A Review on Recent Development of Personal Income Distribution

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Yan He†

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1. Background

The distribution of income has been a central concern in academics since 1990s. The history of study on income distribution follows two mainstreams, that is, the functional distribution of income and the measurement of inequality in income (Dagum, 1999).

The functional distribution of income, which deals with the income distribution among the owner of the factors of production and the price determination of each productive factor, stems from Ricardo (1815, 1817). It accounts for the factor prices formation (e.g. rent, wage and profit) and the share that the corresponding factors of production (e.g. land, labor and capital) have in national income.

The measurement of inequality in income, which is called the size or personal distribution of income, stems from Pareto (1895, 1896, 1897) and studies the shape of income distribution and measurement of inequality. It is mainly concerned with the size distribution of income among individual, household and other unites. The total income received by each economic unit is considered, regardless of the factors of production that contributed to its amount or the income components (e.g. wages, investment income).

The first topic has been focused on for a long time. However, the present concerns are

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concentrated on the size or personal distribution of income. Hence, this paper aims to offer a review on recent study of personal distribution of income. The rest of this paper is arranged as follows. Section 2 recalls the stem of personal income distribution theories. Section 3 represents the main contributions to the development of personal income distribution. Section 4 relates the measurement of personal income distribution. Section 5 gives some suggestions on choosing model. Section 6 displays the future development in this field.

2. Stem of Personal Income Distribution Theories

For explaining the size of income distribution, the various theories have emerged from two schools of thought. The first is called the theoretic statistical school proposed by Gibrat (1931), Roy (1950), Champernowne (1953), Aitchison and Brown (1954), Rutherford (1955) and so on, which is known as stochastic models and contributes to explain the generation of income. The partial explanation of income generation process and the lack of highlighting on economics of the distribution process make stochastic models critical.

To overcome the shortcoming of stochastic models, the second school is called the social-economic school, which contributes to seek the explanation of income distribution with the help of economic and institutional of income distribution. This school has three groups of authors. Firstly, the approach initiated by Mincer (1958) followed with human capital path on the basis of the hypothesis of lifetime income maximization. Subsequently, this theory developed by Becker (1962, 1967), Chiswick (1968, 1971, 1974), Husen (1968) and De Wolf and Van Slijpe (1972). However, human capital approach mainly deal with the supply side of market which provides labor of various levels of education and cannot deal with the demand side of market. Secondly, to solve the above problem, education planning school by Tinbergen represented by Bowles (1969), Dougherty (1971, 1972) and Psacharopoulos and Hinchliffe (1972) emerged. This group denotes that demand for various kinds of labor is derived from production function. Finally, the supply and demand school, which considers income distribution as a result of the supply and demand for different kinds of labor, is suggested by Tinbergen (1975). This theory applies not only to labor income but also to incomes from other factors of production.

3. Development of Models

The present study of personal distribution focuses on several domains: income distribution function; measurement of degree of income inequality and poverty; government policies affecting personal distribution of income. Major concern for academics and social reformers is the statistical measurement of inequality and analysis of income distribution since the phenomenon of income inequality has been a source of worldwide social upheaval. Since Pareto (1895, 1896, 1897) started to explore the field of income distribution and proposed famous models known as the first, second and third Pareto laws with the assumption of "normal distribution" or "the Gaussian distribution", a successive surge of literature on probability functions appeared. A variety of probability functions were proposed as suitable in describing the distribution of income by size using both a combination of known statistical distributions (Nirei and Souma, 2004; Clementi and Gallegati, 2005) and parametric functional forms for the distribution of income as a whole. These functional forms can be grouped in the following three main categories by Dagum (1977).

3.1 Stochastic Process

The first category is functional forms proposed to describe the generation of income distribution by the means of a stochastic process. A statistical description of income distribution was defined by Aichinson and Brown (1954) as: Given for each value of income x, the proportion $F(x)$ of persons in a given population who have an income not greater than x, where $F(x)$ should be given a precise mathematical expression involving known or unknown parameters, including four criteria, that is, the economic meaning of the estimated parameters of the description; the relationship between theoretical description and practical evidence; the tool employed to analyze the data; the consistence between statistical description shown to rest on assumptions and common knowledge if the way in which incomes are generated.

