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# Energy channeling from energetic particles to bulk ions via beam driven geodesic acoustic mode -GAM channeling-

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**Abstract.** A novel mechanism for energy transfer from energetic particles to bulk ions via geodesic acoustic modes (GAMs) is presented. The mechanism involves the excitation of GAMs by energetic particles. The GAMs are damped on ions by the Landau damping, by which wave energy is given to bulk ions. This process of the energy exchange is formulated within the framework of the quasilinear theory. The rate of energy transfer from energetic particles to GAMs can be comparable to that of energy exchange to particles via collisions, if the  $E \times B$  velocity of GAMs reaches the level of the diamagnetic velocity. Under this circumstance, the partition of energy absorptions by bulk ions and electrons is substantially modified due to the selective ion heating by GAMs.

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#### 1. Introduction

An efficient process of transferring energy from energetic ions to bulk ions has been investigated for the nuclear fusion in magnetic confined plasmas. If one takes an example of the neutral beam injection (NBI) heating, the energy of energetic particles is likely to be transferred to electrons through the collision, as the energy of energetic particles becomes higher [1]. It is difficult to heat bulk ions via collision by using energetic particles. This casts a problem for the heating of main ions by fusion-generated alpha particles.

The energy channeling from energetic particles to waves has been studied in order to explore an alternative path of energy exchange from energetic particles to main ions. The energy transfer from energetic particles to main ions by using waves, which are driven externally, has been reported. The concept of this channeling effect has been explored in [2], which is known as alpha channeling. It is known that the ion cyclotron range of frequencies (ICRF) wave is excited during NBI heating [3], which is thought to be an instability in the range of ion cyclotron frequencies [4]. The injection of energy and momentum from beam ions to main ions via ICRF (the seed of which is driven externally, and is amplified by beam ions) has been proposed. This mechanism has been called ICRF catalyst [5]. The energy exchange between energetic particles to waves, which are driven spontaneously in plasmas, has also been investigated. It has been reported that the compressional Alfven waves are excited by wave-particle interaction, and the waves heat main ions by stochastic diffusion in the velocity space [6]. This mechanism is useful under the situation that the velocity of beam ions is comparable to the Alfven velocity. Methods of energy channeling that can be used in wider circumstances for toroidal plasmas have been required.

Recent studies have shown that geodesic acoustic modes (GAMs) [7] can be excited by energetic ions introduced by NBI. The GAMs, which are considered to be driven by beam ions, are observed experimentally [8]-[10], and their theoretical studies have been started [11]-[13]. There are many studies about the linear properties of GAMs [14]-[17], and the Landau damping rate by bulk ions has been studied in detail [14]. Based on the two processes, i.e., the excitation of GAMs by energetic particles and the absorption by bulk ions, we propose here the possibility of energy channeling from beam ions to bulk ions via GAMs.

GAMs can be a new energy channeling from energetic particles to bulk ions, that is, the energetic particles of NBI drive GAMs, and the GAMs heat bulk ions through the Landau damping. The wave channeling of energy transfer is called GAM channeling in this article. We evaluate the rate of energy exchange between beam ions and bulk ions within a framework of quasilinear theory, in which the amplitude of GAMs is treated as a given parameter. The rate of energy exchange by GAM channeling is compared to that by collisional process.

#### 2. Model

The GAM channeling is formulated within the framework of the quasilinear theory. In analyzing the GAM channeling, two energy paths must be calculated; 1) the path from energetic particles to GAMs, and 2) the path from GAMs to bulk ions. In a simple model

of high aspect ratio tokamak with circular cross-section, the energy density of GAMs,  $W_G$ , can be written as

$$W_G = \frac{1}{2} m_i n_i \frac{q_r^2 |\phi_G|^2}{R^2},\tag{1}$$

where B is the strength of the magnetic field,  $m_i$  is the ion mass,  $n_i$  is the ion density. The electrostatic potential and the radial wavenumber of GAMs are denoted by  $\phi_G$  and  $q_r$ , respectively. Here, the electrostatic potential and wavenumber of GAMs are given as parameters. We formulate the problem in two steps. First, the rate of GAM excitation is evaluated (in which the damping by bulk ions are not taken into account [11]-[13]). Then the Landau damping of the GAMs by bulk ions is calculated. By this approach, the rate of energy transfer is estimated.

