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Abstract

This paper discusses new bounds for restricted isometry property in compressed sensing. In the literature, E.J. Candès has proved that $\delta_{2s} < \sqrt{2} - 1$ is a sufficient condition via l_1 optimization having s-sparse vector solution. Later, many researchers have improved the sufficient conditions on δ_{2s} or δ_s. In this paper, we have improved the sufficient condition to $\delta_s < 0.309$ and have given the sufficient condition to δ_k $(s < k)$ using an idea of Q. Mo and S. Li' result. Furthermore, we have improved the sufficient conditions to $\delta_{2s} < 0.593$ and $\delta_s < 0.472$ in special case.

Key Words and Phrases: Compressed sensing, Restricted isometry constants, Restricted isometry property, Sparse approximation, Sparse signal recovery.

1 Introduction

This paper introduces the theory of compressed sensing(CS). For a signal $\boldsymbol{x} \in \boldsymbol{R}^n$, let $\|\boldsymbol{x}\|_1$ be l_1 norm of $\boldsymbol{x}$ and $\|\boldsymbol{x}\|_2$ be l_2 norm of $\boldsymbol{x}$. Let $\boldsymbol{x}$ be a sparse or nearly sparse vector. Compressed sensing aims to recover high-dimensional signal (for example: images signal, voice signal, code signal...etc.) from only a few samples or linear measurements. Formally, one considers the following model:

$$\boldsymbol{y} = A\boldsymbol{x} + \boldsymbol{z}, \tag{1}$$

where A is a $m \times n$ matrix$(m < n)$ and $\boldsymbol{z}$ is a vector of measurement error.

Our goal is to reconstruct an unknown signal $\boldsymbol{x}$ based on A and $\boldsymbol{y}$ are given. Then we consider reconstructing $\boldsymbol{x}$ as the solution $\boldsymbol{x}^\star$ to the optimization problem

$$\min_{\tilde{\boldsymbol{x}}} \|\tilde{\boldsymbol{x}}\|_1, \quad \text{subject to} \quad \|\boldsymbol{y} - A\tilde{\boldsymbol{x}}\|_2 \leq \varepsilon, \tag{2}$$

where ε is an upper bound on the measurement error.

In fact, we can recover the signal $\boldsymbol{x}$ in noiseless case under sufficient conditions. In CS theory, a crucial issue is to research good conditions in order to achieve our goal. One of the most generally known condition for CS theory is the restricted isometry property(RIP) introduced by E.J. Candès and T. Tao [4]. When we discuss our proposed results, it is an important notion. The RIP needs that the subsets of columns of A for all locations in $\{1, 2, \cdots, n\}$ behave nearly orthonormal system. In detail, a matrix A satisfies the RIP of order s if there exists a constant δ with $0 < \delta < 1$ such that

$$(1-\delta)\|\boldsymbol{a}\|_2^2 \leq \|A\boldsymbol{a}\|_2^2 \leq (1+\delta)\|\boldsymbol{a}\|_2^2 \tag{3}$$

for all s-sparse vectors $\boldsymbol{a}$. A vector is said to be s-sparse vector if it has at most s nonzero entries. The minimum δ satisfying the above restrictions is said to be the restricted isometry constant and is denoted by δ_s.

It has been shown that l_1 optimization can recover an unknown signal in noiseless case and noisy case under various sufficient conditions on δ_s or δ_{2s}. For example, E.J. Candès and T. Tao [4] have proved that if $\delta_{2s} < \sqrt{2} - 1$, then an unknown signal can be recovered. Later, S. Foucart and M. Lai [6] have improved the bound to $\delta_{2s} < 0.4531$. Others, $\delta_{2s} < 0.4652$ is used by S. Foucart [5], $\delta_{2s} < 0.4721$ for cases such that s is a multiple of 4 or s is very large by T. Cai. *el.al.* [2], $\delta_{2s} < 0.4734$ for the case such that s is very large by S. Foucart [5] and $\delta_s < 0.307$ by T. Cai *el.al.* [2]. In a resent paper, Q. Mo and S. Li [7] have improved the sufficient condition to $\delta_{2s} < 0.4931$ for general case and $\delta_{2s} < 0.6569$ in some special case.

In this paper, we propose the sufficient condition of δ_s and the sufficient condition to δ_k(each number $k > s$) using the idea of Q. Mo and S. Li. Furthermore, we propose the sufficient condition of δ_s and $\delta_k(k > s)$ in case of $\|A\| \leq 1$, where $\|\cdot\|$ is operator norm. The special case is different from the case($n < 4s$) in Q. Mo and S. Li [7].

There are several benefits for considering the bounds of δ_s and $\delta_k(k > s)$ in case of $\|A\| \leq 1$. First, we suppose that a matrix satisfy the condition to $\|A\| \leq 1$, where $\|\cdot\|$ is operator norm. However, we do not suppose the condition of sparsity. Practically, we can not know the sparsity of $\boldsymbol{x}$ but we can calculate $\|A\|$. Second, it gives better error bounds to recover noisy signal in special case. Thirdly, the assessments of various cases lead to developments for signal analysis or other analysis.

Our analysis is very simple and elementary. We introduce the proposed results using E.J. Candѐs' idea, T. Cai *el.al.*' idea and Q. Mo and S. Li' idea. We regard Theorem 3.1, 3.2, 3.3 and 3.4 as the main results in this paper. Otherwise, in Section 2, we prepare some notions and lemmas to prove main theorems. In Section 3, we introduce new bounds of δ_s and generalizations of δ_{2s} in general cases and new error bounds. In Section 4, we introduce new bounds of δ_s and $\delta_k(k > s)$ in case of $\|A\| \leq 1$ and new error bounds.

