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Numerical and theoretical evaluations of AC losses for single and infinite numbers of superconductor strips with direct and alternating transport currents in external AC magnetic field

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## Abstract

AC losses in a superconductor strip are numerically evaluated by means of a finite element method formulated with a current vector potential. The expressions of AC losses in an infinite slab that corresponds to a simple model of infinitely stacked strips are also derived theoretically. It is assumed that the voltage-current characteristics of the superconductors are represented by Bean's critical state model. The typical operation pattern of a SMES (Superconducting Magnetic Energy Storage) coil with direct and alternating transport currents in an external AC magnetic field is taken into account as the electromagnetic environment for both the single strip and the infinite slab. By using the obtained results of AC losses, the influences of the transport currents on the total

losses are discussed quantitatively.

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## 1. Introduction

Coated conductors with rare-earth-element-based cuprate superconductors have an advantage of significant critical current density in a high magnetic field [1], so that the magnet system for a Superconducting Magnetic Energy Storage (SMES) is one of the promising applications of the coated conductors. Since the superconducting layers in the coated conductors have a cross section typified by a width of 10 mm and a thickness in the order of micrometers, AC losses arising during charge and discharge operations of the SMES coil have an anisotropic property for the direction of an external applied magnetic field. When the coated conductors are exposed to an external field parallel to their wide faces, the AC losses could be estimated as isolated infinite slabs [2–6]. In the case of an perpendicular magnetic field, on the other hand, the AC loss properties become quite complicated such as an isolated superconductor strip [7–10] for gaps between turns much larger than the width of the coated conductors, a homogeneous superconductor with a thickness same as the conductor width for much smaller gaps [11–13], and intermediate situations [13].

The windings for SMES with the coated conductors are usually constructed in the form of pancake-type coils, and they are located with a toroidal arrangement [14,15]. The AC loss properties for the pancake coils with the coated conductors have been evaluated up to now [13,16]. The AC losses in a bundle of superconductor strips exposed to the perpendicular magnetic field have agreed well with those for a homogeneous superconductor with the identical cross section [13]. Since the winding in the SMES coil itself carries a transport current in the external applied magnetic field generated by the other windings, the effect of the transport current on the AC loss has to be investigated quantitatively for the realization of the SMES system.

In this study, the AC losses in single and infinite numbers of superconductor strips are evaluated for the simultaneous applications of the transport current and the external magnetic field. The AC losses in the single strip with direct and alternating transport currents in the external AC magnetic field are calculated numerically by means of a finite element method [17]. The expressions of AC losses in an infinite slab for the similar electromagnetic configuration are also derived analytically, and the obtained results are compared with conventional theoretical curves [3–6].

## 2. Numerical results of AC losses

When a typical operation pattern of the SMES for load fluctuation compensation as shown in Fig. 1 is taken apart in details, one turn of the coil under consideration as a fundamental unit has a direct transport current  $I_d$  and is also exposed to an external DC magnetic field  $H_{ed}$  due to currents flowing in the other turns at a standby mode first. After that, if the SMES shifts to a continuous pulse mode with charge and discharge operations, the alternating transport current  $I_a$  and the external AC magnetic field  $H_{ea}$  are superposed to them. Thus, it can be considered that the windings for SMES coil are generally exposed to the electromagnetic environment with the direct/alternating transport currents and the external DC/AC magnetic fields, but the AC losses in such a situation have not been discussed so far. Since the external DC magnetic field indirectly affects the AC loss through the variation of the critical current density of superconductor, it could be enough to take into account only the AC component of external applied field for the AC loss evaluation.

