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Komatsu, Goro
九州大学大学院経済学府

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Goro Komatsu †

Abstract

No, is the answer we find in an estimated DSGE model of involuntary unemployment for the US economy in which optimal policy shows the considerable tension between stabilizations of inflation and unemployment due to the exogenous movements in workers' market power. Our counterfactual exercises show in the absence of the latter emerges no policy trade-off with effectively minimal welfare losses. Our findings are consistent with some seminal contributions in optimal policy—with no unemployment variable—of the "*divine coincidence*" of Blanchard and Galí (2010) and the "*trinity*" of Justiniano, Primiceri and Tambalotti (2013), but our key contribution lies in its direct measurement of the stability of unemployment and welfare losses conceived by US households.

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1 Introduction

Since Phillips (1958) identified the key negative correlation between inflation and unemployment, this nontrivial trade-off has intrigued many policymakers seeking to stabilize these two primal policy objectives. The US economy in 1970s witnessed, however, both inflation and unemployment remain high throughout 1980's—a severe adverse macroeconomic condition know as *stagflation*. This motivated the US Congress to reform the Federal Reserve Act in November 1977, leading to the Congress Reform Act that explicitly identifies the *dual mandate*—the goals of "maximum employment and stable prices (and moderate long-term interest rate)." Ever since, the Federal Reserve has been constantly pursing one mantra—Is the dual mandate achievable?

Assessing the latter, however, still remains surprisingly less explored by a standard policy evaluation framework. One policy target—unemployment—has been paid surprisingly less

† Graduate School of Economics, Kyushu University.

attention under the current vintages of dynamic stochastic general equilibrium (DGSE) models, due in part to the economic stability known as the *Great Moderation*. However, the 2008 financial crisis and the subsequent Great Recession resurrect this nontrivial trade-off for many policymakers only to find that there is no structural framework that can address this old and new inquiry.

We contribute to fill this gap in the policy trade-off—optimal monetary policy—literature by providing that missing structural model suitable for addressing the Federal Reserve's dual mandate. Following conventions, we define the dual mandate—the stable inflation and the maximum employment—as *stable inflation in prices* and *the minimum unemployment*.

What kinds of model is suitable for the policy trade-off analysis? Since maintaining maximum employment directly translates into maintaining minimum unemployment, it requires a direct modeling of unemployment. Since whether the dual mandate is achieved or not is not independent of dynamic interactions of many other key macro variables and shocks, this requires dynamic and stochastic general equilibrium consideration. Since the attainment of the join stabilization should be judged by some formal measures, it requires a quantitative criteria based on theory. Hence, our choice of the model is an estimated DSGE model of unemployment with some explicit welfare criterion. Specifically, we rely on an involuntary unemployment DSGE model in the spirit of Galí (2011). This choice satisfies direct modeling of unemployment under the DSGE setting, but also allows us to obtain the formal measure of the social welfare—the welfare loss function as a second-order approximation to households' utility, in the spirit of Rotemberg and Woodford (1997) and Clarida, Galí and Gertler (1998). We estimate our business cycle model using the post-war US quarterly macro aggregates spanning from 1967 to 2012. Our estimation is conducted using the Bayesian inference method.

We make three contributions to the policy trade-off literature. First and foremost, we contribute to provide the first DSGE framework that allows for the direct assessment on the achievability of the dual mandate. This framework is made possible by building a class of involuntary unemployment DSGE models of Galí (2011) with the formal development of the optimal monetary policy in an estimated DSGE model consistent with the existence of the balanced growth path. The involuntarily unemployment model of Galí (2011) is convenient since it preserves many conventional assumptions of the representative agent paradigm, and it further allows, on top of the latter paradigm, for obtaining the optimal—often called *Ramsey*—policy by central banks to seek to minimize the welfare losses conceived by a representative household. Our unique modeling strategies make it possible to construct an empirical optimal policy framework that can directly investigate the joint stabilization of inflation and unemployment (i.e., the dual mandate). As no existing literature has yet provided that policy trade-off framework (mostly due to the fact the the unemployment DSGE models are still relatively new and weak as pointed out by Galí (2011)), our analysis can be viewed as providing the first theoretical and empirical underpinnings to the formal criteria of the dual mandate.

Second, we judge the attainment of the dual mandate by restoring to formal criteria based on theory. That is, the policy trade-off, or the lack thereof, is formally evaluated by the use of the *welfare loss function*—the second-order approximation to the households' utility function, following the seminal work of Rotemberg and Woodford (1997) and Clarida, Galí and Gertler (1998). This not only makes our approach consistent with the large literature on optimal design of monetary policy, but also remedies the shortcomings of the existing DSGE literature on policy trade-off. The optimal policy analysis of the output gap as well as the price and wage inflations by Justiniano, Primiceri and Tambalotti (2013), for example, concludes that the joint stabilization of these three variables can be achieved, but that conclusion hinges only on the comparisons of the times series plots of these variables under the Taylor-rule and the counterfactual Ramsey policy. In other words, their decision lacks any theoretical justifications—their judgement does not rely on, for example, any threshold values nor quantitative criteria based on existing theory. We rectify that anomaly by providing that theoretical justifications—we derive the exact welfare loss function as criteria for the policy trade-off on top of the explicit modeling of unemployment. One key advantage of our approach—one example among many—is that it allows for plotting the historical behavior of the welfare losses. This makes it possible to identify the magnitudes and behaviors of the utility losses in any given historical episodes.

Third, and to propose some possibly feasible (e.g., implementable) policy options for the Federal Reserve based on our empirical evidence, we contribute to offer a policy that can be close, or replicate, the optimal allocation suggested by Ramsey policy. Even if the optimal policy can not attain the joint stabilization of price inflation and unemployment slack, much of the optimal monetary policy literature suggests that the latter is still the best allocation for the central banks to pursue in their conduct of monetary policy. That is, there still remains an substantial incentive for any central banks who seek to maximize the welfare of consumers to design policies that are close to the optimal allocation. To offer a policy rule in a tractable and parsimonious way, we develop a simple policy rule called *optimal simple rule* (OSR, hereafter). In that rule, and taking into account the key roles played by wage inflation and unemployment (to be shown later), we add these variables to the original Taylor-type rule specification. We then compute the coefficients of that policy rule so as to minimize the unconditional period utility losses in line with the loss function developed in this text. The impulse responses obtained from the optimized simple rule—together with those computed by the previous rules described so far—help to interpret to what extent that policy reaction rule should react to the extended policy targets to better replicate the Ramsey equilibria, by, in many cases, reducing the volatilities of those policy variables.

1.1 Related Literature

One of the first discussion on the stability of price under the basic New Keynesian framework can be found in detail in Goodfriend and King (1997). The desirability of inflation targeting strategies is proposed and analyzed by Svensson (1997), although much of his discussion is based on models with no explicit microfoundations. Taylor (1993) introduced the simple monetary policy rule now widely known as the Taylor rule, which approximates fairly well the Federal Reserve policy in the early Greenspan years. Some alternative versions to the original Taylor rule are estimated and examined by Judd and Rudebusch (1998) and Clarida, Galí and Gertler (2000), including their (in)stabilities over the postwar era. For the pre-Volcker years, Orphanides (2003) argues that the vast bulk of the deviations of policy rates from the predictions made by the baseline Taylor rule can be attributed to the biases of significant degree in the measures of the real-time output gap development. For the Greenspan era, Taylor (1999a) evaluates the monetary policy based on his calibrated Taylor rule. The influence of Taylor-type rules in both research and policy in terms of some historical episodes is provided by Koenig, Leeson and Kahn (2012).