2 Preliminaries and Some Lemmas

In this section, we prepare some lemmas needed for the proofs of Theorem 3.1 and Theorem 3.2.

Lemma 2.1. Take any $t \geq 1$ and positive integers s', s'' such tha ts' is an integer. Suppose that A obeys the RIP of order $(ts' + s'')$. Then,

$$| < A\boldsymbol{a}, A\boldsymbol{b} > | \leq \sqrt{t}\delta_{s'+s''}\|\boldsymbol{a}\|_2\|\boldsymbol{b}\|_2 \tag{4}$$

for any vectors $\boldsymbol{a}, \boldsymbol{b} \in \boldsymbol{R}^n$ with disjoint supports and sparsity ts' and s'', respectively. In particular, if $\|A\| \leq 1$, then

$$| < A\boldsymbol{a}, A\boldsymbol{b} > | \leq \frac{\sqrt{t}}{2}\delta_{s'+s''}\|\boldsymbol{a}\|_2|\boldsymbol{b}\|_2. \tag{5}$$

Proof. The proof of this lemma can be obtained based on a minor modification of [3].

Lemma 2.2. For any $\boldsymbol{a} \in \boldsymbol{R}^k$, we have

$$\|\boldsymbol{a}\|_2 \leq \frac{1}{\sqrt{k}}\|\boldsymbol{a}\|_1 + \frac{\sqrt{k}}{4}\left(\max_{1\leq i\leq k}|a_i| - \min_{1\leq i\leq k}|a_i|\right). \tag{6}$$

Proof. The proof of this lemma can be obtained by [2, Proposition 2.1.].

Suppose $\boldsymbol{x}$ is an original signal we need to recover and $\boldsymbol{x}^\star$ is the solution of CS optimization problem (2). Let $\boldsymbol{h} \equiv \boldsymbol{x}^\star - \boldsymbol{x}$ and $\boldsymbol{h} = (h_1, \cdots, h_n)$. For simplicity, we assume that the index of $\boldsymbol{h}$ is sorted by $|h_1| \geq |h_2| \geq \cdots \geq |h_n|$. Throughout this paper, let T_0 be an arbitrary location of $\{1, 2, \cdots, n\}$ with $|T_0| = s$ and let $\{T_1, T_2, \cdots, T_l\}$ be a decomposition of $\{1, 2, \cdots, n\}$ with $|T_1| = s$, $|T_k| = s'$ $(2 \le k \le l-1)$ and $1 \le |T_l| \equiv r \le s'$, where $|T|$ is number of elements of T. We consider the decomposition of $\boldsymbol{h}$ as follows:

$$\begin{aligned}
\boldsymbol{h}_{T_1} &= (h_1^{(T_1)}, h_2^{(T_1)}, \cdots, h_s^{(T_1)}, 0, \cdots, 0) \\
\boldsymbol{h}_{T_2} &= (0, \cdots, 0, h_1^{(T_2)}, \cdots, h_{s'}^{(T_2)}, 0, \cdots, 0) \\
&\vdots \\
\boldsymbol{h}_{T_{l-1}} &= (0, \cdots, 0, h_1^{(T_{l-1})}, \cdots, h_{s'}^{(T_{l-1})}, 0, \cdots, 0) \\
\boldsymbol{h}_{T_l} &= (0, \cdots 0, h_1^{(T_l)}, \cdots, h_r^{(T_l)}).
\end{aligned}$$

This is due to the T. Cai *et.al.*' idea [2] in case of $s = s'$. We have the following Lemma 2.3–Lemma 2.9 for the decomposition $(\boldsymbol{h}_{T_1}, \boldsymbol{h}_{T_2}, \cdots, \boldsymbol{h}_{T_l})$ of $\boldsymbol{h}$. By definition of CS optimization (2), we have the following

Lemma 2.3. We have

$$\|\boldsymbol{h}_{T_0^c}\|_1 \le 2\|\boldsymbol{x} - \boldsymbol{x}_{T_0}\|_1 + \|\boldsymbol{h}_{T_0}\|_1. \tag{7}$$

Refer to [3] for the proof of Lemma 2.3. T. Cai *et.al.* [2] have obtained a similar result for the location T_1.

Lemma 2.4. For $|T_0| = |T_1| = s$, we have

$$\|\boldsymbol{h}_{T_1^c}\|_1 \le 2\|\boldsymbol{x} - \boldsymbol{x}_{T_0}\|_1 + \|\boldsymbol{h}_{T_1}\|_1. \tag{8}$$

Proof. Since $|T_0^c \cap T_1| = |T_0 \cap T_1^c|$, we have $\|\boldsymbol{h}_{T_0 \cap T_1^c}\|_1 \le \|\boldsymbol{h}_{T_0^c \cap T_1}\|_1$, which implies by (7) that

$$\begin{aligned}
\|\boldsymbol{h}_{T_1^c}\|_1 &= \|\boldsymbol{h}_{T_0 \cap T_1^c}\|_1 + \|\boldsymbol{h}_{T_0^c}\|_1 - \|\boldsymbol{h}_{T_1 \cap T_0^c}\|_1 \\
&\le 2\|\boldsymbol{x} - \boldsymbol{x}_{T_0}\|_1 + \|\boldsymbol{h}_{T_1}\|_1 + 2\left(\|\boldsymbol{h}_{T_0 \cap T_1^c}\|_1 - \|\boldsymbol{h}_{T_1 \cap T_0^c}\|_1\right) \\
&\le 2\|\boldsymbol{x} - \boldsymbol{x}_{T_0}\|_1 + \|\boldsymbol{h}_{T_1}\|_1.
\end{aligned}$$