First, the energy path from energetic particles to GAMs is analyzed. The rate of energy transfer from energetic particles to GAMs per unit time and unit volume,  $P_{h\to G}$ , can be written, in terms of the excitation rate of GAMs due to energetic particles, as.

$$P_{h\to G} = 2\gamma_h W_G. \tag{2}$$

The excitation rate  $\gamma_h$  has been studied in literatures [11]-[13]. In the model of [12], following assumptions are introduced. The velocity of energetic particles |u| is mono-energetic as  $|u| = u_0$ , and the effect of pitch angle distribution is considered. The pitch angle is defined as  $\Lambda = u_{\parallel}/u_0$ , where  $u_{\parallel}$  is the velocity parallel to magnetic field, and its distribution is given as a quadratic function (the peak angle is denoted by  $\Lambda = \Lambda_0$ , and the width is given by  $\Delta\Lambda$ ). In addition, the loss boundary for the pitch angle is considered, which limits the pitch angle distribution as  $\Lambda_c < \Lambda$ . The dispersion relation was derived, neglecting the Landau damping and the spatial gradient of energetic particle density, as

$$D(\sigma) = 1 - \frac{\omega_G^2}{\omega_b^2} \left( 1 + \sigma \frac{\Delta \Lambda}{\Lambda_0} \right)^{-2} + \frac{3q^2 n_h}{16n_i \Delta \Lambda} \left( \frac{1 + \Lambda_0^2}{\Lambda_0} \right)^2 \left( 2 + \sigma \frac{\Delta \Lambda}{\Lambda_0} \right)^{-1} \times \left\{ \frac{1 - \delta \Lambda_c^2}{2(\sigma + \delta \Lambda_c)} + 1 + \delta \Lambda_c + \sigma \ln \left( \frac{1 - \sigma}{-\delta \Lambda_c - \sigma} \right) \right\}.$$
(3)

(The assumption of neglecting the Landau damping is shown valid, a posteriori, in wide range of parameters, as is illustrated in Fig. 1.) Here q is the safety factor, and  $\omega_G$  is the GAM frequency as  $\omega_G = \sqrt{2/m_i R^2} (7T_i/4 + T_e)^{1/2}$ , where R is the major radius, and  $T_e$  and  $T_i$  are the electron and ion temperature, respectively. The transit frequency of energetic particles at the center of the distribution is denoted as  $\omega_b = u_0 \Lambda_0/qR$ . The density of energetic particles is written as  $n_h$ , and  $\delta \Lambda_c$  is defined as  $\delta \Lambda_c = (\Lambda_0 - \Lambda_c)/\Delta \Lambda$ . The eigenvalue,  $\sigma$ , is defined as  $\sigma = (\omega - \omega_b)\Lambda_0/\omega_b\Delta \Lambda$ , where  $\omega = \omega_r + i\gamma_h$  is the frequency of GAMs which is modified by energetic particles. Equation (3) is an extension of that in [12], whose detail is explained in Appendix A. The excitation rate of GAMs by energetic particles is determined not only by the plasma geometry such as the safety factor and the major radius, but also by the velocity distribution of energetic particles. This instability is the velocity space instability, and the gradient of the velocity distribution of the energetic particles destabilizes GAMs. We note here that the dispersion relation Eq. (3) shows two possibilities of instability. When the condition  $\omega_G < \omega_b$  holds, the GAM branch (i.e., the branch which reduces to  $\omega_r \rightarrow \omega_G$  as

 $n_h/n_i \to 0$ ) becomes unstable. When  $\omega_G > \omega_b$  is satisfied, the beam branch whose frequency is close to  $\omega_b$  becomes unstable. (The latter case is studied in [12] in detail.) In this article, we focus on the GAM branch. Namely, we consider the case where the energy of energetic particles,  $E_h$ , is in the range of  $E_h/T_i > q \sqrt{7/4 + T_e/T_i}/\Lambda_0$ .