Some reference papers on the properties of a rich array of alternative simple rule specifications are contained in the volume edited by Taylor (1999b). In their seminal contribution, Rotemberg and Woodford (1997) derive a second-order approximation to the utility of the representative consumer. Detailed discussion on welfare-based evaluation of policy rules can be found in chapter 6 in Woodford (2011). The comprehensive analysis of the optimal monetary policy in the context of the basic New Keynesian model augmented with an ad-hoc cost-push shocks is given by Clarida, Galí and Gertler (2000). In particular, they compare the outcomes of the optimal policy under either discretion or commitment policy adopted by central banks. Chapter 5 of Galí (2015) closely follows Clarida, Galí and Gertler (2000), but also provides an analysis of the optimal design of monetary policy in the presence of the zero lower bound (ZLB) on the nominal interest rate, following the works of Jung, Teranishi and Watanabe (2005) and Eggertsson and Woodford (2003).

The key contribution in the policy trade-off literature was made by Blanchard and Galí (2010). They show that in environment with flexible wages and no markups shocks, the stabilization of the output gap and of aggregate markups are equivalent to produce no price dispersion, thus delivering the efficient allocation. An explicit normative consideration of monetary policy in medium-scale DSGE models can be found in the optimal policy literature. Levin, Onatski, Williams and Williams (2006), Schmitt-Grohé and Uribe (2004), and Schmitt-Grohé and Uribe (2007) are the reference work, and find that nominal dispersion is the key for the normative implications fo the model. Justiniano, Primiceri and Tambalotti (2013) find, on the other hand, virtually no tension between inflation and output gap stabilization, once they recognize that wage markup shocks are likely to be small. Following the seminal work of Blanchard and Gali (2007), they coin this lack of trade-offs among three policy objectives as *trinity*. Furlanetto, Gelain and Sanjani (2017) extends the model of Justiniano, Primiceri and Tambalotti (2013) to incorporate the financial accelerator mechanism of Bernanke, Gertler and Gilchrist (1999) and risks shocks of Christiano, Motto and Rostagno (2014). They show that the trinity may not hold under the economy in which financial factors play nonnegligible roles in the shape and behavior of the estimated output gap. The important role of wage markup as well as labor supply shocks are first pointed out by Chari, Kehoe and McGrattan (2009), Sala, Ulf Söderström and Trigari (2010), and Galí, Smets and Wouters (2012b), although they are not concerned with the characterization of optimal policy.

This study is also related to a large literature on the estimated (and medium-scale) DSGE models including, but not limited to, Rotemberg and Woodford (1997), Lubik and Schorfheide (2004), Christiano, Eichenbaum and Evans (2005), Boivin and Giannoni (2006), Smets and Wouters (2003), Smets and Wouters (2007), and Lindé, Smets and Wouters (2016). These medium-scale DSGE models, however, do not explicitly incorporate unemployment variable and data.

The rest of the paper is organized as follows. Section 2 provides the model economy. Section 3 describes the approaches to the model solution, measurement, and Bayesian inference. Section 4 explains optimal monetary policy and the relevant welfare loss function. Section 5 investigate the achievability of the dual mandate, the source of the policy trade-off, and the alternative and optimized simple rule. Section 6 concludes.

2 Model

This section outlines our baseline DSGE model of the US business cycles. Following Galí (2011) and Galí, Smets and Wouters (2012b), our DSGE model of involuntary unemployment is built with a neoclassical growth core, augmented with several shocks and frictions in tastes, technology, and monopolistic market with some forms of nominal wage rigidities common in the literature. The economy is populated by five classes of agents: producers of a final good, intermediate goods producers, households, employment agencies, and a government. Below we present their optimization problems.

2.1 Final Goods Producers

In each period t, perfectly competitive firms produce the final good Y_t by combining a continuum of intermediate goods $Y_t(f)$, $f \in [0, 1]$, by using the technology

$$Y_t = \left[\int_0^1 Y_t(f)^{\frac{\epsilon_{p,t}-1}{\epsilon_{p,t}}} df\right]^{\frac{\epsilon_{p,t}}{\epsilon_{p,t}-1}}.$$

Profit maximization and the resulting zero profit condition imply that the price of the final good, P_t, is a CES aggregate of the prices of the intermediate goods, $P_t(f)$:

$$P_t = \left[\int_0^1 P_t(f)^{1-\epsilon_{p,t}} df\right]^{\frac{1}{1-\epsilon_{p,t}}}$$

and that the demand schedule for intermediate good f is

$$Y_t(f)=\left(\frac{P_t(f)}{P_t}\right)^{-\epsilon_{p,t}}Y_t. \tag{1}$$

The curvature of the aggregator $\epsilon_{p,t}$ controls the degree of substitutability across intermediate goods in final goods production, and thus can be interpreted as measuring the elasticity of demand for each intermediate good, as shown in (1).

We further model the term $\frac{\epsilon_{p,t}}{\epsilon_{p,t}-1}\equiv\mathcal{M}^n_{p,t}\equiv\mathcal{M}_p(\varepsilon^p_t)^{100}$ as an exogenous stochastic process:

$$\ln\varepsilon^p_t=\rho_p\ln\varepsilon^p_{t-1}-\vartheta_p\eta^p_{t-1}+\eta^p_t,$$

driven by *i.i.d.* innovation $\eta^p_t\sim N(0,\ \sigma^2_p)$. Since $\mathcal{M}^n_{p,t}$ is the desired (natural) markup of price over marginal cost for intermediate firms as suggested by Lerner's formula, we refer to these innovations as *price markup shocks*. In this regard, $\mathcal{M}^n_{p,t}$ can also be interpreted as a measure of the (lack of) competitiveness in the intermediate goods market. These exogenous movements play major roles in driving the economy away from its efficient frontier.

2.2 Intermediate Goods Producers

A monopolist produces the intermediate good f according to the production function

$$Y_t(f)=Z_tN_t(f)^{1-\alpha}, \tag{2}$$

where $N_t(f)$ is the labor (or hours worked) employed by firm f and parameter α controls returns to scale in production. Z_t represents exogenous technological progress. We assume that the level of neutral technology is nonstationary and its growth rate ($z_t\equiv\Delta Z_t$) follows an $AR(1)$ process

$$\ln z_t=(1-\rho_z)\ln\gamma+\ln z_{t-1}+\eta^z_t,$$

where γ is the growth rate of neutral technology, *i.i.d.* innovation $\eta^z_t\sim N(0,\ \sigma^2_z)$.

As in Calvo (1983) and Yun (1996), every period a fraction ι_p of intermediate firms can not optimally reset their price. If a firm is not allowed to choose its optimal price, the latter is assumed to be adjusted by a weighted combination of the lagged and steady state inflation:

$$P_{t+k|t}=P_{t+k-1|t}\left(\Pi^p_{t+k-1}\right)^{\iota_p}(\Pi^p)^{1-\iota_p},$$

where $\iota_p\in[0,\ 1]$ and $\Pi^p_t\equiv\frac{P_t}{P_{t-1}}$ denotes (gross) inflation, Π^p the steady state (gross) inflation, and $P^*_t\equiv P_{t|t}$. A positive value of the indexation parameter ι_p introduces structural inertia into the inflation dynamics. This particular indexation scheme implies no price dispersion in steady state. In other words, the value of steady state inflation Π^p is inconsequential in terms of welfare. This scheme thus allows for circumventing the issue pertaining the optimal level of inflation (Schmitt-Grohé and Uribe (2004), Schmitt-Grohé and Uribe (2007), and Schmitt-Grohé and Uribe (2010)).

The remaining fraction of firms optimally reset their price by choosing P^*_t, through maximization of the present discounted value of the expected future profits

$$E_t \sum_{k=0}^{\infty} (\theta_p)^k \frac{\beta^k \Lambda_{t+k} P_t}{\Lambda_t P_{t+k}} \left(P_t^* \prod_{s=1}^{k} \left(\Pi_{t+s-1}^p \right)^{\iota_p} \left(\Pi^p \right)^{1-\iota_p} - MC_{t+k|t} \right) Y_{t+k|t}$$

subject to the demand function (1) and the production function (2) associated with this problem, $Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \prod_{s=1}^{k} \left(\Pi_{t+s-1}^p \right)^{\iota_p} \left(\Pi^p \right)^{1-\iota_p} \right)^{-\epsilon_{p,t}} Y_{t+k}$, as well as the marginal cost associated with this problem, $MC_{t+k|t} = \frac{W_t}{(1-\alpha) Z_t [N_{t+k|t}]^{-\alpha}}$. In this objective, Λ_{t+k} is the marginal utility of real income of the representative household that owns the firm.