In this section, the AC losses in a superconductor strip of  $2a$  in width with the direct and alternating transport currents,  $I_d$  and  $I_a$ , exposed to the external AC magnetic

field  $H_{\text{ea}}$  perpendicular to the flat face as shown in Fig. 2(a) are numerically calculated by means of the finite element method formulated with a current vector potential [17]. It is assumed that the transport property of the superconductor strip is represented by the Bean model [2,18] including a flux-flow state with the resistivity of  $1 \times 10^{-7} \Omega\text{m}$  [19]. Fig. 3(a) shows the numerical results of AC losses  $W$  per unit volume per cycle in the single strip without the alternating transport current for the amplitude  $H_{\text{em}}$  of the external AC applied field  $H_{\text{ea}}$  to understand the essential effect of the direct transport current. The vertical and horizontal axes are normalized by  $W_0 = \mu_0 J_c I_c / \pi$  and  $H_c = J_c d / \pi$ , respectively, with the critical current density  $J_c$ , critical current  $I_c$  and thickness  $d$  of the strip. It can be seen that the AC losses sharply increase at a current determined by each direct transport current. Such a property of AC losses has been well known for an infinite slab subject to the similar electromagnetic environment as shown in Fig. 3(b) [4], where  $H_p$  and  $H_{\text{id}}$  represent the full penetration field of the slab and the self-field due to the direct transport current, respectively. When the amplitude of external magnetic field applied to the slab with the direct transport current is small, the total magnetic flux penetrating from each surface in a half cycle returns to the corresponding side in a subsequent half cycle and therefore the AC losses have an exact agreement with those for only the external AC magnetic field. In the relatively large amplitude of external field, on the other hand, a part of the magnetic flux coming from one side of the slab goes into the other side during the cycle, and this causes an effective electric field in the direction of the direct transport current [4]. Such a dynamic resistance results in the large amount of AC losses not only in the infinite slab but also for the single superconductor strip as shown in Fig. 3.

Fig. 4 shows the numerical results of AC losses in the single strip carrying the

alternating transport current in the external AC magnetic field. Fig. 4(a) has no direct transport current, and therefore corresponds to the conventional theoretical expressions of AC losses [10]. On the other hand, Fig. 4(b) represents the numerical results for the constant direct transport current  $I_d$  fixed at sixty percents of the critical current  $I_c$ . It is found in Fig. 4(b) that the AC losses agree well with those for the case of no direct transport current if the amplitude of external applied field is small. For the external-field amplitude larger than a threshold value, on the other hand, the AC loss suddenly grows up. Such a property of AC loss in Fig. 4(b) is similar to the numerical and theoretical results in Fig. 3, so that the dynamic resistance drastically enhances the AC loss in a large range of the external-field amplitude.

### 3. Analytical expressions of AC losses

Let us consider the electromagnetic configuration similar to the previous section for the infinitely stacked superconductor strips as shown in Fig. 2(b). When the gaps  $g$  between the strips located face-to-face at even intervals are much smaller than the width  $2a$  of strip, they can be regarded as a homogeneous superconductor slab with a thickness same as the strip width as shown in Fig. 2(c) [13]. In this case, the filling factor  $\lambda$  of the strips in the slab is given by  $\lambda = d/g$  with the thickness  $d$  of the strip. The Bean model [2,18] is also assumed here, and the full penetration field  $H_p$  of the equivalent slab only for the external applied magnetic field becomes  $H_p = \lambda J_c a = I_c/(2g)$ .

When every strip carries the direct and alternating transport currents,  $I_d$  and  $I_a$  ( $-I_m \leq I_a(t) \leq I_m$ ,  $I_m \leq I_d$ ,  $I_d + I_a(t) \leq I_d + I_m \leq I_c$ ) in the perpendicular AC magnetic field  $H_{ea}$  ( $-H_{em} \leq H_{ea}(t) \leq H_{em}$ ), the magnetic fields on both the surfaces of the slab,  $H_1$  and  $H_2$ , can be expressed as

$$H_1(t) = H_{ea} + H_i = H_{id} + (H_{ea} + H_{ia}), \quad (1)$$

$$H_2(t) = H_{ea} - H_i = -H_{id} + (H_{ea} - H_{ia}), \quad (2)$$

where  $H_i$  is the self-field due to the transport current given by  $H_i = H_{id} + H_{ia}$  with the DC and AC components,  $H_{id} = I_d/(2g)$  and  $H_{ia} = I_a/(2g)$ . In the case of  $H_{em} \leq H_p - H_{id}$ , the expression of the AC loss  $W$  per unit volume per cycle of the slab can be derived analytically,

