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Abstract

This paper discusses the theory for RIPless in compressed sensing (CS). In the literature, E.J. Candès has proved that \( \delta_{2s} < \sqrt{2} - 1 \) is a sufficient condition via \( l_1 \) optimization having \( s \)-sparse vector solution. Later, many researchers have improved the sufficient conditions on \( \delta_{2s} \) or \( \delta_s \). Such researches have supposed that a matrix \( A \) obeys RIP and a signal to recover is sparse. In this paper, we do not suppose that a matrix \( A \) obeys RIP and a signal is sparse. We propose the RIPless theory and the method of any signal recovery for noiseless and noisy cases in CS.

Key Words and Phrases: Compressed sensing, RIPless theory.

1 Introduction

This paper introduces the RIPless theory of compressed sensing (CS). For a signal \( x \in \mathbb{R}^n \), let \( \|x\|_1 \) be \( l_1 \) norm of \( x \) and \( \|x\|_2 \) be \( l_2 \) norm of \( x \). Let \( x \) be not a sparse vector. Compressed sensing aims to recover high-dimensional signal (for example: images signal, voice signal, code signal...etc.) from only a few samples or linear measurements. Formally, one considers the following model in noiseless case:

\[
y = Ax,
\]

where \( A \) is a \( m \times n \) matrix\((m < n)\).

Our goal is to reconstruct an unknown signal \( x \) based on \( A \) and \( y \) are given. Then we consider reconstructing \( x \) as the solution \( x^* \) to the optimization problem

\[
\min_{\tilde{x}} \|\tilde{x}\|_1, \quad \text{subject to} \quad y = A\tilde{x}.
\]

Furthermore, one considers the following model in noisy case:

\[
y = Ax + z,
\]

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where $z$ is an unknown noise term.

In this context, we consider reconstructing $x$ as the solution $x^*$ to the optimization problem

$$
\min_{\hat{x}} \|\hat{x}\|_1, \quad \text{subject to} \quad \|y - A\hat{x}\|_2 \leq \varepsilon,
$$

where $\varepsilon$ is an upper bounded on the size of the noisy contribution.

In CS theory, a crucial issue is to research good conditions in order to achieve our goal. One of the most generally known condition for CS theory is the restricted isometry property (RIP) introduced by E.J. Candès and T. Tao [4]. When we discuss our proposed results, it is an important notion. The RIP needs that the subsets of columns of $A$ for all locations in $\{1, 2, \cdots, n\}$ behave nearly orthonormal system. In detail, a matrix $A$ satisfies the RIP of order $s$ if there exists a constant $\delta$ with $0 < \delta < 1$ such that

$$(1 - \delta)\|a\|_2^2 \leq \|Aa\|_2^2 \leq (1 + \delta)\|a\|_2^2$$

for all $s$-sparse vectors $a$. A vector is said to be $s$-sparse vector if it has at most $s$ nonzero entries. The minimum $\delta$ satisfying the above restrictions is said to be the restricted isometry constant and is denoted by $\delta_s$.

It has been shown that $l_1$ optimization can recover an unknown signal in noiseless case and noisy case under various sufficient conditions on $\delta_s$ or $\delta_{2s}$. For example, E.J. Candès and T. Tao [4] have proved that if $\delta_{2s} < \sqrt{2} - 1$, then an unknown signal can be recovered. Later, S. Foucart and M. Lai [6] have improved the bound to $\delta_{2s} < 0.4531$. Others, $\delta_{2s} < 0.4652$ is used by S. Foucart [5], $\delta_{2s} < 0.4721$ for cases such that $s$ is a multiple of 4 or $s$ is very large by T. Cai. el.al. [2], $\delta_{2s} < 0.4734$ for the case such that $s$ is very large by S. Foucart [5] and $\delta_s < 0.307$ by T. Cai el.al. [2]. In a resent paper, Q. Mo and S. Li [7] have improved the sufficient condition to $\delta_{2s} < 0.4931$ for general case and $\delta_{2s} < 0.6569$ in some special case.