Lemma 2.5. We have

$$\sum_{i\geq 2}\|\boldsymbol{h}_{T_i}\|_2 \leq \frac{2}{\sqrt{s'}}\|\boldsymbol{x}-\boldsymbol{x}_{T_0}\|_1 + \left(\frac{\sqrt{s}}{\sqrt{s'}}+\frac{\sqrt{s'}}{4\sqrt{s}}\right)\|\boldsymbol{h}_{T_1}\|_2. \tag{9}$$

Proof. By using Lemma 2.2, we have

$$\|\boldsymbol{h}_{T_i}\|_2 \leq \frac{1}{\sqrt{s'}}\|\boldsymbol{h}_{T_i}\|_1 + \frac{\sqrt{s'}}{4}\left(|h_1^{(T_i)}|-|h_1^{(T_{i+1})}|\right),\ \ 3\leq i\leq l-1,$$

which implies by Lemma 2.3 that

$$\begin{aligned}\sum_{i\geq 2}\|\boldsymbol{h}_{T_i}\|_2 &\leq \frac{1}{\sqrt{s'}}\sum_{i\geq 2}\|\boldsymbol{h}_{T_i}\|_1 + \frac{\sqrt{s'}}{4}|h_1^{(T_2)}| \\ &\leq \frac{2}{\sqrt{s'}}\|\boldsymbol{x}-\boldsymbol{x}_{T_0}\|_1 + \left(\frac{\sqrt{s}}{\sqrt{s'}}+\frac{\sqrt{s'}}{4\sqrt{s}}\right)\|\boldsymbol{h}_{T_1}\|_2.\end{aligned} \tag{10}$$

Similarly we have the following

Lemma 2.6. Let $T_1=\{T_1',T_1''\}$ be a decomposition of T_1 with $|T_1'|=s'$ and $|T_1''|=s''$. Then, $s'=ts,\ s''=(1-t)s$ for some $t\in(0,1)$ and

$$\sum_{i\geq 2}\|\boldsymbol{h}_{T_i}\|_2 \leq \frac{2}{\sqrt{s(1-t)}}\|\boldsymbol{x}-\boldsymbol{x}_{T_0}\|_1 + \left(\frac{1}{\sqrt{1-t}}+\frac{\sqrt{1-t}}{4}\right)\|\boldsymbol{h}_{T_1}\|_2. \tag{11}$$

We put $\|\boldsymbol{h}_{T_2}\|_1 \equiv p\sum_{i\geq 2}\|\boldsymbol{h}_{T_i}\|_1 = p\|\boldsymbol{h}_{T_1^c}\|_1$. Then $0\leq p\leq 1$ and $\sum_{i\geq 3}\|\boldsymbol{h}_{T_i}\|_1 = (1-p)\|\boldsymbol{h}_{T_1^c}\|_1$. Then the following Lemma 2.7 is easily shown and Lemma 2.8 is also easily shown by using the inequality (10).

Lemma 2.7. We have

$$\sum_{i\geq 3}\|\boldsymbol{h}_{T_i}\|_2^2 < \frac{p(1-p)}{s'}\|\boldsymbol{h}_{T_1^c}\|_1^2. \tag{12}$$

Lemma 2.8. We have

$$\sum_{i\geq 3}\|\boldsymbol{h}_i\|_2 < \frac{1-3p/4}{\sqrt{s'}}\|\boldsymbol{h}_{T_1^c}\|_1. \tag{13}$$

Lemma 2.9. We have

$$\|\sum_{i\geq 3} A\boldsymbol{h}_{T_i}\|_2^2 \leq \frac{1}{s'}\left(p(1-p)+\delta_{2s'}\left(1-\frac{3}{4}p\right)^2\right)\|\boldsymbol{h}_{T_1^c}\|_1^2. \tag{14}$$

Proof. By the definition of RIP, Lemma 2.1, Lemma 2.6 and Lemma 2.7, we have

$$\begin{aligned}
\|\sum_{i\geq 3} A\boldsymbol{h}_{T_i}\|_2^2 &= \sum_{i\geq 3} < A\boldsymbol{h}_{T_i}, A\boldsymbol{h}_{T_i} > +2\sum_{3\leq i<j\leq l} < A\boldsymbol{h}_{T_i}, A\boldsymbol{h}_{T_j} > \\
&\leq \sum_{i\geq 3}(1+\delta_{s'})\|\boldsymbol{h}_{T_i}\|_2^2 + 2\sum_{3\leq i<j\leq l}\delta_{2s'}\|\boldsymbol{h}_{T_i}\|_2\|\boldsymbol{h}_{T_j}\|_2 \\
&\leq \sum_{i\geq 3}\|\boldsymbol{h}_{T_i}\|_2^2 + \delta_{2s'}\left(\sum_{i\geq 3}\|\boldsymbol{h}_{T_i}\|_2\right)^2 \\
&\leq \frac{p(1-p)}{s'}\|\boldsymbol{h}_{T_1^c}\|_1^2 + \frac{\delta_{2s'}}{s'}\left(1-\frac{3}{4}p\right)^2\|\boldsymbol{h}_{T_1^c}\|_1^2 \\
&= \frac{1}{s'}\left(p(1-p)+\delta_{2s'}\left(1-\frac{3}{4}p\right)^2\right)\|\boldsymbol{h}_{T_1^c}\|_1^2.
\end{aligned}$$

Lemma 2.7, 2.8 and 2.9 are due to the Q. Mo and S. Li [7] in case of $s' = s$.

