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An Investigation into Relationships between Exponential Functions and Some Natural Phenomena

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The present study was designed to make an investigation into relationships between exponential functions and some natural phenomena by introducing mathematical interpretations and hypotheses. The results obtained were as follows. The operation of the negative weight problem by introducing the time reversal in basic growth function might look like the operation of the negative energy state by introducing the time reversal in wave function. Relationships between basic growth function and simplified wave function were shown. A relationship between Euler's formula and Bondi K–factor to the power of imaginary unit was shown. Relationships between the massless matter traveling at the speed of ∞ and the property of the wave function were shown in the complex number world. A special relationship between Euler's formula, Bondi K–factor and zeta function was shown through $\pi^2/6$. The present investigation suggested that exponential functions were considered the continuo being played in some natural phenomena.

Key words: Bondi K–factor, Euler's formula, exponential function, wave function, zeta function

INTRODUCTION

Exponential function is a key tool to basic growth analysis of an individual plant (Blackman, 1919) or animal (Brody, 1945), a method that is old–fashioned but still looks attractive due to the easiness to treat. In addition to a modified analysis of growth based on its mechanics (Shimojo *et al.*, 2006, 2009), there is an extended investigation into exponential functions appearing in some mathematical and physical phenomena (Shimojo, 2011a, 2011b, 2011c; Shimojo and Nakano, 2012).

The present study was designed to make an investigation into relationships between exponential functions and some natural phenomena by introducing mathematical interpretations and hypotheses.

EXPONENTIAL FUNCTIONS AND SOME NATURAL PHENOMENA

The second–order differential equation for basic growth function

Basic growth function (1) is normally derived from the first–order differential equation (2),

$$W = W_0 \exp(rt), \quad (1) \qquad \frac{dW}{dt} = rW, \quad (2)$$

where W = weight, t = time, r = relative growth rate, W_0 = weight at t = 0.

Shimojo *et al.* (2006) proposed its second–order differential equation (3) to investigate basic growth mechanics,

$$\left(\frac{dW}{dt}\right)^2 = W\left(\frac{d^2W}{dt^2}\right). \quad (3)$$

Equation (3) is characterized by the terms squared,

$$\left(\frac{dW}{dt}\right)^2 = (W_0)^2 r^2 (\exp(rt))^2 . \quad (4)$$

The operation of the negative weight problem based on an analogy to the case of the negative energy state

Solving equation (4) gives the following functions,

$$\pm W = (\pm W_0)\exp((\pm r)(\pm t)). \quad (5)$$

Since the negative weight does not exist, $-W$ in (5) is replaced by W by introducing the time reversal and sign reversal of r,

$$-W = (-W_0)\exp(rt) \;\rightarrow\; W = W_0\exp((-r)(-t)). \quad (6)$$

If W is the weight of an individual animal, then $-W$ corresponds to the feed that is eaten and converted into the animal body under the conservation of energy. The interpretation of operation (6) is that the existence of the feed precedes the animal growth (Shimojo *et al.*, 2009).

The time reversal introduced into the operation of the negative weight problem in basic growth function (6), boldly writing at the risk of ignoring the conceptual difference, might look like the case of the negative energy state in wave function (7),

$$\psi = \exp\left(\frac{\boldsymbol{i}}{\hbar}(E)(t)\right) \;\rightarrow\; \psi = \exp\left(\frac{\boldsymbol{i}}{\hbar}(-E)(-t)\right), \quad (7)$$

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where ψ = wave function, $\boldsymbol{i}$ = imaginary unit, $\hbar = h/2\pi$ (h: Planck's constant, π: circular constant), E = energy, t = time.
Functions in (7) are also solutions to the second–order differential equation (3) when ψ replaces W.

Relationships between basic growth function and simplified wave function

Both basic growth function (1) and simplified wave function (8) are solutions to the second–order differential equation (3),

$$W = W_0 \exp(rt), \ (1) \qquad \psi = A \exp(\boldsymbol{i}(F)t), \ (8)$$

where A = amplitude, F = frequency, $\boldsymbol{i}$ = imaginary unit.
Shimojo (2011b) suggested that the amplitude (A) might look like the weight (W_0) from the viewpoint of the existence of a unit matter and its accumulation, and the frequency (F) might look like the relative growth rate (r) from the viewpoint of the energy that a unit matter has.

What is directly observed is the value of W in basic growth function (1). When $A = 1$ in simplified wave function (8), even a unit matter might show a wave phenomenon. In addition, when the value of A is directly observed, there might occur the collapse of simplified wave function to result in the existence as matter associated with the absolute value of ψ.

A relationship between Euler's formula and Bondi K–factor to the power of imaginary unit

Exponential functions are related to the factors of Lorentz transformation,

$$\frac{\exp(\theta)+\exp(-\theta)}{2} = \frac{1}{\sqrt{1-(v/c)^2}}, \qquad (9)$$

$$\frac{\exp(\theta)-\exp(-\theta)}{2} = \frac{v/c}{\sqrt{1-(v/c)^2}}, \qquad (10)$$

where v = velocity of a matter, c = speed of light.