In this paper, we propose the RIPless theory and the method of an unknown signal recovery in CS for noiseless case. There are main benefits for considering the RIPless theory. First, we do not suppose that a matrix satisfies the condition of RIP. Moreover, we do not suppose the condition of sparsity. Practically, it is very difficult to know the condition of RIP and the sufficient condition of isometry constants. Likewise, we can not
know the sparsity of $x$. Second, the assessments of various cases lead to developments for signal analysis or other analysis.

Our analysis is very simple and elementary. We introduce the proposed results using most simple approach. We expect that more efficient approaches are suggested as developments for many analysis.

In this paper, suppose $x$ is an original signal we need to recover and $x^* = (x^*_1, \ldots, x^*_n)$ is the solution of CS optimization problem (2) or (4). Let $A = (a_1, a_2, \ldots, a_n)$, where $a_1 = (a_{11}, a_{21}, \ldots, a_{m1})'$, $a_2 = (a_{12}, a_{22}, \ldots, a_{m2})'$, $\ldots, a_n = (a_{1n}, a_{2n}, \ldots, a_{mn})'$.

## 2 RIPless Theory

In this section, we first introduce the RIPless theory in noiseless case. We suppose that $x$ is a $s$-sparse vector and put $x_s = (x_1, x_2, \ldots, x_s, 0, \ldots, 0)$. We have the following:

**Theorem 2.1.** Suppose $\{a_1, a_2, \ldots, a_s\}$ is linear independent, and $\{a_1, a_2, \ldots, a_s\}$ and $\{a_{s+1}, a_{s+2}, \ldots, a_n\}$ are orthogonal. Then, the solution $x^*$ to (2) recovers $x$ exactly, i.e., $x = x^*$.

**Proof.** Since

$$0 = \|Ax^* - Ax\|_2^2 = \|((x^*_1 - x_1)a_1 + \cdots + (x^*_s - x_s)a_s\|_2^2$$

\[ = \|x^*_{s+1}a_{s+1} + \cdots + x^*_na_n\|_2^2, \tag{6}\]

we have

$$(x^*_1 - x_1)a_1 + \cdots + (x^*_s - x_s)a_s = 0.$$ 

Since $\{a_1, a_2, \ldots, a_s\}$ is linear independent, we have $x^*_1 = x_1, \ldots, x^*_s = x_s$. Thus,

$$x^*_s \equiv (x^*_1, x^*_2, \ldots, x^*_s, 0, \ldots, 0) = x \text{ and } Ax^*_s = Ax = y.$$ 

By the definition of CS optimization, we have

$$\|x^*\|_1 \leq \|x^*_s\|_1.$$ 

Furthermore, by the notation of $x^*_s$, we have

$$\|x^*_s\|_1 \leq \|x^*\|_1.$$ 

Thus, we have $x^* = x^*_s$ and $x^*_{s+1} = \cdots = x^*_n = 0$. By these discussions, we have

$$x^* = x^*_s = x.$$ 

This completes the proof.

We next discuss a generalization of Theorem 2.1 in noiseless and noisy cases. We first consider in case that we have the knowledges of data, that is, we know a good location $T_0$. Let $K \equiv \{ x \in \mathbb{R}^n; \| y - Ax \|_2 \leq \varepsilon \}$, $\varepsilon > 0$ and $K_0 \equiv \{ x \in \mathbb{R}^n; y = Ax \}$. Assume that $K_{\varepsilon} \neq \emptyset$.

**Theorem 2.2.** Let $T_0$ be a location in $\{1, 2, \cdots, \}$ with $|T_0| = s$. Suppose

(i) $\{a_k; k \in T_0\}$ is linearly independent;

(ii) $\mu_{T_0} = \max\{|a_k, a_j|; k \in T_0, j \in T^c_0\} < \frac{1}{\mu(A^*_0 A_0)^{-1}} = \frac{\lambda_1}{\lambda_s}$, where $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_s > 0$ are eigenvalues of $A^*_0 A_0$.