2.3 Employment Agencies

Following Erceg, Henderson and Levin (2000), we assume a continuum of monopolistically competitive households indexed as $h \in [0, 1]$. Each household supplies to a continuum of goods-producing firms $f \in [0, 1]$ a differentiated labor service $N_t(f, h)$ which is aggregated as

$$N_t \equiv \int_0^1 \int_0^1 N_t(f, h) df dh. \tag{3}$$

A large number of competitive employment agencies combine these specialized types of labor into a homogenous labor input that is in turn sold to intermediate firms, according to the packer

$$N_t(f) = \left[\int_0^1 N_t(f, h)^{\frac{\epsilon_{w,t}-1}{\epsilon_{w,t}}} dh \right]^{\frac{\epsilon_{w,t}}{\epsilon_{w,t}-1}}.$$

As in the case of the final good production, the curvature of the aggregator $\epsilon_{w,t}$ determines the degree of substitutability across specialized labor h.

We also model $\frac{\epsilon_{w,t}}{\epsilon_{w,t}-1} \equiv \mathcal{M}_{w,t}^n \equiv \mathcal{M}_w (\varepsilon_t^w)^{100}$ as an exogenous stochastic process

$$\ln \varepsilon_t^w = \rho_w \ln \varepsilon_{t-1}^w - \vartheta_w \eta_{t-1}^w + \eta_t^w$$

driven by *i.i.d.* innovation $\eta_t^w \sim N(0, \sigma_w^2)$. We refer to these innovations as *wage markup shocks* corresponding to the desired (natural) markup of wages over households' marginal rate of substitution between consumption and disutility of work (leisure).

Profit maximization by the perfectly competitive employment agencies implies first that the labor demand function is given by

$$N_t(f) = \left(\frac{W_t(h)}{W_t} \right)^{-\epsilon_{w,t}} N_t, \tag{4}$$

where $W_t(h)$ is the wage paid by the employment agencies to the supplier of labor of type h, and second that

$$W_t = \left[\int_0^1 W_t(h)^{1-\epsilon_{w,t}} df \right]^{\frac{1}{1-\epsilon_{w,t}}}$$

is the wage paid by intermediate firms for the homogenous labor input sold to them by the agencies.

2.4 Households

We assume a representative household with a continuum of members represented by a pair $(h, j) \in [0, 1] \times [0, 1]$. The index $h \in [0, 1]$ represents the specialized labor service by household h, while the index $j \in [0, 1]$ determines his disutility from work given by $\varepsilon_t^l \Theta_t j^{\varphi}$ if he is employed, zero otherwise. The model also assumes two preference shifters. The term ε_t^l is an exogenous preference shifter and the term Θ_t represents an endogenous shifter with a disutility parameter $\varphi \geq 0$. Following Galí, Smets and Wouters (2012b), full risk sharing within the household is assumed as in Merz (1995). This implies the same level of consumption for all household members, independently of their occupation or employment status, i.e., $C(h, j) = C_t$, $\forall (h, j) \in [0, 1] \times [0, 1]$ and t. Thus, the utility function of a household member h is given by:

$$E_t \sum_{s=0}^{\infty} \beta^s \varepsilon_{t+s}^b \left[\ln\left(\frac{C_{t+s}}{Z_{t+s}}\right) - \varepsilon_{t+s}^l \Theta_{t+s} \int_0^1 \int_0^{N_{t+s}(h)} j^{\varphi} dj dh \right],$$

where C_t is consumption and employment for each occupation h, $N_t(h)$, is taken as given by the household. The disturbance to the discount factor ε_t^b is an *intertemporal preference shock* and follows the stochastic process

$$\ln \varepsilon_t^b = \rho_b \ln \varepsilon_{t-1}^b + \eta_t^b,$$

with *i. i. d.* innovation $\eta_t^b \sim N(0, \sigma_b^2)$. The exogenous preference shifter ε_t^l has instead an interpretation of the *intratemporal preference* or *labor supply shock*

$$\ln \varepsilon_t^l = \rho_l \ln \varepsilon_{t-1}^l + \eta_t^l, \quad \eta_t^l \sim N(0, \sigma_l^2),$$

with *i. i. d.* innovation $\eta_t^l \sim N(0, \sigma_l^2)$. This shock enters households' first-order conditions for the optimal supply of labor in exactly the same way as the wage markup shock. As a consequence, these two disturbances are not separately identified in the model, when only using data on wages and hours worked. However, as proposed by Galí (2011) and Galí, Smets and Wouters (2012b), a reinterpretation of the model with an explicit treatment of unemployment allows for the separate identification of these two shocks.[1)]

On the other hand, the endogenous preference shifter Θ_t is assumed to follow

$$\Theta_t \equiv \frac{\Xi_t}{\bar{C}_t}$$

$$\Xi_t \equiv (\Xi_{t-1})^{1-\upsilon} (\bar{C}_t)^{\upsilon},$$

where the term Ξ_t represents a *consumption trend*, and $\bar{C}_t$ is a given aggregate consumption. These equations are intended to match the joint behavior of the labor force, consumption, and the wage over the business cycle suggested by the evidence (Christiano, Trabandt and Walentin (2010)). For sufficiently low values of the parameter $\upsilon \in [0, 1]$ that governs the wealth effect, the latter on labor

1) Christiano, Trabandt and Walentin (2011) provides a DSGE model with similar implications, but with alternative microfoundations of unemployment.

supply can be overturned to match the evidence (Galí, Smets and Wouters (2012b)). Consumption is not indexed by j due to the assumption made above, and to the existence of state contingent securities that ensure that, in equilibrium, consumption and asset holdings are the same across all households. The household's flow budget constraint is then given by

$$P_tC_t+\frac{B_t}{R_t}\leq\int_0^1 W_t(h)N_t(h)dh+B_{t-1}+Q_t(h)+\prod{}_t,$$

where B_t is government bonds, R_t is the (gross) nominal interest rate, $Q_t(h)$ is the net cash flow accruing from household's j portfolio of state contingent securities, $\prod_t$ is the profit to households from ownership of the firms.

Households set nominal wages in the form of the staggered nominal contracts of Calvo (1983) and Yun (1996). In each period, each household h faces a constant probability of $1-\theta_w$ to reoptimize the wage $W_t(h)$. This wage resetting probability is assumed to be independent of the history that it reset its wage last. Letting $W_{t+k|t}$ denote the price in period $t+k$ for households that last reoptimized their wage in period t, we assume that a fraction $\iota_w\in[0,\ 1]$ of wages are indexed to past inflation

$$W_{t+k|t}=W_{t+k-1|t}\gamma\left(\prod{}^p_{t+k-1}\right)^{\iota_w}\left(\prod{}^p\right)^{1-\iota_w},$$

where $W_t^*=W_{t|t}$. The indexation parameter ι_w introduces structural inertia into the revolution of wage inflation. The growth rate of neutral technology γ enters into this indexation scheme to ensure the existence of a balanced growth path. These assumptions amount to the following optimization problem for households' wage decision:

$$\max_{W_t^*}E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k\left[\Lambda_{t+k}\frac{W_t^*}{P_{t+k}}\left(\prod_{s=1}^{k}\gamma\left(\prod{}^p_{t+s-1}\right)^{\iota_w}\left(\prod{}^p\right)^{1-\iota_w}\right)N_{t+k|t}-\varepsilon^l_{t+k}\Theta_t\frac{N_{t+k|t}^{1+\varphi}}{1+\varphi}\right]$$

subject to the labor demand function (4) associated with this optimization problem, $N_{t+k|t}=\left[\frac{W_t^*}{W_{t+k}}\left(\prod_{s=1}^{k}\gamma\left(\prod{}^p_{t+s-1}\right)^{\iota_w}\left(\prod{}^p\right)^{1-\iota_w}\right)\right]^{-\epsilon_{w,t+k}}N_{t+k}$. Λ_{t+k} denotes the Lagrange multiplier associated with household's optimization problem in period $t+k$, and $N_{t+k|t}$ denotes the labor service in period $t+k$ for household that last reset its wage in period t.