Next, the energy transfer rate from GAMs to bulk ions per unit time and unit volume,  $P_{G\rightarrow i}$ , are discussed. This term can be expressed as

$$P_{G \to i} = 2\gamma_G W_G. \tag{4}$$

Here  $\gamma_G$  denotes the damping rate of GAMs due to the Landau damping. Once the wave frequency is given, the rate of the Landau damping by bulk ions is calculated. In order to keep the analytic transparency of the argument, we use an approximation that the unstable mode satisfies the relation  $\omega \sim \omega_G$ . This is allowed in the case where the transit frequency of energetic particles  $\omega_b$  is larger than the GAM frequency  $\omega_G$ . It is pointed out higher order correction with respect to  $q_r^2 \rho_i^2$  can have a substantial value, where  $\rho_i$  is the ion gyroradius measured by ion thermal velocity [14]. However, as a first step of the study of GAM channeling, we here consider the first order correction which is introduced by the frequency deviation from GAM frequency. By use of this simplification, the Landau damping by bulk ions is calculated as [14]

$$\gamma_{G}(\omega) = \frac{\sqrt{\pi}}{2R} \sqrt{\frac{T_{i}}{m_{i}}} e^{-\hat{\Omega}_{G}^{2}} \left[ \left\{ \hat{\Omega}_{G}^{4} + \left( 1 + 2\frac{T_{e}}{T_{i}} \right) \hat{\Omega}_{G}^{2} \right\} + 2 \left\{ -\hat{\Omega}_{G}^{5} + \left( 1 - 2\frac{T_{e}}{T_{i}} \right) \hat{\Omega}_{G}^{3} + \left( 1 + 2\frac{T_{e}}{T_{i}} \right) \hat{\Omega}_{G} \right\} \left( \omega q R \sqrt{\frac{m_{i}}{T_{i}}} - \hat{\Omega}_{G} \right) \right], \quad (5)$$

where  $\hat{\Omega}_G$  is defined as  $\hat{\Omega}_G = q \sqrt{7/4 + T_e/T_i}$ . The energy transfer from GAMs to bulk ions is determined not only by the safety factor, the electron and ion temperatures, but also by the energy of energetic particles whose effect is introduced through the frequency shift correction (i. e., the last parenthesis in the RHS of Eq. (5)). Here, we note that the higher order Landau damping which is induced by the poloidal harmonics of the eigenfunction must be taken into account in future when one improves the numerical accuracy of  $\gamma_G$ . The effect of energetic particles on the poloidal eigenfunction must be investigated. In this article, we focus on the order estimation of the energy transfer, so that the derivations of them are out of our scope.

#### 3. Comparison between GAM channeling and collisional energy exchange

The energy of energetic particles is transferred to ions and electrons via Coulomb collision, in addition to the process via GAMs excitation. The rate of energy transfer from energetic particles to ions is a decreasing function of energy of energetic particles, while the one from energetic particles to electrons is an increasing function. However, the existence of GAM channeling changes this situation as explained in this section.

In this section, we discuss the effect of GAM channeling. First, the effectiveness of GAM channeling is formulated by comparing the collisional energy exchange. Then, the evaluations of the effectiveness of GAM channeling are given.

#### 3.1. Formulation of the effectiveness of GAM channeling

The energy transfer rates from energetic particles to ions and to electrons, per unit time and unit volume, can be written as

$$P_{h\to i}^{(c)} = \frac{2^{1/2} n_i n_h Z^2 Z_h^2 e^4 m_h^{1/2} \ln \lambda}{8\pi \epsilon_0^2 m_i E_h^{1/2}},\tag{6}$$

$$P_{h\to e}^{(c)} = \frac{2^{1/2} n_e n_h Z_h^2 e^4 m_e^{1/2} \ln \lambda E_h}{6\pi^{3/2} \epsilon_0^2 m_h T_e^{3/2}},\tag{7}$$

where Z is the charge number of the ion,  $\ln \lambda$  is the Coulomb logarithm,  $n_e$  and  $m_e$  are the density and the mass of electrons,  $m_h$  and  $Z_h$  are the mass and the charge number of energetic particles, and  $E_h$  and  $n_h$  are the energy and the density of energetic particles.