3 Main results

In this section, we introduce the main results of the sufficient condition of δ_s and generalization of sufficient condition of Q. Mo and S. Li' result in general case.

3.1 New Bound for δ_s

We have established the sufficient condition $\delta_s < 0.309$ for CS optimization problem in general case.

Theorem 3.1. Assume that A obeys the RIP of order s and $\delta_s < \frac{1}{1+\sqrt{5}} \approx 0.309$. Then, the solution $\boldsymbol{x}^\star$ to (2) obeys

$$\|\boldsymbol{x}-\boldsymbol{x}^\star\|_2 \leq C_0\|\boldsymbol{x}-\boldsymbol{x}_{T_0}\|_1 + C_1\varepsilon, \tag{15}$$

where

$$C_0 = \frac{3\left(5+\left(3\sqrt{5}-5\right)\delta_s\right)}{5\left(1-\left(\sqrt{5}+1\right)\delta_s\right)\sqrt{s}}, \quad C_1 = \frac{16\sqrt{1+\delta_s}}{3\left(1-\left(\sqrt{5}+1\right)\delta_s\right)}.$$

Proof. Let $T_1 = \{T_1', T_1''\}$ be a decomposition of T_1 with $|T_1'| = s'$ and $|T_1''| = s''$. Then, $s' = ts,\ s'' = (1-t)s$ for some $t \in (0,1)$. By Lemma 2.1, we have

$$\begin{aligned}(1-\delta_s)\|\boldsymbol{h}_{T_1}\|_2^2 &\leq\ < A\boldsymbol{h}_{T_1}, A\boldsymbol{h} - \sum_{j\geq 2} A\boldsymbol{h}_{T_j} > \\ &\leq\ 2\sqrt{1+\delta_s}\varepsilon\|\boldsymbol{h}_{T_1}\|_2 + \frac{1}{\sqrt{t}}\delta_s\|\boldsymbol{h}_{T_1}\|_2\left(\sum_{j\geq 2}\|\boldsymbol{h}_{T_j}\|_2\right).\end{aligned}$$

Thus, by Lemma 2.5 and the above inequality, we have

$$\begin{aligned}(1-\delta_s)\|\boldsymbol{h}_{T_1}\|_2 &\leq\ 2\sqrt{1+\delta_s}\varepsilon + \frac{2\delta_s}{\sqrt{(1-t)ts}}\|\boldsymbol{x}-\boldsymbol{x}_{T_0}\|_1 \\ &\quad + \frac{1}{\sqrt{t}}\left(\frac{1}{\sqrt{1-t}} + \frac{\sqrt{1-t}}{4}\right)\delta_s\|\boldsymbol{h}_{T_1}\|_2. \qquad (16)\end{aligned}$$

Here, put $f(t) = \frac{1}{\sqrt{t}}\left(\frac{1}{\sqrt{1-t}} + \frac{\sqrt{1-t}}{4}\right)$. Then, f is increasing when $\frac{5}{9} < t < 1$ and decreasing when $0 < t < \frac{5}{9}$. Thus, when $t = \frac{5}{9}$, we have

$$(1-\delta_s)\|\boldsymbol{h}_{T_1}\|_2 \leq 2\sqrt{1+\delta_s}\varepsilon + \frac{9}{\sqrt{5s}}\delta_s\|\boldsymbol{x}-\boldsymbol{x}_{T_0}\|_1 + \sqrt{5}\delta_s\|\boldsymbol{h}_{T_1}\|_2, \qquad (17)$$

so that by assumption $\delta_s < \frac{1}{1+\sqrt{5}} \approx 0.309$,

$$\|\boldsymbol{h}_{T_1}\|_2 \leq \frac{1}{1-(\sqrt{5}+1)\delta_s}\left(2\sqrt{1+\delta_s}\varepsilon + \frac{9}{\sqrt{5s}}\delta_s\|\boldsymbol{x}-\boldsymbol{x}_{T_0}\|_1\right). \qquad (18)$$

Furthermore, it follows from Lemma 2.5 that

$$\|\boldsymbol{h}_{T_1^c}\|_2 \leq \frac{3}{\sqrt{s}}\|\boldsymbol{x}-\boldsymbol{x}_{T_0}\|_1 + \frac{5}{3}\|\boldsymbol{h}_{T_1}\|_2, \qquad (19)$$

which implies by (18) that

$$\|\boldsymbol{x}-\boldsymbol{x}^\star\|_2 \leq \|\boldsymbol{h}_{T_1}\|_2 + \|\boldsymbol{h}_{T_1^c}\|_2 \leq C_0\|\boldsymbol{x}-\boldsymbol{x}_{T_0}\|_1 + C_1\varepsilon.$$

This completes the proof.