This kind of model was initiated by Kapetyn (1903) who attempted to be at once a popular presentation of statistical methods and a mathematical derivation of a new theory regarding skew frequency curves. Gibrat (1931)[1] was the first to popularize the idea of stochastic process with the contribution of showing that the distribution of the logarithms of some economic variates, such as the firm size distribution (FSD), is approximately normal, which is known as lognormal distribution or Galton distribution by academics. The following work was done by Kalecki (1945) to question the implications and underlying assumptions[2] of Galton distribution and modify the model. To examine applicability of the lognormal model, Aichinson and Brown (1969) found that the lognormal distribution fit the whole range of income distribution but is quite poor in describing both the upper and lower tails of the actual distribution. By studying 12

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1) Kleiber (2007, p. 1) states that "He looked for a model accommodating the heavy tails present in empirical income and wealth distributions as well as permitting an interior mode. The former aspect is well captured by the Pareto but not by the lognormal distribution, the latter by the lognormal but not the Pareto distribution."

2) Kalecki (1945, p.162) states that "the argument in Gabrit (1931) implies that as time goes by the standard deviation of the logarithm of the variate considered increases continuously. In the case of many economic phenomena, however, no tendency for such an increase is apparent (for instance in distribution of incomes). And also for a priori reasons it is clear that changes in the standard deviation of the logarithm of a given variate are to a great extent determined by economic forces."

examples, Roy (1950) demonstrates that the evidence is favor of the log-normal distribution rather than normal distribution judging by the criterion of humpedness or asymmetry. Rutherford (1955) suggested a modified lognormal which provides a better fit than the lognormal, however, this does not appear to give so good an approximation to observed distributions as Champernowne's distribution, particularly in the tails. Simon (1955) discusses a number of related stochastic processes that lead to a class of highly skewed distributions (the Yule distribution) possessing characteristic properties that distinguish them from such well-known functions as the negative binomial and Fisher's logarithmic series. Mandelbrot (1959, 1961) modifies the Simon's 1955 model for the Pareto-Yule-Zipf distributions.

To overcome the assumption of constant stochastic matrix through time, Champernowne (1953) developed models which can deal with enumerable infinity of income ranges by applying Markov chains following Solow (1951). Fisk (1961) suggests a special case of Champernowne's distribution function known as Log-logistic distribution or Fisk distribution, which may prove useful when examining distributions of incomes which are homogeneous. Following the structure of Arnold and Laguna (1977) calling a Feller-Pareto distribution, Arnold, Robertson and Yeh (1986) provided a characterization of the Fisk distribution known as a Preto (III) distribution. Recently, Zandonatti (2001) suggested a "generalized" Fisk distribution employing the procedure leading to Stoppa's generalized Pareto distribution. Empirical study by Fisk (1961a) concluded that the Fisk distribution may prove useful when income distributions that are homogeneous in at least one characteristic are examined.

A successive surge of research, the field has been stimulated by studies on a reflected a multiplicative process (Levy and Solomon, 1996; Manrubia and Zanette, 1999; Gabaix, 1999) or a closely related Kesten process (Sornette and Cont, 1997; Takayasu *et al.*, 1997) which have revealed the effect of a reflective lower bound on the tail of the stationary distribution. Wagner (1977) builds a CHANCE-model which is a special type of a stochastic process which serves as descriptive models. However, stochastic models have been critized for not providing an explanation for distribution processes in terms of economic variables. Empirical contributions on this category include Gabaix (1999), Levy (2003), Feenberg and Poterba (1992) and Souma (2002).