In order to estimate the effectiveness of the energy channeling from energetic particles to GAMs, the rate of energy exchange to bulk ions via GAMs is compared to that via collisions. The energy flow from energetic particles to GAMs,  $P_{h\to G}$ , is normalized to the total collisional exchange rate,  $P_{h\to e}^{(c)} + P_{h\to e}^{(c)}$  of the energy transfer rate to bulk ions and electrons as follows,

$$\frac{P_{h\to G}}{P_{h\to e}^{(c)} + P_{h\to i}^{(c)}} = \frac{P_{h\to G}}{P_{h\to e}^{(c)}} \left( 1 + \frac{P_{h\to i}^{(c)}}{P_{h\to e}^{(c)}} \right)^{-1}.$$
 (8)

Here,  $P_{h\to i}^{(c)}/P_{h\to e}^{(c)}$  is a known function by Eqs. (6) and (7). The ratio,  $P_{h\to G}/P_{h\to e}^{(c)}$ , which is denoted as  $\Gamma_c$  in this article, represents the impact of energy exchange via GAMs. The analytical expression of  $\Gamma_c$  is given as

$$\Gamma_{c} = \frac{P_{h \to G}}{P_{h \to e}^{(c)}} 
= \frac{6\pi^{3/2} \epsilon_{0}^{2} m_{i} m_{h}}{2^{1/2} Z Z_{h}^{2} e^{4} m_{e}^{1/2} \ln \lambda} \frac{T_{e}^{3/2}}{n_{h} E_{h} B^{2}} q_{r}^{2} |\phi_{G}|^{2} \gamma_{h}.$$
(9)

Here,  $\Gamma_c$  is a decreasing function of the strength of the magnetic field.  $\Gamma_c$  is proportional to  $q_r^2 |\phi_G|^2$ , which is treated as a parameter here. In reality, the electrostatic potential of GAMs has to be determined by nonlinear theory, which is a future work.

Next, the effectiveness of the energy path from GAMs to ions is analyzed. The heating partition between ions and electrons is greatly modified due to the ion heating by GAMs. The heating partition between ions and electrons is defined by the ratio between total energy transfer to ions and electrons, which can be written as

$$\frac{P_{h\to i}^{(c)} + P_{G\to i}}{P_{h\to e}^{(c)}} = \frac{P_{h\to i}^{(c)}}{P_{h\to e}^{(c)}} \left(1 + \frac{P_{G\to i}}{P_{h\to i}^{(c)}}\right). \tag{10}$$

The ratio  $P_{G\rightarrow i}/P_{h\rightarrow i}^{(c)}$ , which is defined as  $\Gamma_h$ , represents the impact of GAMs on ion heating. The analytical expression of  $\Gamma_h$  is given as

$$\Gamma_{h} = \frac{P_{G \to i}}{P_{h \to i}^{(c)}} 
= \frac{8\pi \epsilon_{0}^{2} m_{i}^{2}}{2^{1/2} Z^{2} Z_{h}^{2} e^{4} m_{h}^{1/2} \ln \lambda} \frac{E_{h}^{1/2}}{n_{h} B^{2}} q_{r}^{2} |\phi_{G}|^{2} \gamma_{G}.$$
(11)

We can evaluate the effectiveness of GAM channeling by using Eqs. (9), (11), whose evaluations are given in the next subsection. The order estimation of GAM channeling is described in Appendix B.

#### 3.2. Evaluation of the effectiveness of GAM channeling

The evaluations of the effectiveness of GAM channeling, Eqs. (9), (11), are carried out by using following plasma parameters.

$$Z = 1, B = 1[T], R = 1[m], L_n = 0.1[m], T_e = 1[\text{keV}], n_i = 10^{20}[\text{m}^{-3}].$$

The parameters related to energetic particles are given as

$$Z_h = 1, E_h = 42 \text{[keV]}, n_h/n_i = 2.4 \text{[\%]}, \Lambda_0 = 0.5, \Delta \Lambda = 0.1, \delta \Lambda_c = 0.2.$$

The amplitude and the radial wavenumber of GAMs are chosen based on the observations reported in [8, 10, 18] as

$$|\phi_G| = \frac{T_e}{eq_r L_n}, q_r = 10[\text{m}^{-1}].$$
 (12)

Here, the amplitude of the GAM is determined by considering the case where the  $E \times B$  velocity of GAMs is equal to the diamagnetic drift velocity as  $v_G/v_d=1$ . The value of  $q_r$  is evaluated from the radial gradient of the envelope of the potential fluctuation [10], so that this value is lower limit in the case of [10]. In this situation, the ratio between collisional energy transfer to ions and to electrons is  $P_{h\to i}^{(c)}/P_{h\to e}^{(c)}=0.21$ . Since the ratio between transit frequency of energetic particles and GAM frequency is  $\omega_b/\omega_G=1.3>1$ , the GAM branch becomes unstable.