3.2 Generalization of Q. Mo and S. Li' result

Using the E.J. Candès decomposition $\{T_1, T_2, \cdots, T_q\}$ of T_0^c with $|T_k| = s$ $(k = 1, \cdots, q)$ and $|h_1^{(T_1)}| \geq |h_2^{(T_1)}| \geq \cdots \geq |h_s^{(T_1)}| \geq |h_1^{(T_2)}| \geq |h_2^{(T_2)}| \geq \cdots$, Q. Mo and S. Li [7] have obtained a new bound of the isometry constant δ_{2s}. In this section, using the decomposition of $\{T_1, T_2, \cdots, T_l\}$ stated in Section 2 and taking an arbitrary natural number s', we have obtained a new bound of the isometry constant δ_k $(s < k)$.

Theorem 3.2. (1) Let $\frac{s}{8} < s' \leq s$. We assume A obeys the RIP of order $(s+s')$ and $\frac{\sqrt{s}}{\sqrt{s'}}\theta_{s,s'} < 1$, equivalently

$$\delta_{s+s'} < \frac{20s + 57s' - \sqrt{656s^2 + 632ss' + 49s'^2}}{2(16s + 25s')}.$$

Then,

$$\|\boldsymbol{x} - \boldsymbol{x}^\star\|_2 \leq D_0 \|\boldsymbol{x} - \boldsymbol{x}_{T_0}\|_1 + D_1 \varepsilon, \tag{20}$$

where

$$\begin{aligned}
\theta_{s,s'} &= \sqrt{\frac{4\left(1 + 5\delta_{s+s'} - 4\delta_{s+s'}^2\right)}{\left(1 - \delta_{s+s'}\right)\left(32 - 25\delta_{s+s'}\right)}}, \\
D_0 &= \frac{4\sqrt{2}}{\sqrt{s'} - \sqrt{s}\theta_{s,s'}} \sqrt{\frac{2 - \delta_{s+s'}}{(1 - \delta_{s+s'})(32 - 25\delta_{s+s'})}}, \\
D_1 &= \frac{2}{\sqrt{1 - \delta_{s+s'}}} \left(1 + \frac{4\sqrt{s}}{\sqrt{s'} - \sqrt{s}\theta_{s,s'}} \sqrt{\frac{2 - \delta_{s+s'}}{(1 - \delta_{s+s'})(32 - 25\delta_{s+s'})}}\right).
\end{aligned}$$

(2) Let $s' \geq s$. We assume that A obeys the RIP of order $2s'$ and $\frac{\sqrt{s}}{\sqrt{s'}}\theta_{s'} < 1$, equivalently

$$\delta_{2s'} < \frac{20s + 57s' - \sqrt{656s^2 + 632ss' + 49s'^2}}{2(16s + 25s')}.$$

Then,

$$\|\boldsymbol{x} - \boldsymbol{x}^\star\|_2 \leq D_0' \|\boldsymbol{x} - \boldsymbol{x}_{T_0}\|_1 + D_1' \varepsilon, \tag{21}$$

where

$$\begin{aligned}
\theta_{s'} &= \sqrt{\frac{4\left(1 + 5\delta_{2s'} - 4\delta_{2s'}^2\right)}{\left(1 - \delta_{2s'}\right)\left(32 - 25\delta_{2s'}\right)}}, \\
D_0' &= \frac{4\sqrt{2}}{\sqrt{s'} - \sqrt{s}\theta_{s'}} \sqrt{\frac{2 - \delta_{2s'}}{(1 - \delta_{2s'})(32 - 25\delta_{2s'})}}, \\
D_1' &= \frac{2}{\sqrt{1 - \delta_{2s'}}} \left(1 + \frac{4\sqrt{s}}{\sqrt{s'} - \sqrt{s}\theta_{s'}} \sqrt{\frac{2 - \delta_{2s'}}{(1 - \delta_{2s'})(32 - 25\delta_{2s'})}}\right).
\end{aligned}$$

Proof. Let $\frac{s}{8} < s' \le s$. By the definition of RIP and Lemma 2.7, we have

$$\begin{aligned}
(1-\delta_{s+s'})\,\|\boldsymbol{h}_{T_1}\|_2^2 &= (1-\delta_{s+s'})\|\boldsymbol{h}_{T_1\cup T_2}\|_2^2 - (1-\delta_{s+s'})\|\boldsymbol{h}_{T_2}\|_2^2 \\
&\le \|A\boldsymbol{h} - \sum_{j\ge 3} A\boldsymbol{h}_{T_j}\|_2^2 - \frac{(1-\delta_{s+s'})}{s'} p^2 \|\boldsymbol{h}_{T_1^c}\|_1^2 \\
&\le \left(2\varepsilon + \|\sum_{j\ge 3} A\boldsymbol{h}_{T_j}\|_2\right)^2 - \frac{(1-\delta_{s+s'})}{s'} p^2 \|\boldsymbol{h}_{T_1^c}\|_1^2 \\
&\le 4\varepsilon^2 + 4\varepsilon \frac{1}{\sqrt{s'}} \sqrt{p(1-p) + \delta_{s+s'}\left(1-\frac{3}{4}p\right)^2}\,\|\boldsymbol{h}_{T_1^c}\|_1 \\
&\quad + \frac{1}{s'}\left(p(1-p) + \delta_{s+s'}\left(1-\frac{3}{4}p\right)^2 - (1-\delta_{s+s'})p^2\right)\|\boldsymbol{h}_{T_1^c}\|_1.
\end{aligned}$$

