

MI2009-5 Toshiro HIRANOUCHI & Yuichiro TAGUCHII
Flat modules and Groebner bases over truncated discrete valuation rings

MI2009-6 Kenji KAJIWARA & Yasuhiro OHTA
Bilinearization and Casorati determinant solutions to non-autonomous 1+1 dimensional discrete soliton equations

MI2009-7 Yoshiyuki KAGEI
Asymptotic behavior of solutions of the compressible Navier-Stokes equation around the plane Couette flow

MI2009-8 Shohei TATEISHI, Hidetoshi MATSUI & Sadanori KONISHI
Nonlinear regression modeling via the lasso-type regularization

MI2009-9 Takeshi TAKAISHI & Masato KIMURA
Phase field model for mode III crack growth in two dimensional elasticity

MI2009-10 Shingo SAITO
Generalisation of Mack's formula for claims reserving with arbitrary exponents for the variance assumption

MI2009-11 Kenji KAJIWARA, Masanobu KANEKO, Atsushi NOBE & Teruhisa TSUDA
Ultradiscretization of a solvable two-dimensional chaotic map associated with the Hesse cubic curve

MI2009-12 Tetsu MASUDA
Hypergeometric τ-functions of the q-Painlevé system of type $E_8^{(1)}$

MI2009-13 Hidenao IWANE, Hitoshi YANAMI, Hirokazu ANAI & Kazuhiro YOKOYAMA
A Practical Implementation of a Symbolic-Numeric Cylindrical Algebraic Decomposition for Quantifier Elimination

MI2009-14 Yasunori MAEKAWA
On Gaussian decay estimates of solutions to some linear elliptic equations and its applications

MI2009-15 Yuya ISHIHARA & Yoshiyuki KAGEI
Large time behavior of the semigroup on L^p spaces associated with the linearized compressible Navier-Stokes equation in a cylindrical domain

MI2009-16 Chikashi ARITA, Atsuo KUNIBA, Kazumitsu SAKAI & Tsuyoshi SAWABE
Spectrum in multi-species asymmetric simple exclusion process on a ring

MI2009-17 Masato WAKAYAMA & Keitaro YAMAMOTO
Non-linear algebraic differential equations satisfied by certain family of elliptic functions

MI2009-18 Me Me NAING & Yasuhide FUKUMOTO
Local Instability of an Elliptical Flow Subjected to a Coriolis Force

MI2009-19 Mitsunori KAYANO & Sadanori KONISHI
Sparse functional principal component analysis via regularized basis expansions and its application

MI2009-20 Shuichi KAWANO & Sadanori KONISHI
Semi-supervised logistic discrimination via regularized Gaussian basis expansions

MI2009-21 Hiroshi YOSHIDA, Yoshihiro MIWA & Masanobu KANEKO
Elliptic curves and Fibonacci numbers arising from Lindenmayer system with symbolic computations

MI2009-22 Eiji ONODERA
A remark on the global existence of a third order dispersive flow into locally Hermitian symmetric spaces

MI2009-23 Stjepan LUGOMER & Yasuhide FUKUMOTO
Generation of ribbons, helicoids and complex scherk surface in laser-matter Interactions

MI2009-24 Yu KAWAKAMI
Recent progress in value distribution of the hyperbolic Gauss map

MI2009-25 Takehiko KINOSHITA & Mitsuhiro T. NAKAO
On very accurate enclosure of the optimal constant in the a priori error estimates for H_0^2-projection

MI2009-26 Manabu YOSHIDA
Ramification of local fields and Fontaine's property (Pm)

MI2009-27 Yu KAWAKAMI
Value distribution of the hyperbolic Gauss maps for flat fronts in hyperbolic three-space

MI2009-28 Masahisa TABATA
Numerical simulation of fluid movement in an hourglass by an energy-stable finite element scheme

MI2009-29 Yoshiyuki KAGEI & Yasunori MAEKAWA
Asymptotic behaviors of solutions to evolution equations in the presence of translation and scaling invariance

MI2009-30 Yoshiyuki KAGEI & Yasunori MAEKAWA
On asymptotic behaviors of solutions to parabolic systems modelling chemotaxis