3.2 Goodness of Fit

The second type is functional forms proposed solely by the practical bearing upon the encountered empirical distributions on the grounds of presenting a satisfactory goodness of fit. The Beta distribution (three-parameter model) was proposed by Thurow (1970) and by Kakwani and Podder (1976) is very versatile. The following works on extending the beta distribution offered three non-nested generalizations including the Gauss hypergeometric distribution (Armero and Bayarri, 1994), the generalized beta distribution (McDonald and Xu, 1995), and the confluent hypergeometric distribution (Gordy, 1998). The most well-known in the literature is the generalized beta distribution (GB, five-parameter distribution) encompassing both the first (GB1) and second (GB2) kind. Relatively, the generalized beta distribution of the first kind (GB1), for which Johnson *et al.* (1995) and Gupta and Nadarajah (2004) offered detailed basic properties, is more flexible. However, Empirical studies appear to suggest that GB1 or B1 did not provide a better fit, see e.g. Brachmann *et al.* (1996) estimated German income data using both the B1 and GB1 dsitributions and concluded that both models tend to underestimate the mean for the data because the ML estimation of the GB1 proved to be rather difficult since the gradient of the log likelihood in the parameter b was rather small. Nadarajah and Kotz (2006) contributed to GB2 distribution. McDonald (1984) fit the models including gamma, beta, Singh-Maddala (or Burr), Pareto, Weibull, and generalized beta of first and second kinds to the income data and concluded that the generalized beta of the second type provided the best relative fit and that the Singh-Maddala (SM) distribution provided a better fit than the generalized beta of the first kind.

The Gamma model proposed by Ammon (1895), which was applied to fit income data by March (1898) and further promoted by the work from Salem and Mount (1974) and Bordley *et al.* (1996) for USA and Bartels and van Metelen (1975) for the Netherlands showing that empirical evidence favors the Gamma over the lognormal distribution. The generalized Gamma distribution deduced by Amoroso (1924-1925) contains the special case of the Gamma and Pearson Type V distributions. The models proposed by Amoroso and Vinci partly overlap with the first category. An outstanding synthesis by D'Addario (1949) specified a differential equation which includes the particular cases among the Pareto, the Lognormal, the Gamma and the Pearson Type V models. In the study of Brachman *et al.* (1996) shows that the gamma distribution emerges as the best two-parameter model by utilizing German household incomes data.

The Lomax distribution, originally proposed as Pareto II distribution by Lomax (1954), is a further two-parameter special case of GB2 distribution. A generalization of the Lomax distribution was recently suggested by Zandonatti (2001) following the approach proposed by Stoppa (1990a, b) leading to a generalized Pareto I distribution. Empirical work, such as Hogg and Klugman (1983) fit the Lomax distribution on malpractice losses, shows which is preferable over lognormal, Weibull, Singh-Maddala and the beta II distributions. In recent investigation, Burnecki *et al.* (2000) obtained a tail index q in the vicinity of 2.7 by applying a Pareto type II. However, on the basis of application on two sets of liability data, Cummins *et al.* (1990) debates the performance of Lomax distribution is not impressive since it ranks only 12^{th} and 13^{th} out of 16 distributions of gamma and beta type.

Following the work of Pareto (1896, 1897), Benini (1897) confirmed the Pareto law holds for incomes as well as various other economic variables. Subsequently, Benini (1905-1906) suggested a distribution function known as Benini distribution, size-of-loss distribution (Head, 1968), approximate lognormal distribution (Dumouchel and Olshen, 1975), or the quasi-lognormal distribution (Shpilberg, 1977). By fitting the model to UK fire losses, it is preferable to Pareto distribution (Ramachandran, 1969). Later, Turroni (1914) and Mortara (1917, 1949) employed the idea of Benini distribution and presented higher-order terms. Winkler (1950) empirically suggested that a higher-order term may provide an even better fit that Pareto and original Benini distributions.

A number of size distributions, which may not be in the mainstream of current research, definitely offer potential applications. In this category are included the Pearson Type V distribution proposed by Vinci (1921); the hyperbolic distribution proposed by Champernowne (1952); the Weibull distribution proposed by Bartels and van Metelen (1975); the log t (where t is the Student distribution) studies by Kloek and van Dijk (1976); Paralogistic distribution proposed by Klugman et al. (1998); Davis distributions proposed by Davis (1941) which is not merely the upper tail of the distribution of income named. Moreover, Benktander (1970) discussed two new loss models, where one is following from lognormal distribution (Watson and Wells, 1961) and Weibull distribution for the other one (Beirlant and Teugels, 1992).

3.3 Specification

Specification of differential equations that purport to capture the characteristics of regularity and permanence observed in the empirical distributions of income. The functional form is the solution of the corresponding differential equation. The contribution of this field is initiated by Pareto (1896). The apparent attractions of the Pareto distribution evaporate somewhat when one considers its implications for the distribution of income amongst the population as a whole (Clementi *et al.*, 2008). However, empirical studies showed that the Pareto distribution accurately models only high levels of income, but do a poor job in describing the lower end of the distribution since the income distribution is right-skewed and has a fat right-hand tail.