First, the energy transfer from energetic particles to the GAMs on the density of the energetic particles is evaluated. Figure 1 illustrates the growth rate and the damping rate of the GAM against the density of the energetic particles. The excitation condition,  $\gamma_h > \gamma_G$ , is satisfied so that the GAM is excited, and GAM channeling appears. The dependence of the effectiveness of the energy exchange from energetic particles to the GAM is shown in Fig. 2. The effectiveness  $\Gamma_c$  becomes unity in this case when  $v_G/v_d=1$ , which means that the rate of the energy transfer to GAMs can be comparable to the rate of the energy exchange to bulk particles through collision. Figure 3 shows the dependence of  $\Gamma_c$  on the energy of energetic particles. The parameter  $\Gamma_c$  decreases with the increase of the energy  $E_h$ . In order to investigate the accurate parameter dependence, the model in which the excitation rate and the Landau damping rate are included self-consistently is required, which is a future work.

Next, the effectiveness of the energy transfer from GAMs to ions is evaluated. Figure 4 illustrates the dependence of  $\Gamma_h$  on the density of energetic particles. The rate of energy transfer from GAMs to ions can be comparable to the rate of energy exchange to ions through collision if GAM amplitude reaches the level of  $v_G/v_d=1$ . GAMs have a substantial impact on ion heating. Therefore, the heating partition between ions and electrons is greatly modified.

#### 4. Summary

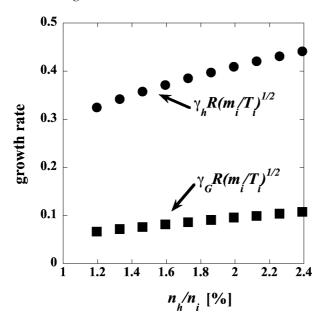
In summary, the energy transfer from energetic particles to bulk ions via beam driven GAMs, GAM channeling, is investigated. The channeling via excitation of GAMs is formulated within the framework of the quasilinear theory. The results are summarized as follows.

- (i) The energy transfer rate from energetic particles to GAMs can be comparable to the rate of energy exchange to particles via collision, if the amplitude of GAMs reaches the level of  $v_G \sim v_d$ .
- (ii) Under this circumstance, the partition of energy by bulk ions and electrons is substantially modified due to the selective ion heating by GAMs.

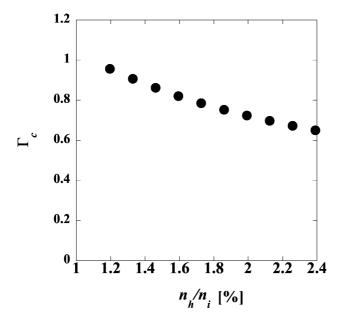
The mechanism of the GAM channeling has several advantages. First, GAMs are excited spontaneously in the presence of beam ions, therefore the extra excitation power is not required. Second, GAMs extract energy from energetic particles and transfer them to bulk ions, improving the energetic particle confinement. Third, the heating mechanism is robust, since there are many resonant particles whose velocity is  $v \sim \omega_G qR$ . Finally, GAMs are universal modes in toroidal plasmas, therefore GAM channeling can be useful for every toroidal plasmas. In order to improve the quantitative evaluation of the rate of energy transfer, we have to understand the poloidal eigenfunction of the GAMs more precisely, and its effect on the Landau damping must be taken into account. In addition, the amplitude of GAMs should be analyzed by nonlinear theory. These extensions will be carried out in the near future.

#### Acknowledgement

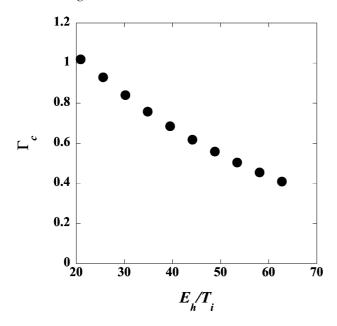
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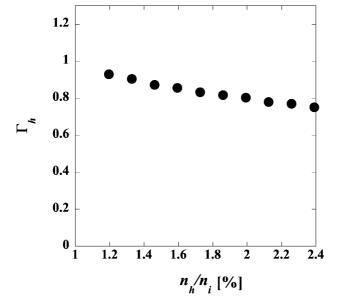
**Figure 1.** The dependence of the growth rate  $\gamma_h$  and the damping rate  $\gamma_G$  on the density of the energetic particles. Here, the energy of energetic particles is fixed,  $E_h = 42 [\text{keV}]$ .