2.5 Unemployment

To introduce unemployment into otherwise standard monetary DSGE model, consider individual $(h,\ j)$ specialized in labor type of index h, with her disutility of work given by $\varepsilon^l_t\Xi_t j^{\varphi}$ that is heterogenous in index j. Based on its household's welfare as a criterion, and regarding the prevailing wage for her labor type h as a summary for the current conditions of the labor market and taking it given, in each period t she will find it optimal to participate in the labor market if and only if the following condition is satisfied:

$$\frac{W_t(h)}{P_t}\geq\varepsilon^l_t\Xi_t j^{\varphi}$$

in other words, as long as the relevant real wage is above her disutility from work. Evaluating the condition above at the symmetric equilibrium results in the labor participation condition that determines the marginal supplier of labor service specialized in type h defined as $L_t(h)$:

$$\frac{W_t(h)}{P_t}=\varepsilon_t^l \Xi_t L_t(h)^{\varphi}. \tag{5}$$

After detrending, taking logs, and aggregating over all occupations of equation 5 (integrating over h), we derive the following (log) aggregate participation equation:

$$w_t=\varepsilon_t^l+\xi_t+\varphi l_t, \tag{6}$$

where $w_t \equiv \int_0^1 w_t(h)dh$ is the average (log) real wage, $l_t \equiv \int_0^1 l_t(h)dh$ can be interpreted as the (log) labor force, and $\xi_t \equiv \ln(\Xi_t/Z_t)$.

The marginal rate of substitution between consumption and employment for type h workers in period t, on the other hand, is defined by a reciprocal between marginal utility of consumption and the marginal disutility of work:

$$\begin{aligned} MRS_t(h) &\equiv -\frac{U_{N(h),t}}{U_{C,t}} \\ &=\varepsilon_t^l \Xi_t N_t(h)^{\varphi}. \end{aligned}$$

Equilibrium in labor markets under monopolistic competition implies that the real wage for type h labor is set to be equal to the wage markup times the relevant marginal rate of substitution:

$$\begin{aligned} \frac{W_t(h)}{P_t} &=\mathcal{M}_{w,t} MRS(h)_t \\ &=\mathcal{M}_{w,t}\varepsilon_t^l \Xi_t N(h)_t^{\varphi}. \end{aligned} \tag{7}$$

Defining $\mu_{w,t} \equiv \log \mathcal{M}_{w,t}$ as the (average) log wage markup and analogous to the log linearization of (5), log-linear approximation to (7) can be obtained as the following relation:

$$w_t=\mu_{w,t}+\varepsilon_t^l+\xi_t+\varphi n_t, \tag{8}$$

where $n_t \equiv \int_0^1 n_t(h)dh$ is the average (log) employment.

Following Galí (2011), we define the *unemployment rate* as the following (log) difference between the labor force and employment[2)]

$$u_t \equiv l_t-n_t. \tag{9}$$

Combining (6), (8), (9) relates the unemployment rate to the wage markup via the Frisch labor elasticity parameter φ:

$$u_t=\frac{1}{\varphi}\mu_{w,t}.$$

2) This definition of the unemployment rate is very close to the conventional one defined by $1-\frac{N_t}{L_t}$, for low values that are typical in the observed unemployment rate. This is due to the fact that the following approximation holds for the unemployment rate near zero: $1-N_t/L_t=1-e^{-u_t} \simeq u_t$.

2.6 Monetary Policy

We assume that the short-term nominal interest rate follows a Taylor-type feedback rule that has been shown in the literature to provide a good description of actual monetary policy of the Federal Reserve (Taylor (1993)). We assume this rule when we estimate our baseline model to compute our flexible price and wage economy with no markup disturbances that characterize the model's potential output. Our generalized Taylor rule features: interest rate smoothing to the lagged policy rate, systematic reactions to deviations of (annual) inflation from a time-varying inflation target (to be described below), and to discrepancy of observed annual output growth from its steady state

$$\frac{R_t}{R}=\left(\frac{R_{t-1}}{R}\right)^{\phi_r}\left[\left(\frac{\left(\prod_{t=s}^{3}\prod_{t-s}^{p}\right)^{1/4}}{\prod_t^*}\right)^{\phi_\Pi}\left(\frac{(Y_t/Y_{t-4})^{1/4}}{\gamma}\right)^{\phi_{\Delta y}}\right]^{1-\phi_r}\eta_t^r, \tag{10}$$

where R is the steady state gross nominal interest rate, with monetary policy shock given by *i.i.d.* innovation $\eta_t^r \sim N(0,\ \sigma_b^2)$.

The inflation target $\prod_t^*$, on the other hand, evolves following the exogenous process

$$\ln \prod_t^* = (1-\rho_\pi)\ln \prod^* + \ln \prod_{t-1}^* + \eta_t^\pi,$$

with *i.i.d.* innovation $\eta_t^\pi \sim N(0,\ \sigma_\pi^2)$. This inflation target is intended to capture the low frequency development of inflation, following Ireland (2007) and Justiniano, Primiceri and Tambalotti (2013). As argued by Cogley and Sargent (2005) and Primiceri (2006), the slow evolution of policymakers' beliefs and their consequences to the monetary policy conduct can be reflected in this secular movement of inflation. When we characterize the optimal policy in Section 4, this rule is replaced by Ramsey policy by which central bank sets the interest rate to directly maximize the utility of the representative consumer.

3 Model Solution and Estimation

This section briefly describes the solution and estimation methodologies of our DSGE model. For the solution, we first collect the first-order conditions and constraints of optimization problems of our agents in a system of rational expectation (stochastic) difference equations which characterize the equilibrium of our economy. As real quantities of this baseline economy are nonstationary because the technological progress has a unit root, we normalize the equilibrium conditions of our real variables by the nonstationary technology process Z_t. Specifically, we let

$$E_t[\mathbf{f}(\boldsymbol{\zeta}_{t+1},\ \boldsymbol{\zeta}_t,\ \boldsymbol{\zeta}_{t-1},\ \mathbf{e}^{\eta_t},\ \boldsymbol{\theta})]=\mathbf{0}, \tag{11}$$

collect these detrended equilibrium conditions, in which $\boldsymbol{\zeta}_t$, $\boldsymbol{\eta}_t$, and $\boldsymbol{\theta}$ are, respectively, the vectors of endogenous variables, exogenous *i.i.d.* innovations, and unknown structural parameters.

To characterize the variables that would prevail under optimal monetary policy—our main objectives of interest—(11) must also include the equilibrium conditions of the corresponding counterfactual economies with frictionless prices and wages resulting in the constant markups and optimal monetary policy. This suggests that the vectors of endogenous quantities ζ_t must also include the variables required to construct the counterfactual economies in which the dynamics of unobservable variables, for example, potential output and natural rate of interest, are characterized.

We then log-linearize (11) around the nonstochastic steady state and solve the resulting linear system of rational expectation equations by solution methodologies now common in the literature.[3] The solution leads to the following system of transition equations:

$$\hat{\xi}_t = \mathbf{G}(\boldsymbol{\theta})\hat{\xi}_{t-1} + \mathbf{M}(\boldsymbol{\theta})\varepsilon_t,$$

where the hat denotes log deviations from the steady state, $\hat{\xi}_t$ is an extended version of $\hat{\zeta}_t$ that also includes the expectational variables necessary to characterize the solution of the model, and $\mathbf{G}(\boldsymbol{\theta})$ and $\mathbf{M}(\boldsymbol{\theta})$ are conformable matrices whose elements are functions of $\boldsymbol{\theta}$.