MI2009-31 Masato WAKAYAMA & Yoshinori YAMASAKI
Hecke's zeros and higher depth determinants

MI2009-32 Olivier PIRONNEAU & Masahisa TABATA
Stability and convergence of a Galerkin-characteristics finite element scheme of lumped mass type

MI2009-33 Chikashi ARITA
Queueing process with excluded-volume effect

MI2009-34 Kenji KAJIWARA, Nobutaka NAKAZONO & Teruhisa TSUDA
Projective reduction of the discrete Painlevé system of type$(A_2 + A_1)^{(1)}$

MI2009-35 Yosuke MIZUYAMA, Takamasa SHINDE, Masahisa TABATA & Daisuke TAGAMI
Finite element computation for scattering problems of micro-hologram using DtN map

MI2009-36 Reiichiro KAWAI & Hiroki MASUDA
Exact simulation of finite variation tempered stable Ornstein-Uhlenbeck processes

MI2009-37 Hiroki MASUDA
On statistical aspects in calibrating a geometric skewed stable asset price model

MI2010-1 Hiroki MASUDA
Approximate self-weighted LAD estimation of discretely observed ergodic Ornstein-Uhlenbeck processes

MI2010-2 Reiichiro KAWAI & Hiroki MASUDA
Infinite variation tempered stable Ornstein-Uhlenbeck processes with discrete observations

MI2010-3 Kei HIROSE, Shuichi KAWANO, Daisuke MIIKE & Sadanori KONISHI
Hyper-parameter selection in Bayesian structural equation models

MI2010-4 Nobuyuki IKEDA & Setsuo TANIGUCHI
The Itô-Nisio theorem, quadratic Wiener functionals, and 1-solitons

MI2010-5 Shohei TATEISHI & Sadanori KONISHI
Nonlinear regression modeling and detecting change point via the relevance vector machine

MI2010-6 Shuichi KAWANO, Toshihiro MISUMI & Sadanori KONISHI
Semi-supervised logistic discrimination via graph-based regularization

MI2010-7 Teruhisa TSUDA
UC hierarchy and monodromy preserving deformation

MI2010-8 Takahiro ITO
Abstract collision systems on groups

MI2010-9 Hiroshi YOSHIDA, Kinji KIMURA, Naoki YOSHIDA, Junko TANAKA & Yoshihiro MIWA
An algebraic approach to underdetermined experiments

MI2010-10 Kei HIROSE & Sadanori KONISHI
Variable selection via the grouped weighted lasso for factor analysis models

MI2010-11 Katsusuke NABESHIMA & Hiroshi YOSHIDA
Derivation of specific conditions with Comprehensive Groebner Systems

MI2010-12 Yoshiyuki KAGEI, Yu NAGAFUCHI & Takeshi SUDOU
Decay estimates on solutions of the linearized compressible Navier-Stokes equation around a Poiseuille type flow

MI2010-13 Reiichiro KAWAI & Hiroki MASUDA
On simulation of tempered stable random variates

MI2010-14 Yoshiyasu OZEKI
Non-existence of certain Galois representations with a uniform tame inertia weight

MI2010-15 Me Me NAING & Yasuhide FUKUMOTO
Local Instability of a Rotating Flow Driven by Precession of Arbitrary Frequency

MI2010-16 Yu KAWAKAMI & Daisuke NAKAJO
The value distribution of the Gauss map of improper affine spheres

MI2010-17 Kazunori YASUTAKE
On the classification of rank 2 almost Fano bundles on projective space

MI2010-18 Toshimitsu TAKAESU
Scaling limits for the system of semi-relativistic particles coupled to a scalar bose field

MI2010-19 Reiichiro KAWAI & Hiroki MASUDA
Local asymptotic normality for normal inverse Gaussian Lévy processes with high-frequency sampling

MI2010-20 Yasuhide FUKUMOTO, Makoto HIROTA & Youichi MIE
Lagrangian approach to weakly nonlinear stability of an elliptical flow

MI2010-21 Hiroki MASUDA
Approximate quadratic estimating function for discretely observed Lévy driven SDEs with application to a noise normality test