Solving simultaneous equations (9) and (10) gives expression (11),

$$\exp(\pm\theta) = \sqrt{\frac{1\pm v/c}{1\mp v/c}} = \frac{1}{\sqrt{1-(v/c)^2}}(1\pm v/c), \qquad (11)$$

where double–sign corresponds.
Thus, exponential functions are related to Bondi K–factor including Lorentz factor (Bondi, 1964; Shimojo, 2011a, 2011c; Shimojo and Nakano, 2012).

Since expression (12) is derived from expression (11),

$$\pm\theta = \ln\sqrt{\frac{1\pm v/c}{1\mp v/c}}, \qquad (12)$$

Euler's formula is Bondi K–factor to the power of imaginary unit as shown in expression (13),

$$\exp(\pm\boldsymbol{i}\theta) = \cos\theta \pm \boldsymbol{i}\sin\theta = \left(\sqrt{\frac{1\pm v/c}{1\mp v/c}}\right)^{\boldsymbol{i}}, \qquad (13)$$

where double–sign corresponds.

Relationships between the massless matter traveling at the speed of ∞ and the property of the wave function

The mathematical application of $v \to \infty$ to Bondi K–factor (Shimojo, 2011a, 2011c) gives,

$$\lim_{v\to\infty}\sqrt{(1\pm v/c)/(1\mp v/c)} = \lim_{v\to\infty}\sqrt{(c/v\pm1)/(c/v\mp1)} = \sqrt{-1} = \boldsymbol{i} = \exp\left(\boldsymbol{i}\frac{\pi}{2}\right), \qquad (14)$$

where double–sign corresponds.
The mathematical application of $v \to \infty$ to Lorentz transformation (Shimojo, 2011c) gives,

$$x' = \frac{x-vt}{\sqrt{1-(v/c)^2}} = \frac{cx/v-ct}{\sqrt{(c/v)^2-1}} \to \frac{-ct}{\sqrt{-1}} = \boldsymbol{i}ct,$$

$$t' = \frac{t-vx/c^2}{\sqrt{1-(v/c)^2}} = \frac{ct/v-x/c}{\sqrt{(c/v)^2-1}} \to \frac{-x/c}{\sqrt{-1}} = \frac{\boldsymbol{i}x}{c},$$

$$m' = \frac{m}{\sqrt{1-(v/c)^2}} = \frac{mc/v}{\sqrt{(c/v)^2-1}} \to \frac{0}{\sqrt{-1}} = 0.$$

Thus,

$$x' \to \boldsymbol{i}ct, \quad ct' \to \boldsymbol{i}x, \quad m' \to 0. \qquad (15)$$

Mathematical phenomena (14) and (15) suggest the following hypotheses. (i) The massless matter traveling at the speed of ∞, its existence is prohibited because of breaking Lorentz transformation, might be associated with the wave function in the complex number world. (ii) The speed is defined by the distance traveled over the time elapsed. Besides the ∞ distance traveled in a certain amount of time, the speed of ∞ might be reinterpreted as the simultaneous existence of different states, as the nonlocal correlation between states that exist apart, or as the phenomenon observed in the delayed choice experiment. (iii) The breakdown of Lorentz transformation transforms the real space and time into the imaginary time and space, respectively. This leads to the phenomenon that the momentum and energy in the real number world might look like the energy and momentum in the complex number world, respectively.

Hypotheses (i) ~ (iii) might be associated with the matter–wave duality and the difficulty in observing the wave function itself. This is caused by a mystery of the complex number that is a combination of the two mutually exclusive mathematical rules, $a^2 > 0$ and $(b\boldsymbol{i})^2 < 0$ in $a+b\boldsymbol{i}$. There might still be problems at the back of Lorentz transformation. Anyway, the above hypotheses must be criticized severely.

A special relationship between Euler's formula, Bondi K–factor and zeta function

Euler's formula, Bondi K–factor and zeta function are tools of importance to the investigation into natural phenomena. When related to π, they are described as follows, respectively.

$$\exp(\boldsymbol{i}\pi) = -1. \quad (16)$$

$$\exp(\pi) = \sqrt{\frac{1+v_\pi/c}{1-v_\pi/c}}\ , \quad (17)$$

$$\text{wher } v_\pi = \frac{c(\exp(2\pi) - 1)}{\exp(2\pi) + 1}\ .$$

$$\frac{\pi^2}{6} = \sum_{k=1}^{\infty} \frac{1}{k^2}\ . \quad (18)$$

Thus, through $\pi^2/6$, there is a special relationship between them,

$$\frac{\pi^2}{6} = \sum_{k=1}^{\infty} \frac{1}{k^2}$$

$$= \frac{1}{6}(\ln(-1)^{-\boldsymbol{i}})^2$$

$$= \frac{1}{6}\left(\ln\sqrt{\frac{1+v_\pi/c}{1-v_\pi/c}}\right)^2. \quad (19)$$

Conclusions

The present investigation suggests that exponential functions are considered the continuo being played in some natural phenomena.

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