3.1 Data and Measurement

We employ for estimation six series of US macro quarterly data including: the inflation rate, the unemployment rate, the nominal interest rate, the logarithm of per capital hours, the log- difference of real per capita GDP and real wages, and we do not demean or detrend any series. The full sample period spans from 1967Q1 to 2012Q4.[4]

The corresponding measurement equations are given by:

$$Y_t^{obs} = \begin{bmatrix} \Delta\ln GDP_t \\ \Delta\ln WAGE_t \\ \Delta\ln DEF_t \\ \ln HOURS_t \\ FEDFUNDS\ RATE_t \\ UNEMP.\ RATE_t \end{bmatrix} = \begin{bmatrix} \bar{\gamma} \\ \bar{\gamma} \\ \bar{\pi}^* \\ \bar{n} \\ \bar{r} \\ \bar{u} \end{bmatrix} + \begin{bmatrix} \hat{y}_t - \hat{y}_{t-1} + \hat{\varepsilon}_t^Z \\ \hat{w}_t - \hat{w}_{t-1} + \hat{\varepsilon}_t^Z \\ \hat{\pi}_t^p \\ \hat{n}_t \\ \hat{R}_t \\ \hat{u}_t \end{bmatrix},$$

where $\bar{\gamma} \equiv 100(\gamma - 1)$ is the trend growth, $\bar{n}$ denotes steady state hours worked, $\bar{\pi}^* \equiv 100(\Pi^* - 1)$ is defined as the steady state inflation, $\bar{r} \equiv 100(\beta^{-1}\gamma\Pi^* - 1)$ gives the steady state nominal interest rate, and $\bar{u}$ is the steady state unemployment rate, respectively.

3.2 Bayesian Inference and Priors

We characterize the posterior distribution of the coefficients of our DSGE model by combining the likelihood function with priors. The likelihood is evaluated by the Kalman filter algorithm. The latter and its smoother can also be used to estimate the historical developments of the model's endogenous variables, $\{\hat{\xi}_t\}_{t=1}^T$, which, as mentioned above, include potential and optimal output.

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3) Blanchard and Kahn (1980) and Sims (2002) are often used in solving the rational expectation system.
4) The detailed description of the data set is provided in the Appendix.

Table 1 displays the specification of the prior distributions used for Bayesian estimation. The covariance matrix of the vector of shocks is diagonal. Note that the intertemporal preference shocks are normalized so that the latter enter with a unit coefficient in consumption. Following much of the literature, this normalization often improves the convergence properties of the Markov Chain Monte Carlo (MCMC) algorithm for Bayesian estimation.

TABLE 1. PRIOR DISTRIBUTIONS AND POSTERIOR PARAMETER ESTIMATES IN THE BASELINE MODEL

		Prior Distribution			Posterior Distribution			
		Type[a]	Mean	S.D.	Mode	Mean	5%	95%
Structural Parameters								
υ	Wealth effects	$\mathcal{N}$	0.100	0.050	0.0377	0.0630	0.0089	0.1161
α	Capital share	$\mathcal{N}$	0.100	0.050	0.1463	0.1417	0.0781	0.2045
φ	Labor utility	$\mathcal{N}$	2.000	0.750	4.5160	4.8554	3.9146	5.8265
ι_p	Price indexation	$\mathcal{B}$	0.500	0.150	0.1758	0.1957	0.0726	0.3103
ι_w	Wage indexation	$\mathcal{B}$	0.500	0.150	0.2048	0.2123	0.1076	0.3093
θ_p	Price stickiness	$\mathcal{B}$	0.500	0.100	0.5824	0.5794	0.4902	0.6730
θ_w	Wage stickiness	$\mathcal{B}$	0.500	0.100	0.1209	0.1364	0.0949	0.1778
ϕ_π	Taylor rule inflation	$\mathcal{N}$	1.500	0.250	2.4035	2.4416	2.1664	2.7031
$\phi_{\Delta y}$	Taylor rule output growth	$\mathcal{N}$	0.125	0.050	0.2672	0.2660	0.1910	0.3398
$\bar{\gamma}$	Trend growth	$\mathcal{N}$	0.200	0.100	0.3801	0.3783	0.2966	0.4534
$100(\beta^{-1}-1)$	Discount rate	$\mathcal{G}$	0.250	0.100	0.1780	0.2025	0.0821	0.3201
$100(\Pi^*-1)$	SS inflation	$\mathcal{G}$	0.625	0.100	0.6578	0.6902	0.5124	0.8526
$\bar{n}$	SS labor supply	$\mathcal{N}$	0.000	2.000	-1.2490	-1.5592	-3.9952	0.7996
$\mathcal{M}_p$	SS price markup	$\mathcal{N}$	1.250	0.125	1.2396	1.2538	1.0685	1.4231
$\mathcal{M}_w$	SS wage markup	$\mathcal{N}$	1.250	0.125	1.3552	1.3879	1.2654	1.5092
Persistence of the Exogenous Process								
ρ_b	Intertemporal shock	$\mathcal{B}$	0.500	0.200	0.9492	0.9468	0.9201	0.9752
ρ_a	Productivity shock	$\mathcal{B}$	0.500	0.200	0.0343	0.0488	0.0103	0.0814
ρ_p	Price markup shock	$\mathcal{B}$	0.500	0.200	0.9687	0.9632	0.9364	0.9907
ρ_w	Wage markup shock	$\mathcal{B}$	0.500	0.200	0.9612	0.9610	0.9322	0.9903
ρ_π	Inflation target shock	$\mathcal{B}$	0.500	0.200	0.9703	0.9637	0.9399	0.9888
ϑ_p	Price markup shock, $MA(1)$	$\mathcal{B}$	0.500	0.200	0.6918	0.6772	0.5628	0.7889
ϑ_w	Wage markup shock, $MA(1)$	$\mathcal{B}$	0.500	0.200	0.1234	0.1575	0.0307	0.2710
S.D. of the Innovations								
σ_b	Intertemporal shock	$\mathcal{IG}$	0.100	2.000	0.1728	0.1779	0.1465	0.2074
σ_a	Productivity shock	$\mathcal{IG}$	0.100	2.000	0.6364	0.6450	0.5899	0.7031
σ_l	Labor supply shock	$\mathcal{IG}$	0.100	2.000	2.2511	2.4356	1.9485	2.9452
σ_p	Price markup shock	$\mathcal{IG}$	0.100	2.000	0.0197	0.0206	0.0155	0.0259
σ_w	Wage markup shock	$\mathcal{IG}$	0.100	2.000	0.0251	0.0290	0.0214	0.0366
σ_π	Inflation target shock	$\mathcal{IG}$	0.100	2.000	0.0540	0.0579	0.0427	0.0728
σ_r	Monetary policy shock	$\mathcal{IG}$	0.100	2.000	0.2381	0.2388	0.2144	0.2624

[a] $\mathcal{B}/\mathcal{G}/\mathcal{IG}/\mathcal{N}$ stand for Beta/Gamma/Inverse Gamma/Normal distribution, respectively.

3.3 Posterior Estimates of the Parameters

Table 1 also shows the posterior estimates of the parameters in our baseline DSGE model. The data are quite informative for these structural parameters. Our estimates are also largely in line with those reported in previous studies. Given these similarities and our main interest on the implications of these estimates for both the inefficiency of the economy and for optimal monetary policy, here we only briefly discuss on the estimated parameters most relevant to nominal rigidities and to monetary policy.

The estimated parameters θ_p and θ_w imply that the stickiness of prices and wages are, approximately and respectively, every 3 quarters and every 1.3 quarter. The parameters for structural inertia, ι_p and ι_w indicate, on the other hand, very low levels of backward indexation. Steady-state (annualized) inflation is about 2.8 percent. As for monetary policy, it displays a considerable degree of policy reaction to inflation development, with a long-run coefficient for inflation of more than 2 to inflation, while the policy does not respond to the fluctuations in the output growth (the estimated coefficient is only 0.27).

4 Optimal Monetary Policy

The discrepancies between the observed allocation of the economy and the one made by the optimal policy in our model economy typically emerge in two forms of inefficiencies—the one characterized by the *monopoly powers* enjoyed by intermediate goods producers in goods market and households in labor markets, and the other created by the *dispersions* in prices and wages whose natures are exogenous. Below, we illustrate these two sources of inefficiencies.