MI2010-22 Toshimitsu TAKAESU
A Generalized Scaling Limit and its Application to the Semi-Relativistic Particles System Coupled to a Bose Field with Removing Ultraviolet Cutoffs

MI2010-23 Takahiro ITO, Mitsuhiko FUJIO, Shuichi INOKUCHI & Yoshihiro MIZOGUCHI
Composition, union and division of cellular automata on groups

MI2010-24 Toshimitsu TAKAESU
A Hardy's Uncertainty Principle Lemma in Weak Commutation Relations of Heisenberg-Lie Algebra

MI2010-25 Toshimitsu TAKAESU
On the Essential Self-Adjointness of Anti-Commutative Operators

MI2010-26 Reiichiro KAWAI & Hiroki MASUDA
On the local asymptotic behavior of the likelihood function for Meixner Lévy processes under high-frequency sampling

MI2010-27 Chikashi ARITA & Daichi YANAGISAWA
Exclusive Queueing Process with Discrete Time

MI2010-28 Jun-ichi INOGUCHI, Kenji KAJIWARA, Nozomu MATSUURA & Yasuhiro OHTA
Motion and Bäcklund transformations of discrete plane curves

MI2010-29 Takanori YASUDA, Masaya YASUDA, Takeshi SHIMOYAMA & Jun KOGURE
On the Number of the Pairing-friendly Curves

MI2010-30 Chikashi ARITA & Kohei MOTEGI
Spin-spin correlation functions of the q-VBS state of an integer spin model

MI2010-31 Shohei TATEISHI & Sadanori KONISHI
Nonlinear regression modeling and spike detection via Gaussian basis expansions

MI2010-32 Nobutaka NAKAZONO
Hypergeometric τ functions of the q-Painlevé systems of type $(A_2 + A_1)^{(1)}$

MI2010-33 Yoshiyuki KAGEI
Global existence of solutions to the compressible Navier-Stokes equation around parallel flows

MI2010-34 Nobushige KUROKAWA, Masato WAKAYAMA & Yoshinori YAMASAKI
Milnor-Selberg zeta functions and zeta regularizations

MI2010-35 Kissani PERERA & Yoshihiro MIZOGUCHI
Laplacian energy of directed graphs and minimizing maximum outdegree algorithms

MI2010-36 Takanori YASUDA
CAP representations of inner forms of $Sp(4)$ with respect to Klingen parabolic subgroup

MI2010-37 Chikashi ARITA & Andreas SCHADSCHNEIDER
Dynamical analysis of the exclusive queueing process

MI2011-1 Yasuhide FUKUMOTO& Alexander B. SAMOKHIN
Singular electromagnetic modes in an anisotropic medium

MI2011-2 Hiroki KONDO, Shingo SAITO & Setsuo TANIGUCHI
Asymptotic tail dependence of the normal copula

MI2011-3 Takehiro HIROTSU, Hiroki KONDO, Shingo SAITO, Takuya SATO, Tatsushi TANAKA & Setsuo TANIGUCHI
Anderson-Darling test and the Malliavin calculus

MI2011-4 Hiroshi INOUE, Shohei TATEISHI & Sadanori KONISHI
Nonlinear regression modeling via Compressed Sensing

MI2011-5 Hiroshi INOUE
Implications in Compressed Sensing and the Restricted Isometry Property

MI2011-6 Daeju KIM & Sadanori KONISHI
Predictive information criterion for nonlinear regression model based on basis expansion methods

MI2011-7 Shohei TATEISHI, Chiaki KINJYO & Sadanori KONISHI
Group variable selection via relevance vector machine

MI2011-8 Jan BREZINA & Yoshiyuki KAGEI
Decay properties of solutions to the linearized compressible Navier-Stokes equation around time-periodic parallel flow
Group variable selection via relevance vector machine

MI2011-9 Chikashi ARITA, Arvind AYYER, Kirone MALLICK & Sylvain PROLHAC
Recursive structures in the multispecies TASEP

MI2011-10 Kazunori YASUTAKE
On projective space bundle with nef normalized tautological line bundle