**Figure 2.** The dependence of  $\Gamma_c$ , the parameter which characterizes the ratio of the rate of the energy transfer from energetic particles to GAMs and to particles, on the density of the energetic particles. Here, the energy of energetic particles is fixed,  $E_h = 42[\text{keV}]$ .



**Figure 3.** The dependence of  $\Gamma_c$ , the parameter which characterizes the ratio of the rate of the energy transfer from energetic particles to GAMs and to particles, on the energy of the energetic particles. Here, the density of energetic particles is fixed,  $n_h/n_i = 2.4[\%]$ .



**Figure 4.** The dependence of  $\Gamma_h$ , the parameter which characterizes the ion heating by GAMs normalized to the collisional ion heating, on the density of the energetic particles. Here, the energy of energetic particles is fixed,  $E_h = 42[\text{keV}]$ .

#### Appendix A. Extension of the dispersion relation of GAMs

We describe the detail of the extension that leads to the dispersion relation Eq.(3). In [12], the dispersion relation of the GAMs, which are driven by the energetic particles, has been derived as

$$D(\omega) = 1 - \frac{\omega_G^2}{\omega^2} + \frac{n_h}{2n_i R^2} \int \frac{d^3 u u^3}{n_h} \left(\frac{1 + \Lambda^2}{2}\right)^2 \left[\frac{\partial F(u, \Lambda, r)}{\partial u} + \frac{1 - \Lambda^2}{u \Lambda} \frac{\partial F(u, \Lambda, r)}{\partial \Lambda} + sgn \frac{Rq}{r\omega_{ci}\Lambda} \frac{\partial F(u, \Lambda, r)}{\partial r}\right] \left(\omega^2 - \omega_b^2\right)^{-1} = 0, \tag{A.1}$$

where  $F(u, \Lambda, r)$  is the distribution function of the energetic particles. Then, the approximation of the distribution  $F(u, \Lambda, r)$  was used as explained in section 2. The model distribution function was given as

$$F(u, \Lambda, r) = \frac{3n_h}{8\pi\Delta\Lambda} \left[ 1 - \left(\frac{\Lambda - \Lambda_0}{\Delta\Lambda}\right)^2 \right] \times \Theta(\Lambda - \Lambda_0 + \Delta\Lambda)\Theta(\Lambda_0 + \Delta\Lambda - \Lambda)\Theta(\Lambda - \Lambda_c). \tag{A.2}$$

In addition, the assumption was adopted that the real frequency of the unstable mode is close to the bounce frequency of the energetic particles,  $\omega \sim \omega_b$ , and the  $\omega$  was replaced by  $\omega_b$ . Finally, the simplified dispersion relation has been derived as

$$D(\sigma) = -\frac{1}{\tau} + \frac{\delta \Lambda_c^2}{2(\sigma + \delta \Lambda_c)} + 1 + \delta \Lambda_c + \sigma \left( \ln \left| \frac{1 - \sigma}{-\delta \Lambda - \sigma} \right| + i\pi \right), \tag{A.3}$$

where  $\tau$  is defined as  $3q^2n_h(1+\Lambda_0^2/\Lambda_0)^2/(32n_i\Delta\Lambda^2(\omega_G^2/\omega_b^2-1))$ . This simplified dispersion relation can not describe the instability in the case of  $\omega_b > \omega_G$ , and the connection to the solution of  $\omega = \omega_G$  at  $n_h = 0$  is obscure.