Then, for every $x \in K_{\varepsilon}$ we have

$$\|x^* - x\|_2 \leq C_{0}(T_0)\|x - x_{T_0}\|_1 + C_{1}(T_0)\varepsilon,$$

where

$$C_{0}(T_0) = \frac{2(1 + \mu_{T_0} s\| (A^*_0 A_0)^{-1}\|)}{1 - \mu_{T_0} s\| (A^*_0 A_0)^{-1}\|},$$

$$C_{1}(T_0) = \frac{2(1 + \sqrt{s})\| (A^*_0 A_0)^{-1}\|}{1 - \mu_{T_0} s\| (A^*_0 A_0)^{-1}\|}.$$ (8)

**Proof.** Let $x \in K_{\varepsilon}$ and $h \equiv x^* - x$. We may take $T_0 = \{1, 2, \cdots, s\}$ without loss of generality. We have

$$\|Ah_{T_0}\|_2^2 = \left\langle Ah_{T_0}, Ah - \sum_{j \geq 1} Ah_{T_j} \right\rangle$$

$$\leq 2\varepsilon \|Ah_{T_0}\|_2 + \left\langle Ah_{T_0}, \sum_{j \geq 1} Ah_{T_j} \right\rangle$$ (9)
\[
\begin{align*}
\left\langle A h_{T_0}, \sum_{j \geq 1} A h_{T_j} \right\rangle &= \sum_{k=1}^{s} \sum_{j=s+1}^{n} h_k h_j \left\langle a_k, a_j \right\rangle \\
&\leq \mu T_0 \sum_{k=1}^{s} \sum_{j=s+1}^{n} |h_k h_j| \\
&\leq \mu T_0 \|h_{T_0}\|_1 \|h_{T_0}'\|_1 \\
&\leq \mu T_0 \sqrt{s} \|h_{T_0}\|_2 (2\|x - x_{T_0}\|_1 + \sqrt{s} \|h_{T_0}\|_2). 
\end{align*}
\]

We also have
\[
\|h_{T_0}\|_2 \leq \sqrt{\| (A_{T_0}^* A_{T_0})^{-1} \| \|A h_{T_0}\|_2}. 
\]

Indeed, this follows from
\[
\begin{align*}
\|h_{T_0}\|_2^2 &= \left\langle (A_{T_0}^* A_{T_0})^{-1} (A_{T_0}^* A_{T_0}) h_{T_0}, h_{T_0} \right\rangle \\
&= \left\langle (A_{T_0}^* A_{T_0})^{-1} (A_{T_0}^* A_{T_0})^{1/2} h_{T_0}, (A_{T_0}^* A_{T_0})^{1/2} h_{T_0} \right\rangle \\
&\leq \| (A_{T_0}^* A_{T_0})^{-1} \| \| (A_{T_0}^* A_{T_0})^{1/2} h_{T_0}\|_2^2 \\
&= \| (A_{T_0}^* A_{T_0})^{-1} \| \|A h_{T_0}\|_2^2.
\end{align*}
\]

Using (9), (10), (11), we get
\[
\|A h_{T_0}\|_2 \leq 2\varepsilon + \mu T_0 \sqrt{s} \sqrt{r_{T_0}} \|A h_{T_0}\|_2 (2\|x - x_{T_0}\|_1 + \sqrt{s} \|h_{T_0}\|_2),
\]

where \( r_{T_0} \equiv \| (A_{T_0}^* A_{T_0})^{-1} \| \). By using (11) again, we have
\[
\|h_{T_0}\|_2 \leq 2\sqrt{r_{T_0}} \varepsilon + 2\mu T_0 \sqrt{s} r_{T_0} \|x - x_{T_0}\|_1 + \mu T_0 s r_{T_0} \|h_{T_0}\|_2.
\]

Therefore, we obtain
\[
(1 - \mu T_0 s r_{T_0}) \|h_{T_0}\|_2 \leq 2\sqrt{r_{T_0}} \varepsilon + 2\mu T_0 \sqrt{s} r_{T_0} \|x - x_{T_0}\|_1.
\]