Since

$$\sqrt{p(1-p) + \delta_{s+s'}\left(1-\frac{3}{4}p\right)^2} \le \sqrt{\frac{4(1+\delta_{s+s'})}{16-9\delta_{s+s'}}},$$

$$p(1-p) + \delta_{s+s'}\left(1-\frac{3}{4}p\right)^2 - (1-\delta_{s+s'})p^2 \le \frac{4(1+5\delta_{s+s'} - 4\delta_{s+s'}^2)}{32-25\delta_{s+s'}}$$

and

$$\frac{4(1+\delta_{s+s'})}{16-9\delta_{s+s'}} \le 2\frac{4(1+5\delta_{s+s'} - 4\delta_{s+s'}^2)}{32-25\delta_{s+s'}},$$

we have

$$(1-\delta_{s+s'})\|\boldsymbol{h}_{T_1}\|_2^2 \le \left(2\sqrt{2}\varepsilon + \sqrt{\frac{1}{s'}}\sqrt{\frac{4(1+5\delta_{s+s'} - 4\delta_{s+s'}^2)}{32-25\delta_{s+s'}}}\|\boldsymbol{h}_{T_1^c}\|_1\right)^2,$$

which implies by Lemma 2.3 that

$$\|\boldsymbol{h}_{T_1}\|_2 \le \frac{2\sqrt{2}}{\sqrt{1-\delta_{s+s'}}}\varepsilon + \frac{\theta_{s,s'}}{\sqrt{s'}}\left(2\|\boldsymbol{x} - \boldsymbol{x}_{T_0}\|_1 + \|\boldsymbol{h}_{T_1}\|_1\right).$$

By the assumption $\frac{\sqrt{s}}{\sqrt{s'}}\theta_{s,s'} < 1$, we have

$$\|\boldsymbol{h}_{T_1}\|_2 \le \frac{2\sqrt{2}}{\sqrt{1-\delta_{s+s'}}}\frac{\sqrt{s'}}{\left(\sqrt{s'} - \sqrt{s}\theta_{s,s'}\right)}\varepsilon + \frac{2\theta_{s,s'}}{\sqrt{s'} - \sqrt{s}\theta_{s,s'}}\|\boldsymbol{x} - \boldsymbol{x}_{T_0}\|_1. \tag{22}$$

By the above calculations and Lemma 2.7 we have

$$\begin{aligned}
\|\boldsymbol{x}-\boldsymbol{x}^\star\|_2^2 &\le \|\boldsymbol{h}_{T_1\cup T_2}\|_2^2+\sum_{j\ge 3}\|\boldsymbol{h}_{T_j}\|_2^2 \\
&\le \frac{1}{1-\delta_{s+s'}}\left(4\varepsilon^2+\frac{4\varepsilon}{\sqrt{s'}}\sqrt{p(1-p)+\delta_{s+s'}(1-\frac{3}{4}p)^2}\|\boldsymbol{h}_{T_1^c}\|_1\right. \\
&\quad \left.+\frac{1}{s'}\left(\left(-2+\frac{5}{16}\delta_{s+s'}\right)p^2+\left(2-\frac{5}{2}\delta_{s+s'}\right)p+\delta_{s+s'}\right)\|\boldsymbol{h}_{T_1^c}\|_1^2\right) \\
&\le \frac{1}{1-\delta_{s+s'}}\left(2\varepsilon+\frac{1}{\sqrt{s'}}\sqrt{\frac{8(2-\delta_{s+s'})}{32-25\delta_{s+s'}}}\|\boldsymbol{h}_{T_1^c}\|_1\right)^2,
\end{aligned}$$

which implies by Lemma 2.4 and (22) that

$$\|\boldsymbol{x}-\boldsymbol{x}^\star\|_2 \le D_0\|\boldsymbol{x}-\boldsymbol{x}_{T_0}\|_1+D_1\varepsilon.$$

Let $s'\ge s$. Similarly, we can show that

$$\|\boldsymbol{x}-\boldsymbol{x}^\star\|_2 \le D_0'\|\boldsymbol{x}-\boldsymbol{x}_{T_0}\|_1+D_1'\varepsilon.$$

This completes the proof.

By taking various numbers s', we have new bounds for $\delta_{s+s'}$ and $\delta_{2s'}$.

Example

(1) Let $s'=s$, then $\delta_{2s}<0.4931$. This is due to Q. Mo and S. Li [7].
(2) Let $s'=\left[\frac{40}{71}s\right]$, then $\delta_{\left(1+\frac{40}{71}\right)s}\approx\delta_{1.56s}<\frac{1}{3}$.
(3) Let $s'=\left[\frac{59}{75}s\right]$, then $\delta_{\left(1+\frac{59}{75}\right)s}\approx\delta_{1.78s}<\frac{2}{5}$.
(4) Let $s'=\left[\frac{40}{39}s+1\right]$, then $\delta_{2\left(1+\frac{40}{39}s\right)}<\frac{1}{2}$.
(5) Let $s'=\left[\frac{128}{85}s+1\right]$, then $\delta_{2\left(1+\frac{128}{85}s\right)}<\frac{3}{5}$.
Here $[\cdot]$ is a floor function.