In this study, we extend Eq. (A.3) without using the replacement of  $\omega$  to  $\omega_b$ . If one does not employ the approximation  $\omega \approx \omega_b$ , the extended dispersion relation is given as

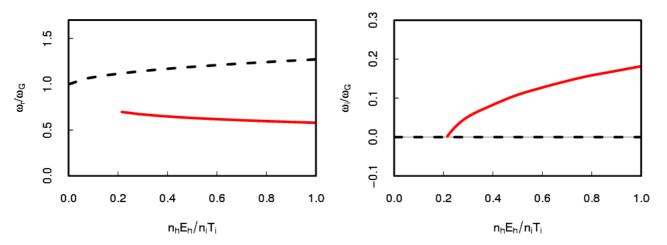
$$D(\sigma) = 1 - \frac{\omega_G^2}{\omega_b^2} \left( 1 + \sigma \frac{\Delta \Lambda}{\Lambda_0} \right)^{-2} + \frac{3q^2 n_h}{16n_i \Delta \Lambda} \left( \frac{1 + \Lambda_0^2}{\Lambda_0} \right)^2 \left( 2 + \sigma \frac{\Delta \Lambda}{\Lambda_0} \right)^{-1} \times \left\{ \frac{1 - \delta \Lambda_c^2}{2(\sigma + \delta \Lambda_c)} + 1 + \delta \Lambda_c + \sigma \ln \left( \frac{1 - \sigma}{-\delta \Lambda_c - \sigma} \right) \right\}.$$
(A.4)

If we take the limit of

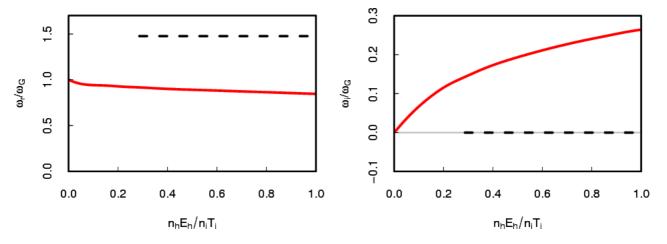
$$\omega \to \omega_b : \sigma \frac{\Delta \Lambda}{\Lambda_0} \to 0$$

$$\ln \left( \frac{1 - \sigma}{-\delta \Lambda_c - \sigma} \right) \to \ln \left| \frac{1 - \sigma}{-\delta \Lambda_c - \sigma} \right| + i\pi,$$

for Eq. (A.4), it reduces to Eq. (A.3). The behavior of the solution of Eq. (A.4) is shown in Figs. A1, and A2. There are two branches which can be unstable ( $\gamma_h > 0$ ). When  $\omega_G < \omega_b$  holds, GAM branch with a real frequency close to  $\omega_G$  becomes unstable. When  $\omega_G > \omega_b$  holds, the beam branch whose frequency is close to  $\omega_b$  becomes unstable. These features are consistent with those in [11]. In this article, we focus on the GAM branch. The GAM branch can be driven ( $\gamma_h > \gamma_G$ , where  $\gamma_G$  is the ion Landau damping rate) in the wide range of parameters as is shown in Fig. 1.



**Figure A1.** The behavior of the solution of Eq. (A.4) in the case of  $\omega_b/\omega_G = 0.71$ . The left figure shows the real frequency, and the right one shows the imaginary part of the frequency. The strongly-damped branch is not shown. The solid line and the dashed line indicate the unstable branch and the weakly stable branch, respectively.



**Figure A2.** The behavior of the solution of Eq. (A.4) in the case of  $\omega_b/\omega_G = 1.21$ . The left figure shows the real frequency, and the right one shows the imaginary part of the frequency. The strongly-damped branch is not shown. The solid line and the dashed line indicate the unstable branch and the weakly stable branch, respectively.

#### Appendix B. Order estimation of GAM channeling

The order estimations of  $P_{h\to G}$ ,  $P_{G\to i}$  are given. The energy density of GAM,  $W_G$ , can be rewritten as

$$W_G = \frac{1}{2} m_i n_i v_G^2$$

$$= \frac{1}{2} \frac{T_e}{T_i} \left( 1 + \frac{T_i}{T_e} \right)^{-1} \left( \frac{\rho_i}{L_n} \right)^2 \left( \frac{v_G}{v_d} \right)^2 W_p,$$
(B.1)

where  $W_p$  is the energy density of the plasma, which is defined as  $W_p = n_i(T_i + T_e)$ ,  $L_n$  is the gradient length of the density,  $v_G$  is the  $E \times B$  velocity of the GAMs,  $v_G \equiv q_r |\phi_G|/B$ , and