$$W = \frac{\mu_0 H_p^2}{3} \left\{ (1+k)h_{em} \right\}^3 + |(1-k)h_{em}|^3 \quad (3)$$

where  $k = H_{im}/H_{em}$  with  $H_{im} = I_m/(2g)$  and  $h_{em} = H_{em}/H_p$ . It can be seen in Eq. (3) that the AC loss is independent of the DC component  $H_{id}$  of transport current, and this means that the AC losses in a small range of the external-field amplitude have a good agreement with those for only the alternating transport current in the external AC magnetic field [3,5,6]. In the case of  $H_{em} > H_p - H_{id}$ , on the other hand, the analytical expression of the AC loss becomes

$$\begin{aligned} W = \frac{2\mu_0 H_p^2}{3(1-k^2)^2} \bigg\{ & k^2(1+k^2)(1+3k^2)h_{em}^3 + 6k^2[(1+3k^2)h_{id} - 4k^2]h_{em}^2 \\ & + 3[(1+k^2)(1+3k^2)h_{id}^2 - 8k^2(1+k^2)h_{id} + (1+7k^4)]h_{em} \\ & + 2[(1+3k^2)h_{id}^3 - 12k^2h_{id}^2 + 3k^2(3+k^2)h_{id} - (1+3k^4)] \bigg\} \end{aligned} \quad (4)$$

where  $h_{id} = H_{id}/H_p$ . Although the case of  $I_m > I_d$  is not intended for the present study, the analytical expression for  $H_{em} > H_p/(1 - H_{id}/H_{im})$  is represented by

$$\begin{aligned} W = \frac{2\mu_0 H_p^2}{3(1-k)^2} \bigg\{ & k^2(1+3k^2)h_{em}^3 - 12k^3h_{em}^2 \\ & + 3[(1+3k^2)h_{id}^2 + (1-2k+3k^2+2k^3)]h_{em} - 2[6kh_{id}^2 + (1-2k+3k^2)] \bigg\} \end{aligned} \quad (5)$$

When  $h_{id} = 0$  is applied to Eq. (5), the conventional theoretical expression of AC loss for



the slab with the alternating transport current in the external AC magnetic field can be obtained [3,5,6], and they are plotted in Fig. 5(a). An example for the AC loss curves with Eqs. (3) and (4) is drawn in Fig. 5(b), where the direct transport current  $I_d$  is set at  $0.6I_c$  and therefore  $H_{id}/H_p = 0.6$ . It is found that both the AC losses agree well with each other in the small range of external-field amplitude. For the field amplitude exceeding a threshold value ( $H_p - H_{id}$ ) determined by the direct transport current, however, the AC losses in Fig. 5(b) become much larger than those for Fig. 5(a). This enlargement is due to the effect of the dynamic resistance for the application of the direct transport current.

#### 4. Conclusions

The AC losses in the single superconductor strip with the direct and alternating transport currents in the external AC magnetic field were numerically evaluated by means of the one-dimensional finite element method. The expressions of AC losses in the infinite slab, which corresponded to the superconductor strips stacked infinitely with the small gaps at even intervals, were also derived theoretically for the similar electromagnetic environment. In both the cases, the AC losses had a good agreement with those without the direct transport current in the small range of the amplitude of external applied field. For the external-field amplitude larger than the threshold value, on the other hand, the AC losses sharply increased due to the effect of the dynamic resistance for the direct transport current.

Although the present study focused only on the perpendicular-field losses of the stacked coated conductors constituting a pancake coil for SMES, the theoretical expressions derived for the infinite slab would be applicable to the evaluation of the parallel-field losses of each individual coated conductor. Therefore, the obtained results

indicate that the total AC losses in the SMES coils for load fluctuation compensation are significantly enhanced by the component of direct transport current. The actual electromagnetic configuration including the direct transport current should be taken into account in order to suppress the total AC losses of windings with the coated conductors and realize an efficient SMES system.