4.1 Sources of Inefficient Allocations: Monopoly Powers and Dispersions

First, the presence of monopoly powers in goods and labor markets in our model environment leads to the observed outcomes deviating from those under perfect competition, thus giving rise to the discrepancies from unobserved efficient allocations as well. These monopoly powers stem from the imperfect substitutability of intermediate goods and of specialized labor services—the monopoly power allows firms to set price on their final output above marginal cost on the one hand, and households to set wage on their labor above the marginal rate of substitution.

This imperfect substitutability results in a wedge—*price markup* or *wage markup*—in the *intratemporal* efficiency conditions, i.e., the equality of the marginal product of labor (*MPN*) between labor and final consumption, and the marginal rate of substitution (*MRS*) between consumption and leisure. Formally, and in consistent with our mode presentation, monopolistic competitions in goods and labor markets define the aggregate price markup, $\mathcal{M}_{p,t}$ and the wage markup, $\mathcal{M}_{w,t}$, as

$$P_t = \mathcal{M}_{p,t} MC_t,$$

and

$$\frac{W_t}{P_t} = \mathcal{M}_{w,t} MRS_t,$$

so that

$$MRS_t = \frac{1}{\mathcal{M}_{w,t}\mathcal{M}_{p,t}} MPN_t, \tag{12}$$

where the use of the definition of the nominal marginal cost $MC_t = W_t/MPN_t$ has been made.

The equilibrium price and wage markups, $\mathcal{M}_{p,t}$ and $\mathcal{M}_{w,t}$, vary over time due mainly to two reasons. The first source of the time-varying variation comes from the exogenous shifts in the substitutability of goods and labor services. The latter result in the elasticity of their demand, hence affecting firms and workers' market powers and desire markups. In algebraic terms, these shifts are captured by the stochastic innovations corresponding to the price markup shocks, $\mathcal{M}^n_{p,t}$, and the wage markup shocks, $\mathcal{M}^n_{w,t}$. The second source of markup variations in equilibrium stems from the presence of nominal rigidities, which prevent firms and workers from achieving their desired markups in their relevant optimality conditions at any given point in time. The consequent endogenous (average) markups are thus triggered by any types of shock that hits the economy.

Second, the cross-sectional price and wage dispersions also cause the inefficiencies in our economy. In equilibrium in which output is stabilized around its potential counterpart, desired prices and wages—those that agents would set in the absence of nominal rigidities—are in general time-varying. Due to markup shocks, for instance, desired markup changes, leading to the mechanical changes in the desired prices as well. Another reason for the time-varying desired prices is the coexistence of staggered nominal contracts. With sticky prices, an increase in the marginal product of labor, due for example to an increase in productivity, in turn results in a fall in firms' marginal costs. The changes in marginal costs in production eventually affect their desired price, leading to the difference repricing decisions by firms at different times.

The resulting cross-sectional price and wage dispersions go hand in hand overtime, leading to unstable and inefficient in price and wage inflation. This is due to the fact that as workers and firms have access to identical tastes and technologies to supply different amounts of hours and of intermediate goods.

To see why price and wage dispersions cause the inefficiency of an asymmetric distribution of the intermediate goods supply, we aggregate the labor service (3) by using labor demand (4) and goods demand (1)

$$\begin{aligned} N_t &\equiv \int_0^1 \int_0^1 N_t(f,\ h)\, df\, dh \\ &= \int_0^1 N_t(f) \int_0^1 \frac{N_t(f,\ h)}{N_t(f)}\, df\, dh \end{aligned}$$

$$=\Delta_{w,t}\int_0^1 N_t(f)df$$

$$=\Delta_{w,t}\Delta_{p,t}\left(\frac{Y_t}{Z_t}\right)^{\frac{1}{1-\alpha}}, \tag{13}$$

where

$$\Delta_{w,t}\equiv\int_0^1\left(\frac{W_t(h)}{W_t}\right)^{-\epsilon_{w,t}}dh,$$

and

$$\Delta_{p,t}\equiv\int_0^1\left(\frac{P_t(f)}{P_t}\right)^{\frac{-\epsilon_{p,t}}{1-\alpha}}df,$$

are measure of wage and price dispersions, respectively. Increase in wage and price dispersions have the same effect as a fall in aggregate productivity. The latter lowers the output of the final good and rising the unemployment rate, for any given level of labor input. Wage dispersion also reduces the utility of the average household due to the concavity of the labor aggregator given by (4).

All told, a stable unemployment is in general incompatible with the absence of cross-sectional dispersions in price and wage, and therefore with stable price and wage inflation. As a result, stabilization policy faces a trade-off between inflation and unemployment. Formally assessing the that policy trade-off requires the investigation of the economy characterized by the optimal monetary policy, and we will discuss the latter and the sources behind the trade-off in the next section.

4.2 The Welfare Loss Function

To investigate a formal and quantitative insight into the policy trade-off issues, in this section we turn to the model's optimal equilibrium allocation. The latter corresponds to the equilibrium chosen by a social planner that maximizes the utility of the representative household under commitment, subject to the constraints represented and obtained by the behavior of private agents. The instrument, and the only one available to that planner, is the short-term nominal interest rate, which thus defines our problem as one of optimal monetary policy.

Our optimal equilibrium is characterized by the seminal works of Rotemberg and Woodford (1997) and Clarida, Galí and Gertler (1998), both of which rely on a linear-quadratic approximation of the planner's problem suitable for economies with an inefficient steady state. We then compute the path of macro quantities that would have been observed if policy had always been optimal and the economy had also been perturbed by the same sequences of shocks that are estimated in the baseline specificaiton under the historical interest rate rule, except for ε_t^{π} and η_t^{r}. These two policy shocks are irrelevant to the optimal equilibrium as they only perturbe to the interest rate (Taylor) rule, which is now replaced by optimal policy.

Appendix to this paper derives a second-order approximation to households' discounted utility in the economy characterized by sticky wages and prices with a (small) steady state distortion. This amounts to the following *welfare loss function*:

$$\mathbb{W}=E_0\sum_{t=0}^{\infty}\left\{\frac{1}{2}\left[\frac{\epsilon_p}{\lambda_p}(\hat{\pi}_t^p-\iota_p\hat{\pi}_{t-1}^p)^2+\frac{\epsilon_w(1-\alpha)}{\lambda_w}(\hat{\pi}_t^w-\iota_w\hat{\pi}_{t-1}^p)^2+\left(\frac{1+\varphi}{1-\alpha}\right)(\widehat{ygap}_t)^2\right]-\Phi\hat{y}_t\right\}$$

$$+t.i.p.+\mathbb{O}(\|\xi\|^3), \quad (14)$$

where $\Phi\equiv1-\frac{1}{\mathcal{M}_p\mathcal{M}_w}>1$ is the steady state distortion, which implies that the latter is the increasing function of markups, $\lambda_p\equiv\frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}$, and $\lambda_w\equiv\frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w}\frac{1}{1+\varphi\epsilon_w}$. In the optimal policy, the Federal Reserve seeks to minimize the loss function (14) subject to the agents' equilibrium conditions described in Section 2. The Taylor rule (10) is replaced by the Ramsey policy accordingly.

5 Findings

With our estimated structural model in hand, this section explores relation between the observed 45 years of the US economy and the unobserved optimal frontier computed under the optimal policy.

After directly answering our research question: Is the Dual Mandate Achievable?, we compare the historical developments of the utility losses under the observed and unobserved policies, the sources behind the policy trade-off, and welfare loss fluctuations by making extensive use of the counterfactual exercises. Finally, the optimized simple rule—an alternative candidate policy rule that seeks to to attain the optimal allocation as much as possible—is compared in the form of the impulse response analysis.

5.1 Is the Dual Mandate Achievable?

Figure 1 plots observed price inflation and the unemployment rate of the US economy over our sample period of 1967 through 2012 (bold lines), as well as their counterfactual evolutions under the optimal monetary policy (dashed lines). The vertical axis for the price inflation denote the quarterly inflation rate. The figure (bottom) also displays the evolutions of the utility losses experienced by households as second order approximation of the latter mentioned in Section 6.4.1. The sold line corresponds to the actual utility losses, while the dashed line shows the values of the utility losses if monetary policy would have been conducted following the Ramsey policy.