 $v_d$  is the diamagnetic drift velocity,  $v_d \equiv T_e/(eBL_n)$ . The rate of mean energy loss from the plasma,  $\tau_E^{-1}W_p$ , is estimated as  $(\omega_*\rho_i^2/L_n^2)W_p$ , where  $\tau_E$  is the energy confinement time which is calculated from the gyro-Bohm diffusion,  $\tau_E = L_n^2/\omega_*\rho_i^2$ . The energy transfer rate from energetic particles to the GAMs can be related to  $\tau_E^{-1}W_p$  as,

$$P_{h\to G} = \gamma_h \frac{T_e}{T_i} \left( 1 + \frac{T_i}{T_e} \right)^{-1} \left( \frac{\rho_i}{L_n} \right)^2 \left( \frac{\mathbf{v}_G}{\mathbf{v}_d} \right)^2 W_p$$

$$= \frac{\gamma_h}{\omega_*} \frac{T_e}{T_i} \left( 1 + \frac{T_i}{T_e} \right)^{-1} \left( \frac{\mathbf{v}_G}{\mathbf{v}_d} \right)^2 \tau_E^{-1} W_p, \tag{B.2}$$

In the steady state, the input power,  $P_{\text{in}}$ , and the transport  $\tau_E^{-1}W_p$  are balanced as  $P_{\text{in}} = \tau_E^{-1}W_p$ . Finally, the order of the energy transfer rate  $P_{h\to G}$  can be estimated as

$$\frac{P_{h\to G}}{P_{\rm in}} \sim \frac{\gamma_h}{\omega_*} \left(\frac{\mathbf{v}_G}{\mathbf{v}_d}\right)^2. \tag{B.3}$$

The energy transfer to GAMs is proportional to the squared amplitude of GAMs. When the  $E \times B$  velocity of the GAMs reaches to the level of the diamagnetic drift velocity, the energy transfer rate from energetic particles to the GAMs becomes not negligible compared to the input power to the plasma. In the same manner, the energy transfer rate from GAMs to bulk ions  $P_{G\rightarrow i}$  is evaluated as

$$\frac{P_{G\to i}}{P_{\rm in}} \sim \frac{\gamma_G}{\omega_*} \left(\frac{\mathbf{v}_G}{\mathbf{v}_d}\right)^2. \tag{B.4}$$

The rate of energy transfer from the GAMs to bulk ions can be important compared to the input power to the plasma when the GAM amplitude comes closer to the level of  $v_G/v_d = 1$ . GAMs can have a substantial impact on ion heating in the case of  $v_G/v_d \sim O(1)$ .

- [1] T. H. Stix, Plasma Phys. 14, 367 (1972).
- [2] N. J. Fisch, et. al., Nucl. Fusion, **34**, 1541 (1994).
- [3] H. Ohtsuka, et al. Bull. Phys. Soc. Jpn. 4, 127 (1983).
- [4] A. B. Mikhailovskii, Theory of Plasma Instabilities, Vol.2, Consultants Bureau, New York, (1974).
- [5] S.-I. Itoh, K. Itoh, A. Fukuyama, Plasma Phys. Control. Fusion 26, 1311 (1984).
- [6] D. A. Gates, et. al., Phys. Rev. Lett. 87, 205003 (2001).
- [7] N. Winsor, et. al., Phys. Fluids, 11, 2448 (1968).
- [8] R. Nazikian, et. al., Phys. Rev. Lett., 101, 185001 (2008).
- [9] K. Toi, et. al., 35th European Physical Society conference on Plasma Physics, EX/P8-4 (2008).
- [10] T. Ido, et. al., Plasma Phys. Control. Fusion, 52, 124025 (2010).
- [11] G. Y. Fu, Phys. Rev. Lett., 101, 185002 (2008).
- [12] H. L. Berk, et. al., Nucl. Fusion, 50, 035007 (2010).
- [13] Z. Qiu, et. al., Plasma Phys. Contrl. Fusion, **52**, 095003 (2010).
- [14] H. Sugama, T.-H. Watanabe, Phys. Plasmas 13, 012501 (2006).
- [15] Z. Gao, K. Itoh, H. Sanuki, J. Q. Dong, Phys. Plasmas 13, 100702 (2006).
- [16] M. Sasaki, K. Itoh, A. Ejiri, Y. Takase, Plasma Fusion Res. 3, 009 (2008).
- [17] M. Sasaki, K. Itoh, A. Ejiri, Y. Takase, Journal of Plasma Phys. 75, 721 (2009).
- [18] A. Fujisawa, Nucl. Fusion, 49, 013001 (2009).