MI2011-11 Hisashi ANDO, Mike HAY, Kenji KAJIWARA & Tetsu MASUDA
An explicit formula for the discrete power function associated with circle patterns of Schramm type

MI2011-12 Yoshiyuki KAGEI
Asymptotic behavior of solutions to the compressible Navier-Stokes equation around a parallel flow

MI2011-13 Vladimír CHALUPECKÝ & Adrian MUNTEAN
Semi-discrete finite difference multiscale scheme for a concrete corrosion model: approximation estimates and convergence

MI2011-14 Jun-ichi INOGUCHI, Kenji KAJIWARA, Nozomu MATSUURA & Yasuhiro OHTA
Explicit solutions to the semi-discrete modified KdV equation and motion of discrete plane curves

MI2011-15 Hiroshi INOUE
A generalization of restricted isometry property and applications to compressed sensing

MI2011-16 Yu KAWAKAMI
A ramification theorem for the ratio of canonical forms of flat surfaces in hyperbolic three-space

MI2011-17 Naoyuki KAMIYAMA
Matroid intersection with priority constraints

MI2012-1 Kazufumi KIMOTO & Masato WAKAYAMA
Spectrum of non-commutative harmonic oscillators and residual modular forms

MI2012-2 Hiroki MASUDA
Mighty convergence of the Gaussian quasi-likelihood random fields for ergodic Levy driven SDE observed at high frequency

MI2012-3 Hiroshi INOUE
A Weak RIP of theory of compressed sensing and LASSO

MI2012-4 Yasuhide FUKUMOTO & Youich MIE
Hamiltonian bifurcation theory for a rotating flow subject to elliptic straining field

MI2012-5 Yu KAWAKAMI
On the maximal number of exceptional values of Gauss maps for various classes of surfaces

MI2012-6 Marcio GAMEIRO, Yasuaki HIRAOKA, Shunsuke IZUMI, Miroslav KRAMAR, Konstantin MISCHAIKOW & Vidit NANDA
Topological Measurement of Protein Compressibility via Persistence Diagrams

MI2012-7 Nobutaka NAKAZONO & Seiji NISHIOKA
Solutions to a q-analog of Painlevé III equation of type $D_7^{(1)}$

MI2012-8 Naoyuki KAMIYAMA
A new approach to the Pareto stable matching problem

MI2012-9 Jan BREZINA & Yoshiyuki KAGEI
Spectral properties of the linearized compressible Navier-Stokes equation around time-periodic parallel flow

MI2012-10 Jan BREZINA
Asymptotic behavior of solutions to the compressible Navier-Stokes equation around a time-periodic parallel flow

MI2012-11 Daeju KIM, Shuichi KAWANO & Yoshiyuki NINOMIYA
Adaptive basis expansion via the extended fused lasso

MI2012-12 Masato WAKAYAMA
On simplicity of the lowest eigenvalue of non-commutative harmonic oscillators

MI2012-13 Masatoshi OKITA
On the convergence rates for the compressible
Navier- Stokes equations with potential force

MI2013-1 Abuduwaili PAERHATI & Yasuhide FUKUMOTO
A Counter-example to Thomson-Tait-Chetayev's Theorem

MI2013-2 Yasuhide FUKUMOTO & Hirofumi SAKUMA
A unified view of topological invariants of barotropic and baroclinic fluids and their application to formal stability analysis of three-dimensional ideal gas flows

MI2013-3 Hiroki MASUDA
Asymptotics for functionals of self-normalized residuals of discretely observed stochastic processes

MI2013-4 Naoyuki KAMIYAMA
On Counting Output Patterns of Logic Circuits

MI2013-5 Hiroshi INOUE
RIPless Theory for Compressed Sensing

MI2013-6 Hiroshi INOUE
Improved bounds on Restricted isometry for compressed sensing

MI2013-7 Hidetoshi MATSUI
Variable and boundary selection for functional data via multiclass logistic regression modeling

MI2013-8 Hidetoshi MATSUI
Variable selection for varying coefficient models with the sparse regularization

MI2013-9 Naoyuki KAMIYAMA
Packing Arborescences in Acyclic Temporal Networks