Figure 1 makes clear that the monetary policy can not achieve the dual mandate. That is, monetary policy faces a trade-off between stabilization of inflation and unemployment over business cycles. Although the policy manages to stabilize price inflation to a large extent, unemployment shows larger fluctuations than those under the actual economy. Thus, monetary policy faces a nontrivial trade-off between price inflation and unemployment.

The one of the striking findings in this comparison exercise is that fluctuations in price inflation in stagflation—the plagued periods of 1970's and 1980's that experienced both high inflation and high unemployment rate at the same time—are greatly reduced under the Ramsey policy. The optimal allocation shows that the highest inflation of the US economy during 1970's, and its following surge in inflation in 1980's are both dampened so that the movements in this policy objective is stabilized around the target (quarterly) inflation value of about 0.7 percent. The subsequent inflation development keeps this less fluctuating behavior and keeps steadily stabilizing around this inflation target value.

A large reduction in volatility of inflation stems from no time-varying inflation target π_t^* under optimal policy. To what extent does this omittance of inflation target development affect the inflation volatility? To isolate the contribution of the former, we compute the standard deviation of price inflation in a version of the baseline model with the monetary policy rule but with a constant inflation target, so that the only shocks that enter into the policy rule are monetary policy shocks. The standard deviation of (quarterly) inflation in this counterfactual exercise suggests the value of 0.57, compared to 0.61 under the actual economy and 0.19 under the optimal policy. This evidence suggests that the estimated movements in the inflation target are the key source behind the suboptimality of observed inflation. This evidence is consistent with the same counterfactual simulation conducted by Justiniano, Primiceri and Tambalotti (2013), who also investigate the policy trade-offs under a mediums-scale DSGE model, but with no explicit treatment of the unemployment variable and data, to argue that the inflation target shocks are "integral part of the model's description of historical policy by Federal Reserve".

Unemployment, on the other hand, hinders the Federal Reserve to attain the dual mandate—in stark contrast to this stable price inflation under the optimal allocation, the unemployment development under that same counterfactual simulation economy shows no stabilization in that variable. That counterfactual variable shows the higher movements until 1980's, and it turns out to show larger fluctuations than those of the observed unemployment rates throughout the rest of the sample periods. The standard deviation of the unemployment rate in data is 1.66 (the actual economy), but the corresponding value under the Ramsey equilibrium increase to 1.89. That is, the optimal monetary policy identifies that there is tension between stabilizing inflation and unemployment. Our comparison exercise of actual and optimal allocations implies that, although the policy manages to stabilize price inflation to a large extent, but this is achieved at the cost of larger fluctuations in the unemployment rate. In other words, the Ramsey policy concludes that the dual mandate is not achievable.

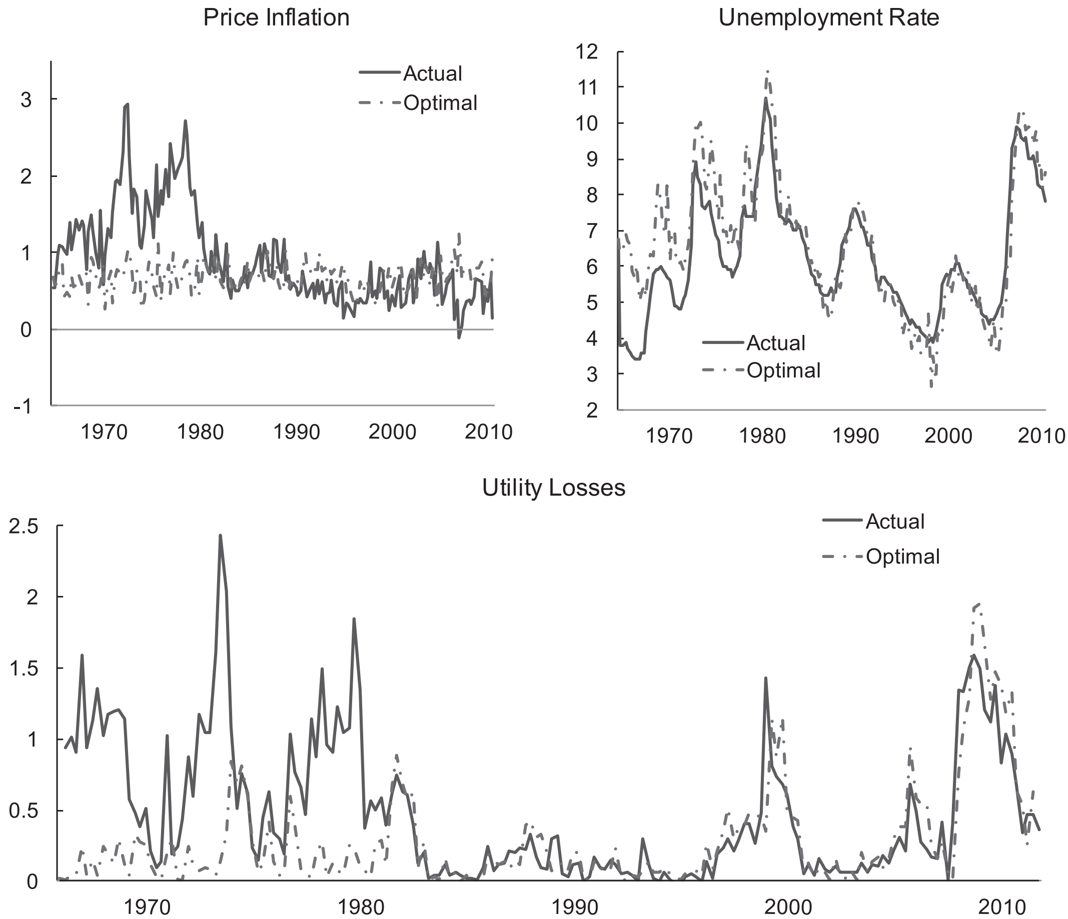

FIG. 1. ACTUAL AND OPTIMAL INFLATION AND THE UNEMPLOYMENT RATE, AND THE UTILITY LOSSES

5.2 How did historical welfare losses evolve over business cycles?

The previous subsection clarifies that, on the one hand the monetary policy can achieve stabilization in price inflation, but on the other hand it does so at the price of unemployment fluctuations. The reduced volatility in quarterly inflation (from 0.61 down to 0.19) and the increased volatility in unemployment (from 1.66 up to 1.89) under the optimal policy, however, makes it difficult for welfare-maximizing Federal Reserve to assess its eventual impact for monetary policy—does the lower volatility in inflation have more impact to effectively reduce the entire social welfare, or does much higher volatility in unemployment offset that impact? Answering this critical policy question requires a formal use of a quantitative measure of the welfare losses.

For that comprehensive assessment and quantitative insight into the trade-off, we resort to the welfare loss function obtained from the second-order Taylor approximation of the representative household. We plot the actual welfare losses experienced by the household as values computed by the loss function derived in Section 4.2. For comparison, we also plot the counterfactual welfare losses by assuming that the policy had been fully optimal over the sample period. This counterfactual experiment helps to identify when and to what extent the lower volatility in inflation under optimal policy contributes to reduce the welfare losses, and when and to what extent the higher volatility in unemployment, to increase losses.

Figure 1 compares the historical welfare losses under actual and optimal allocations. For the simulation for the optimal allocation, the counterfactual price and wage inflations with the corresponding values for the output gap are used to generate the relevant welfare losses.[5)] These values are particularly useful to assess the eventual outcome of optimal policy compared to the actual one since they quantitatively identify the outcomes of those conflicting results. The existing literature on monetary policy trade-offs (e. g., Justiniano, Primiceri and Tambalotti (2013) and Furlanetto, Gelain and Sanjani (2017)) just plots the time paths of policy targets under Ramsey policy, and they arbitrarily assess the policy-trade-offs without any quantitative criteria based on theory.