To find a satisfactory parametric form which can be fitted income distribution better, Singh and Maddala (1976) presented a celebrated model and paid impressive attention on academic. The Singh-Maddala distribution is also known as the Pareto IV distribution (Arnold, 1983), Burr VII distribution (Hogg and Klugman, 1983, 1984), the beta-P distribution (Mielke and Johnson, 1974) or as a generalized log-logistic distribution (El-Saidi *et al.*, 1990). Empirical works have been done to test if the model can be suitable in describing family incomes. For U.S. case, comparing with the gamma distribution employed by Salem and Mount's (1974), Singh and Maddala (1976) shows that Singh-Maddala model provides a better fit than either lognormal or the gamma functions. Dagum (1983b) also gives the same comments. Mcdonald and Ransom (1979a) represents that Singh-Maddala distribution generally outperforms the logmormal, gamma, and beta type I models, with only the beta type I being slightly better in a few cases. However, Cronin (1979) questioned the conclusion of Singh and Maddala (1976) since the implied Gini indices almost always fall outside the Gastwirth (1972) bounds calculated by Salem and Mount (1974) for their data. Furthermore, MacDonald (1984) states that Singh-Maddala distribution ranks second out of 11 considered models being inferior only to GB2 distribution.

Successively, Dagum (1975) experimented with a shifted log-logistic distribution which is a generalization of a distribution previously considered by Fisk (1961) and represented a three-parameter specification (the Dagum type I distribution). This type of distribution can be derived as a special case of the Generalized Beta II (GB2) distribution (a generalization of the Beta prime distribution) with the shape parameter p=1. Undert certain assumptions on its infinitesimal mean and variance, and Mielke (1973), Fattorini and Lemmi (1979) and later Dagum and Lemmi (1989) arrived at Dagum distribution as the equilibrium distribution of continuous-tinme stochastic process, independently.

Following McDonald (1984) and a Dagum Type I distribution over the positive halfline, Dagum (1977, 1980) introduces two further variants of four-parameter generalizations (the Dagum type II distribution) by adding a point mass at the origin and refers to his system as the generalized logistic-Burr system, which was proposed as a model for income distributions with null and negative incomes. This approach was further developed in a series of papers on generating systems for income distributions (Dagum 1980b, 1980c, 1983, 1990). To seek an appropriate representation of sample income distribution of total income receiver, Dagum (1983a) proposed a Type III distribution starting accumulating income of a populayion with initial positive earnings. Dagum distribution is also known as the inverse burr distribution (Klugman, Panjer, and Willmot, 1998), the three-parameter kappa distribution (Mielke, 1973), beta-K distribution (Mielke and Johnson, 1974), income distribution (Fattotini and Lemmi, 1979; Lemmi, 1987), Burr III distribution (Burr, 1942) and Dagum distribution in the income distribution literature (see as, Kleiber, 2007).

To investigate model flexibility to observed family income data, Dagum (1977, 1980a) applied his models (Type II) to U.S. family incomes and concluded that the model outperforms the lognormal, gamma and the Singh-Maddala distributions. In the following work, Dagum (1983) applied his types I and III to U.S. family income data and stated that the performance is as well as the lognormal, gamma and the Singh-Maddala distributions. By fitting French data, Espinguet and Terraza (1983) verified that Dagum type II distribution is superior to the Weibull, Singh-Maddala, the Box-Cox-transformed logistic and three-parameter lognormal distributions, as well as a four-parameter beta type I model.

Dagum type I distribution is improved upon all other two- and three-parameter models by Majumder and Chakravarty (1990) modeling U.S. income data. The conclusions are also confirmed by McDonald and Mantrala (1993, 1995). Only four-parameter GB2 and a five-parameter generalized beta distribution outperformed the Dagum type I (MacDonald and Xu, 1995). Studying on UK household income case shows a better fit of Dagum Type I distribution which is preferred over the gamma distribution. Bantilan *et al.* (1995) fitting the Dagum type I distribution to Philippines family income data found that the model fits the data very well, particularly in tails. Dagum type I distribution is turned to be the best three-parameter model for all data, outperforming three- and four-parameter models (such as the generalized gamma and GB1 distributions) and being inferior only to the GB2 distribution (Bordley *et al.*, 1996).