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## References

- [1] M. Inoue, T. Kiss, T. Kuga, M. Ishimaru, M. Takeo, T. Matsushita, Y. Iijima, K. Kakimoto, T. Saitoh, S. Awaji, K. Watanabe, Y. Shiohara, *Physica C* 392–396 (2003) 1078.
- [2] H. London, *Phys. Lett.* 6 (1963) 162.
- [3] W.J. Carr, Jr., *IEEE Trans. Magn.* 15 (1979) 240.
- [4] T. Ogasawara, Y. Takahashi, K. Kanbara, Y. Kubota, K. Yasohama, K. Yasukochi, *Cryogenics* 19 (1979) 736.
- [5] K. Funaki, M. Iwakuma, K. Kajikawa, M. Takeo, J. Suehiro, M. Hara, K. Yamafuji, M. Konno, Y. Kasagawa, K. Okubo, Y. Yasukawa, S. Nose, M. Ueyama, K. Hayashi, K. Sato, *Cryogenics* 38 (1998) 211.
- [6] K. Kawasaki, K. Kajikawa, M. Iwakuma, K. Funaki, *Physica C* 357–360 (2001) 1205.
- [7] M.R. Halse, *J. Phys. D Appl. Phys.* 3 (1970) 717.
- [8] E.H. Brandt, M. Indenbom, *Phys. Rev. B* 48 (1993) 12893.
- [9] E. Zeldov, J.R. Clem, M. McElfresh, M. Darwin, *Phys. Rev. B* 49 (1994) 9802.
- [10] N. Schönborg, *J. Appl. Phys.* 90 (2001) 2930.
- [11] Y. Mawatari, *Phys. Rev. B* 54 (1996) 13215.
- [12] J.R. Clem, J.H. Claassen, Y. Mawatari, *Supercond. Sci. Technol.* 20 (2007) 1130.
- [13] K. Kajikawa, K. Funaki, K. Shikimachi, N. Hirano, S. Nagaya, *Physica C* 469 (2009) 1436.
- [14] K. Higashikawa, T. Nakamura, M. Sugano, K. Shikimachi, N. Hirano, S. Nagaya, *IEEE Trans. Appl. Supercond.* 18 (2008) 758.
- [15] K. Shikimachi, N. Hirano, S. Nagaya, H. Kawashima, K. Higashikawa, T.

Nakamura, IEEE Trans. Appl. Supercond. 19 (2009) 2012.

[16] E. Pardo, Supercond. Sci. Technol. 21 (2008) 065014.

[17] K. Kajikawa, Y. Mawatari, Y. Iiyama, T. Hayashi, K. Enpuku, K. Funaki, M. Furuse, S. Fuchino, Physica C 445–448 (2006) 1058.

[18] C.P. Bean, Phys. Rev. Lett. 8 (1962) 250.

[19] K. Kajikawa, T. Hayashi, R. Yoshida, M. Iwakuma, K. Funaki, IEEE Trans. Appl. Supercond. 13 (2003) 3630.

**Figure captions:**

Fig. 1 A typical time evolution of transport current  $I_t$  or applied magnetic field  $H_e$  in winding for SMES.

Fig. 2 Schematic diagram of cross-sectional configurations for (a) single strip, (b) infinite number of strips spaced equally and (c) its slab approximation.

Fig. 3 AC loss properties for simultaneous applications of direct transport current and external AC magnetic field. (a) represents the numerical results in single strip and (b) is the theoretical results in an infinite slab [4].

Fig. 4 Numerical results of AC losses in single strip for simultaneous applications of transport current and external AC magnetic field. (a) has no component of direct transport current, and (b) is for both the direct and alternating transport currents.

Fig. 5 Theoretical results of AC losses in an infinite slab for simultaneous applications of transport current and external AC magnetic field. (a) has no component of direct transport current, and (b) is for both the direct and alternating transport currents.

