By the assumption (ii), we have
\[
\|h_{T_0}\|_2 \leq \frac{2\sqrt{r_{T_0}} \varepsilon}{1 - \mu T_0 s r_{T_0}} + \frac{2\mu T_0 \sqrt{s} r_{T_0}}{1 - \mu T_0 s r_{T_0}} \|x - x_{T_0}\|_1. 
\]

Since
\[
\|h_{T_0}\|_2 \leq \|h_{T_0}'\|_1 \leq 2\|x - x_{T_0}\|_1 + \|h_{T_0}\|_1 \\
\leq 2\|x - x_{T_0}\|_1 + \sqrt{s} \|h_{T_0}\|_2,
\]

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it follows from (12) that
\[ \|x^* - x\|_2 \leq \|h_{T_0}\|_2 + \|h_{T_0}\|_2 \]
\[ \leq 2\|x - x_{T_0}\|_1 + (1 + \sqrt{s})\|h_{T_0}\|_2 \]
\[ \leq \frac{2(1 + \sqrt{s})\sqrt{T_{T_0}}}{1 - \mu_{T_0}s^r_{T_0}}\varepsilon + \frac{2(1 + \mu_{T_0}\sqrt{s^r_{T_0}})}{1 - \mu_{T_0}s^r_{T_0}}\|x - x_{T_0}\|_1. \]

This completes the proof.

By Theorem 2.2 we have the following:

**Corollary 2.1.** Suppose that \( A \) satisfies the conditions (i) and (ii) in Theorem 2.2. Then, we have

**Noiseless case:** If \( x \) is a \( T_0 \)-sparse vector in \( K_0 \), then
\[ x = x^* . \]

**Noisy case:** If \( x \) is a \( T_0 \)-sparse vector in \( K_\varepsilon \), then
\[ \|x^* - x\|_2 \leq C_1(T_0)\varepsilon . \]

For case that we do not know any good location we have the following.

**Theorem 2.3.** Suppose that \( A = (a_1, a_2, \cdots, a_n) \) satisfies the following

(i) \( \{a_k; k \in T\} \) is linearly independent for each \( T \subset \{1, 2, \cdots, n\} \) with \( |T| = s; \)

(ii) \( \mu = \max_T \mu_T = \max_{1 \leq i \neq j \leq n} | < a_i, a_j > | < \frac{1}{s \max(\|A^*_T A_T\|^{-1}, \|T\|s)} = \frac{\lambda_1^T}{s}, \) where
\( \lambda_1^T \geq \lambda_2^T \geq \cdots \geq \lambda_s^T > 0 \) are eigenvalues of \( A^*_T A_T . \)

Then, for any \( x \in K_\varepsilon \) and \( T \subset \{1, 2, \cdots, n\} \) with \( |T| = s \), we have
\[ \|x^* - x\|_2 \leq C_0\|x - x_T\|_1 + C_1\varepsilon , \]

where
\[ C_0 = \frac{4 + 5\mu rs}{2\sqrt{s}(1 - \mu rs)} ; \]
\[ C_1 = \frac{9\sqrt{r}}{2(1 - \mu rs)} . \]

**Proof.** The proof of this theorem can be obtained based on Theorem 2.2. We show the modifications (13)-(16). For simplicity, we assume that the index of \( h \) is sorted by
$|h_1| \geq |h_2| \geq \cdots \geq |h_n|$. Take an arbitrary location $T_0$ of $\{1, 2, \cdots, n\}$ with $|T_0| = s$ and let $\{T_1, T_2, \cdots, T_l\}$ be a decomposition of $\{1, 2, \cdots, n\}$ with $|T_1| = s$, $|T_k| = s'$ ($2 \leq k \leq l - 1$) and $1 \leq |T_l| = r \leq s'$, where $|T|$ is number of elements of $T$. We consider the decomposition of $h$ as follows:

$$h_{T_1} = (h_1^{(T_1)}, h_2^{(T_1)}, \cdots, h_s^{(T_1)}, 0, \cdots, 0)$$

$$h_{T_2} = (0, \cdots, 0, h_1^{(T_2)}, \cdots, h_s^{(T_2)}, 0, \cdots, 0)$$

$$\vdots$$

$$h_{T_{l-1}} = (0, \cdots, 0, h_1^{(T_{l-1})}, \cdots, h_s^{(T_{l-1})}, 0, \cdots, 0)$$

$$h_{T_l} = (0, \cdots, 0, h_1^{(T_l)}, \cdots, h_s^{(T_l)}).$$

This is due to the T. Cai et al. idea [2] in case of $s = s'$. Then we have the following:

$$\|h_{T_1}\|_2 \leq \sqrt{\| (A_{T_1}^T A_{T_1})^{-1} \|} \|Ah_{T_1}\|_2 \leq \sqrt{r}\|Ah_{T_1}\|_2. \quad (13)$$

$$< Ah_{T_1}, \sum_{j \geq 2} Ah_{T_j} > \leq \mu \sqrt{s}\|h_{T_1}\|_2 (2\|x - x_{T_0}\|_1 + \sqrt{s}\|h_{T_1}\|_2) \leq \mu \sqrt{s} \sqrt{r}\|Ah_{T_1}\|_2 (2\|x - x_{T_0}\|_1 + \sqrt{s}\|h_{T_1}\|_2). \quad (14)$$

$$\frac{\|h_{T_1}\|_2^2}{\sqrt{r}} \leq ||Ah_{T_1}\|_2 \leq 2\varepsilon + \mu \sqrt{s} + \mu \sqrt{s} \sqrt{r} (2\|x - x_{T_0}\|_1 + \sqrt{s}\|h_{T_1}\|_2). \quad (15)$$

Therefore, we have

$$(1 - \mu rs)\|h_{T_1}\|_2 \leq 2\sqrt{r}\varepsilon + 2\mu r \sqrt{s}\|x - x_{T_0}\|_1.$$ 

By the assumption (ii), we have

$$\|h_{T_1}\|_2 \leq \frac{2\sqrt{r}}{1 - \mu rs} \varepsilon + \frac{2\mu r \sqrt{s}}{1 - \mu rs} \|x - x_{T_0}\|_1. \quad (16)$$

Since

$$\|h_{T_1}\|_2 \leq \sum_{j \geq 2} h_{T_j} \|_2 \leq \frac{1}{\sqrt{s}} \sum_{j \geq 2} \|h_{T_j}\|_1 + \frac{\sqrt{s}}{4} |h_1^{(T_2)}| \leq \frac{2}{\sqrt{s}} \|x - x_{T_0}\|_1 + \frac{5}{4} \|h_{T_1}\|_2,$$
we have by using (13)-(16)
\[
\|x^* - x\|_2 \leq \|h_{T_1}\|_2 + \|h_{T_2}\|_2 \\
\leq \frac{2}{\sqrt{s}}\|x - x_{T_0}\|_1 + \frac{9}{4}\|h_{T_1}\|_2 \\
\leq \frac{2}{\sqrt{s}}\|x - x_{T_0}\|_1 + \frac{9}{4} \left( \frac{2\sqrt{r}}{1-\mu rs} + \frac{2\mu r\sqrt{s}}{1-\mu rs}\|x - x_{T_0}\|_1 \right) \\
= \frac{4 + 5\mu rs}{2\sqrt{s}(1-\mu rs)}\|x - x_{T_0}\|_1 + \frac{9\sqrt{r}}{2(1-\mu rs)}\varepsilon.
\]

This completes the proof.

**Corollary 2.2.** Suppose that \(A\) satisfies the conditions (i) and (ii) in Theorem 2.3. Then, we have

**Noiseless case:** If \(x\) is a \(T_0\)-sparse vector in \(K_0\), then
\[
x = x^*.
\]

**Noisy case:** If \(x\) is a \(T_0\)-sparse vector in \(K_\varepsilon\), then
\[
\|x^* - x\|_2 \leq C_1\varepsilon.
\]

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