Dagum and Lemmi (1989) achieved a quite satisfactory fit in general by empirically studying Italian income data using Dagum type I-III distributions. Successively, similar work done by Botargues and Petrecolla (1997, 1999a, b) for the case of Buenos Aires region, Dagum type I-III distributions were proved to outperform the lognormal and Singh-Maddala distribution. Furthermore, the log-Gompertz distribution appears to be used mainly in income and size distributions and was noticed by Dagum (1980), which has been proved to be an excellent two-parameter model by Cummins et al. (1990)[3].

4. The Measurement of Size Distribution

The measurement of personal income distribution is also main concern in the literature. Various kinds of index have been employed to analyze income inequality including generalized entropy index, Atkinson index, Gini coefficient, Hoover index, Theil index, Income inequality metrics, Suits index, Wealth condensation, Diversity index and so on. Among the "satisfactory measures", the Gini coefficient proposed by Gini (1914), the two Theil indexes (the Theil income-weighted and the Theil population-weighted) proposed by Theil (1967) and the Atkinsons (1970) index have been the more widely recommended. Each one emphasizes in a different way the income changes at various points in the income distribution. Consequently, the picture provided by these inequality indexes can be not coincident.

However, more popular, Theil index and Gini coefficient are wildely used in the literature. The Gini coefficient is more sensitive to the income changes occurred at the middle of the income distribution, treating symmetrically the lower and the upper tails of the incomes ranking. On the other hand, the Theil population-weighted index is more sensitive to the transfers occurring at the bottom of the income distribution. The Theil income-weighted index is, however, less sensitive to the lowest observations than the previous index (Duro, 2004).

Comparing to Gini coefficient, there are some advantages of Theil index such as the solution of income inequality is sole; it is easier decomposable than the Gini Coefficient; it is a weighted average of inequality within subgroups, plus inequality among those subgroups. However, the most widely used tool for measuring inequality is the Gini coefficient, for which is more intuitive

3) Some contents of this part referred to Kleiber, Christian; Kotz, Samuel (2003). "Chapter 7.1: Benini Distribution". Statistical Size Distributions in Economics and Actuarial Sciences. Wiley. ISBN 978-0-471-15064-0.

since it is based on the Lorenz curve (Lorenz 1905; Kleiber, 2008). Traditional calculation of Gini coefficient exist several problems, that is, it is hard to define and interpret surplus term R (Mookherjee and Shorrocks, 1982), although many scholars tried to give reasonable interpretation for R (Siber, 1989; Yitzhaki and Lerman, 1991; Lambert and Aronson, 1993). Another debate of Gini coefficient is that the subgroups decompositions have variable types on weight, such as Rao (1969) demonstrates weights should be population share of subgroup, while Mangahas (1975) think it is proper to use income share of subgroup for weights. Moreover, the Gini index only reflects some aspects of the underlying income distribution: A large amount of information is lost. That is, Two Lorenz curves with the same Gini value may have different shapes. Thus, welfare implication from comparing Gini coefficients (or other summary statistics) may be ambiguous (Wu and Perloff, 2004).

Due to the indices are directly related to the proposed models, such as The Pietra index and the (first) Theil coefficient are associated with the B1 distribution (McDonald, 1981; Pham-Gia and Turkkan, 1992). Using some aspect of the general statistical concept of a probability distribution, the specification forms of distribution models offer a convenient path to analyze and visualize income inequality by the means of the Lorenz curve and correspondingly calculate Gini coefficient accuracy.

5. A Judgment to Choose Suitable Model

The problem of choosing a model for describing the distribution of income over a population has been frequently considered in the literature. To applying to fit personal income data, the models most frequently chosen are the classical distributions including Pareto, the lognormal, the beta family, the Gamma, Singh-Maddala and Dagum distribution. The Pareto model is proved to be the best one to describe high income groups with the limitation of only being useful in describing the lo tail of the distribution according to the criterion of functional simplicity, goodness of fit and the economic interpretation of its parameters. The lognormal and the Gamma fit the whole range of income distributions but are poor in describing both the upper and lower tails of the actual distribution. Judging by goodness of fit, empirical evidence, such as Salem and Mount (1974) and Bartels and van Metelen (1975), favors the Gamma over lognormal distribution. Comparing with gamma, beta, Singh-Maddala (or Burr), Pareto, Weibull, and generalized beta of first and second kinds, the generalized beta of the second type provided the best relative fit (McDonald, 1984).