4 Special Case

In this section, we introduce the sufficient condition of Theorem 3.1 and Theorem 3.2 in special case. We suppose that a matrix satisfies the condition $\|A\|\le 1$, where $\|\cdot\|$ is operator norm. Many researchers have improved the sufficient condition to δ_{2s} in special cases. For example, $\delta_{2s} < 0.4721$ for cases such that s is a multiple of 4 or s is very large by T. Cai. *el.al* [2], $\delta_{2s} < 0.4734$ for the case such that s is very large by S. Foucart [5]. In a resent paper, Q. Mo and S. Li [7] have improved the sufficient condition to $\delta_{2s} < 0.6569$ in some special cases. Above researches are concerned with sparsity level of $\boldsymbol{x}$. However, we proposed results about the case of restricted matrix with respect to A. We do not suppose the condition of sparsity. Practically, we can not know the sparsity of $\boldsymbol{x}$ but we can calculate $\|A\|$.

4.1 Theorem 3.1 in Special Case

Theorem 3.3. Assume that $\|A\| \leq 1$ and A obeys the RIP of order s and $\delta_s < \frac{2}{2+\sqrt{5}} \approx 0.472$. Then the solution $\boldsymbol{x}^\star$ to (2) obeys

$$\|\boldsymbol{x} - \boldsymbol{x}^\star\|_2 \leq C_0 \|\boldsymbol{x} - \boldsymbol{x}_{T_0}\|_1 + C_1 \varepsilon, \tag{23}$$

where

$$C_0 = \frac{2 + \left(14\sqrt{5} + 8\right)\delta_s}{1 - \left(\frac{2+\sqrt{5}}{2}\right)\delta_s}, \quad C_1 = \frac{4\sqrt{1+\delta_s}}{1 - \left(\frac{2+\sqrt{5}}{2}\right)\delta_s}.$$

Proof. The proof of Theorem 3.3 can be obtained based on a modification of the proof of Theorem 3.1. We introduce the modified formulas. By Lemma 2.1, we have

$$(1 - \delta_s)\|\boldsymbol{h}_{T_1}\|_2^2 \;\; \leq \;\; 2\sqrt{1+\delta_s}\varepsilon\|\boldsymbol{h}_{T_1}\|_2 + \frac{1}{2\sqrt{t}}\delta_s\|\boldsymbol{h}_{T_1}\|_2\left(\sum_{j\geq 2}\|\boldsymbol{h}_{T_j}\|_2\right)$$

and

$$\left(1 - \left(\frac{2+\sqrt{5}}{2}\right)\delta_s\right)\|\boldsymbol{h}_{T_1}\|_2 \leq 2\sqrt{1+\delta_s}\varepsilon + \frac{9\sqrt{5}}{\sqrt{s}}\delta_s\|\boldsymbol{x} - \boldsymbol{x}_{T_0}\|_1. \tag{24}$$

By assumption $\delta_s < \frac{2}{2+\sqrt{5}} \approx 0.472$, we have

$$\|\boldsymbol{h}_{T_1}\|_2 \leq \frac{1}{1 - (\frac{2+\sqrt{5}}{2})\delta_s}\left(2\sqrt{1+\delta_s}\varepsilon + \frac{9\sqrt{5}}{\sqrt{s}}\delta_s\|\boldsymbol{x} - \boldsymbol{x}_{T_0}\|_1\right), \tag{25}$$

which implies by Lemma 2.4 that

$$\|\boldsymbol{x} - \boldsymbol{x}^\star\|_2 \leq C_0\|\boldsymbol{x} - \boldsymbol{x}_{T_0}\|_1 + C_1\varepsilon$$

This completes the proof.

4.2 Theorem 3.2 in Special Case

Theorem 3.4. Assume that $\|A\| \le 1$. Then we have the following

(1) Let $\frac{s}{8} < s' \le s$. If A obeys the RIP of order $(s+s')$ and

$$\theta_{s,s'} \equiv 2\sqrt{\frac{-3\delta_{s+s'}^2 + 7\delta_{s+s'} + 2}{(1-\delta_{s+s'})(64-25\delta_{s+s'})}} < \sqrt{\frac{s'}{s}},$$

then

$$\|\boldsymbol{x} - \boldsymbol{x}^\star\|_2 \le D_0\|\boldsymbol{x} - \boldsymbol{x}_{T_0}\|_1 + D_1\varepsilon,$$

where

$$\begin{aligned} D_0 &= \frac{2\sqrt{6}}{\sqrt{s'} - \sqrt{s}\theta_{s,s'}}\sqrt{\frac{3-\delta_{s+s'}}{(1-\delta_{s+s'})(48-25\delta_{s+s'})}}, \\ D_1 &= \frac{2\sqrt{3}}{\sqrt{1-\delta_{s+s'}}}\left(\sqrt{\frac{2}{5}} + \frac{2\sqrt{s}}{\sqrt{s'} - \sqrt{s}\theta_{s,s'}}\sqrt{\frac{3-\delta_{s+s'}}{(1-\delta_{s+s'})(48-25\delta_{s+s'})}}\right). \end{aligned}$$

(2) Let $s \le s'$. If A obeys the RIP of order $2s'$ and

$$\theta_{s'} \equiv 2\sqrt{\frac{-3\delta_{2s'}^2 + 7\delta_{2s'} + 2}{(1-\delta_{2s'})(64-25\delta_{2s'})}} < \sqrt{\frac{s'}{s}},$$

then

$$\|\boldsymbol{x} - \boldsymbol{x}^\star\|_2 \le D_0'\|\boldsymbol{x} - \boldsymbol{x}_{T_0}\|_1 + D_1'\varepsilon,$$

where

$$\begin{aligned} D_0' &= \frac{2\sqrt{6}}{\sqrt{s'} - \sqrt{s}\theta_{s'}}\sqrt{\frac{3-\delta_{2s'}}{(1-\delta_{2s'})(48-25\delta_{2s'})}}, \\ D_1' &= \frac{2\sqrt{3}}{\sqrt{1-\delta_{2s'}}}\left(\sqrt{\frac{2}{5}} + \frac{2\sqrt{s}}{\sqrt{s'} - \sqrt{s}\theta_{s'}}\sqrt{\frac{3-\delta_{2s'}}{(1-\delta_{2s'})(48-25\delta_{2s'})}}\right). \end{aligned}$$

Proof. The proof of Theorem 3.4 can be obtained based on a modification of the proof of Theorem 3.2.