Not satisfied with the classical distributions used to summarize empirical income and wealth distributions, Singh and Maddala (1976) and Dagum (1977a) devoted to look for a model accommodating the heavy tails present in empirical income and wealth distributions as well as permitting an interior mode. Singh-Maddala distribution and Dagum type I distribution are closely related models (Rodriguez, 1983; Tadikamalla, 1980) since both distributions allow for various degrees of positive skewness and leptokurtosis, and even for a considerable degree of negative skewness.

As far as the goodness of fit is concerned, the model deduced by Singh and Maddala (1976), more widely known Singh-Maddala distribution, outperforms both the lognormal and the Gamma distributions. Singh-Maddala (SM) distribution also provided a better fit than the generalized beta of the first kind (McDonald, 1984). The new model of person income distribution proposed by (Dagum, 1977) is preferable to the Lognormal, the Gamma, and the Singh-Maddala models in applications to income data. However, Dagum distribution is less widely known due to the language barrier. However, in recent years there are indications that the Dagum distribution is proved to be a more appropriate choice in many applications.

For better visualizing the relationship among various models, Bandourian *et al.* (2002) offered a convenient way by drawing a relationship tree in figure 1, where GB denotes the generalized beta model and GG denotes the generalized Gamma model.

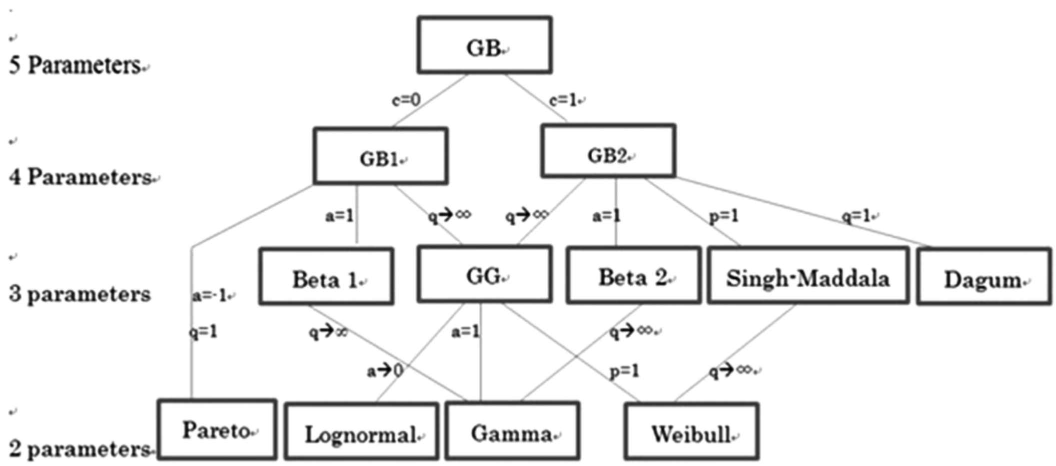

Figure 1　Relationship Tree among Various Models

6. Prospect

In summary, it would seem that most data on size distribution require a more flexible distribution. Although there have been a number of models proposed to fit personal income distribution, which Ord (1975) and Dagum (1983b) offers a summary, none seems to have provided much of an improvement in terms of explanatory power. More recent works, taking Bordley *et al.* (1996) and Dagum (1996) for example, have eschewed process modeling. Instead, they focus on finding alternative parametric distributions which provide a good fit to wide range of observed data (Kitov, undated). Exceptions are the studies of Parker (1999) and Solomom and Richmond (2001).

On the methodological side, there are still some unresolved issues including aspects of likelihood inference (Chotikapanich, 2008). When the distribution celebrates its golden jubilee in economics, these problems no doubt will be solved. Further studies should conclude other relevant social and economic characteristics of the population, such as education, sex and race in order to explain more accurately the movements in the mass of the personal income distribution of individuals.

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