(1) Let $\frac{s}{8} < s' \le s$. Then it follows from Lemma 2.1, 2.7 and 2.8 that

$$\begin{aligned} \|\sum_{i\ge 3} A\boldsymbol{h}_{T_i}\|_2^2 &\le \left(1 + \frac{\delta_{s+s'}}{2}\right)\sum_{i\ge 3}\|\boldsymbol{h}_{T_i}\|_2^2 + \frac{\delta_{s+s'}}{2}\left(\sum_{i\ge 3}\|\boldsymbol{h}_{T_i}\|_2\right)^2 \\ &\le \frac{\|\boldsymbol{h}_{T_1^c}\|_1^2}{2s'}\left((2+\delta_{s+s'})\,p(1-p) + \delta_{s+s'}\left(1 - \frac{3}{4}p\right)^2\right), \end{aligned}$$

which implies that

$$
\begin{aligned}
(1-\delta_{s+s'})\|\boldsymbol{h}_{T_1}\|_2^2 \quad &\leq \quad 4\varepsilon^2 + 4\varepsilon\frac{\|\boldsymbol{h}_{T_1^c}\|_1}{\sqrt{2s'}}\sqrt{(2+\delta_{s+s'})p(1-p)+\delta_{s+s'}\left(1-\frac{3}{4}p\right)^2} \\
&\quad + \frac{\|\boldsymbol{h}_{T_1^c}\|_1^2}{2s'}\left(-\left(4-\frac{25}{16}\delta_{s+s'}\right)p^2+\left(2-\frac{1}{2}\delta_{s+s'}\right)p+\delta_{s+s'}\right). \quad (26)
\end{aligned}
$$

Since

$$
-\left(4-\frac{25}{16}\delta_{s+s'}\right)p^2+\left(2-\frac{1}{2}\delta_{s+s'}\right)p+\delta_{s+s'} \leq \frac{8\left(-3\delta_{s+s'}^2+7\delta_{s+s'}+2\right)}{64-25\delta_{s+s'}}
$$

and

$$
\begin{aligned}
(2+\delta_{s+s'})p(1-p)+\delta_{s+s'}\left(1-\frac{3}{4}p\right)^2 \quad &\leq \quad \frac{8\left(\delta_{s+s'}^2+3\delta_{s+s'}+2\right)}{32+7\delta_{s+s'}} \\
&< \quad 2\cdot\frac{-3\delta_{s+s'}^2+7\delta_{s+s'}+2}{64-25\delta_{s+s'}},
\end{aligned}
$$

we have by (26)

$$
\|\boldsymbol{h}_{T_1}\|_2^2 \leq \left(\frac{2\sqrt{2}}{\sqrt{1-\delta_{s+s'}}}\varepsilon + \frac{2\sqrt{-3\delta_{s+s'}^2+7\delta_{s+s'}+2}}{\sqrt{(1-\delta_{s+s'})(64-25\delta_{s+s'})}}\frac{\|\boldsymbol{h}_{T_1^c}\|_1}{\sqrt{s'}}\right)^2,
$$

which implies by Lemma 2.4 that

$$
\|\boldsymbol{h}_{T_1}\|_2 \leq \frac{2\sqrt{2}}{\left(1-\sqrt{\frac{s}{s'}}\theta_{s,s'}\right)\sqrt{1-\delta_{s+s'}}}\varepsilon + \frac{2\theta_{s,s'}}{\sqrt{s'}\left(1-\sqrt{\frac{s}{s'}}\theta_{s,s'}\right)}\|\boldsymbol{x}-\boldsymbol{x}_{T_0}\|_1. \quad (27)
$$

Hence it follows from (27) and Lemma 2.7 that

$$
\|\boldsymbol{x}-\boldsymbol{x}^\star\|_2 \leq D_0\|\boldsymbol{x}-\boldsymbol{x}_{T_0}\|_1 + D_1\varepsilon
$$

(2) This is shown similarly to (1).

Example

(1) Let $s' > \left[\frac{76}{103}s\right]$, then $\delta_{\left(1+\frac{76}{103}\right)s} \approx \delta_{1.74s} < \frac{1}{2}$.
(2) Let $s' = s$, then $\delta_{2s} < \frac{117-\sqrt{5177}}{76} \approx 0.59$.
(3) Let $s' > \frac{256}{245}s$, then $\delta_{\frac{512}{245}s} \approx \delta_{2.09s} < 0.6$.
(4) Let $s' > \frac{724}{465}s$, then $\delta_{\frac{1448}{465}s} \approx \delta_{3.13s} < 0.7$.

We may consider the bound for the other δ_k $(k > s)$.

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