Figure 1, overall, makes clear that the actual economy experiences higher welfare losses during 1970's and 1980's, and 2000 and 2008. The optimal allocation, however, shows that the losses are substantially smaller in the former period, but the values are still high—even higher in 2008-2009—during the latter.

What makes that nontrivial discrepancy? To see why, we highlight three historical episodes of particular importance in terms of monetary policy conducts and debates—stagflation of 1970's and 1980's, the Great Moderation of mid-1980's throughout 2007, and the 2008 financial crisis. First, Figure 1 implies that the period of severe adverse macroeconomic periods in 1970's and 1980's can be remedies by means of optimal monetary policy. The twin peaks of utility losses that resemble those of inflation and unemployment during that period are greatly reduced under Ramsey equilibria. It is reasonably concluded that this damped utility losses can be attributed to the low volatility in inflation. As the observed unemployment rate in this stagflation period actually shows higher patterns, the impact of much muted inflation volatility is seen to outweigh the uprising pressure arising from unemployment.

Second, the periods that follow the stagnation—the Great Moderation spanning from mid-1980's to 2007—observe that the overall welfare losses are effectively close to zero, even under the actual allocation. The reduction in volatilities of business cycle fluctuations of many macro variables, including the policy targets of our concern in this study, are presumably the source of this near-zero utility losses enjoyed by the households. The end of the millennium and mid-2000 observe, however, a couple of hiccups in welfares. For example, the surge in the losses in 1999 could be attributed to the

5) The values of the welfare losses are computed by normalizing the coefficient for the price inflation to one, following many of the conventions such as Woodford (2011) and Galí (2015).

heating-ups in labor markets by the so-called dot coms bubbles. The striking feature of the rising welfare losses, however, is that the optimal allocation also implies large utility losses, and the magnitude is as the same order as the actual allocation. This is in stark constraint to the stagflation period of low utility losses, suggesting that the sources behind the welfare development could be substantially different from that period.

Third, and finally, the 2008 financial crisis displays again the upturn of the losses, but the latter under optimal policy shows even greater utility losses than that of the observed economy. This rather unexpected finding suggests that in effect the price stability achieved by the Ramsey policy during the financial devastation plays negligible roles in reducing social welfare. Together with a couple of hiccups in the utility losses discusses previously, the welfare losses identify that the sources behind the latter may alter significantly before and after 2000. The discussion of these three historical episodes thus demonstrate the importance of quantifying the welfare losses to measure to assess its eventual impact for monetary policy—while the reduced volatility in inflation effectively reduces the losses in stagnation, but the higher volatility in unemployment may play key roles during the post-2000 US economy.

5.3 What is the main source of the policy trade-off?

So far, we observe that the comparisons between actual and optimal allocations in price inflation and the unemployment rate show the monetary policy's inability to achieve its dual mandate. At the same time, the historical utility losses under both actual and optimal allocations suggest that optimal policy can not achieve small utility losses during the recessions in 2000 and the 2008 financial crisis. The nature questions that arise, then, are: (1) what is the main source of this nontrivial policy trade-off?, and (2) why optimal monetary policy can not deliver small utility losses after the 2000, compared to the much dampened losses during stagflation? In what follows, we show that among the potential sources of the policy trade-off, wage markup shocks are the ones that matter most quantitatively. If the volatility of desired wage markups is small, the trade-off is negligible. If the movements in desired markups are large, the trade-off becomes significant.

Figure 2 plots price inflations and the unemployment rates in both actual and optimal allocations of an economy consistent with previous figures, but now each of shocks has been arbitrarily set to zero, one at a time. Specifically, from the first row to the fifth, the panels show the optimal allocations with (1) no intertemporal shocks, (2) no intratemporal (labor supply) shocks, (3) no neutral technology shocks, (4) no price markup shocks, and lastly, (5) no wage markup shocks. The actual allocations are also depicted to facilitate comparisons. These counterfactual experiments are very similar to ones performed in Levin, Onatski, Williams and Williams (2006) and Justiniano, Primiceri and Tambalotti (2013).

Figure 2 confirms that the dual mandate can be achieved under the economy characterized by the

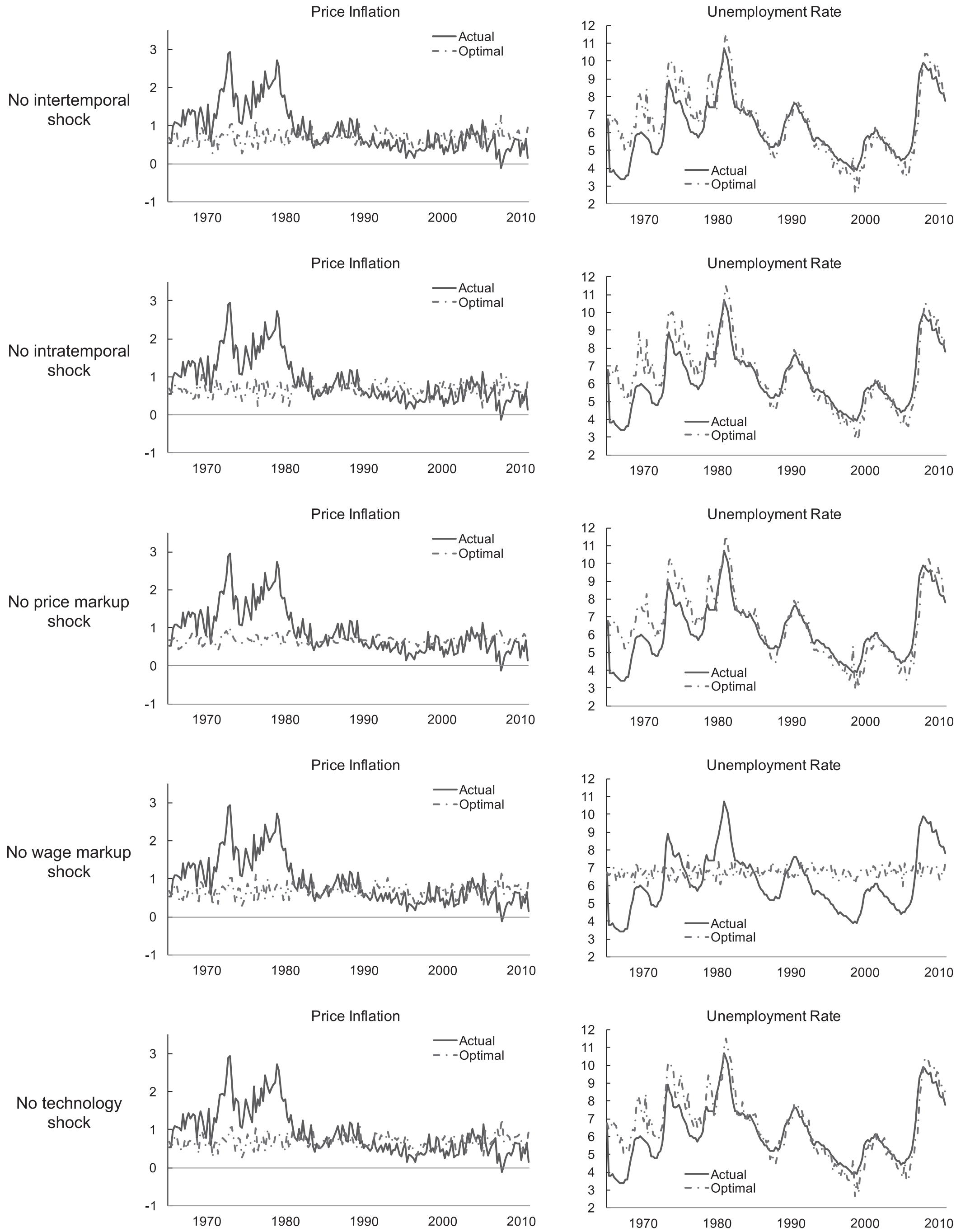

FIG. 2. COUNTERFACTUAL EXPERIMENTS

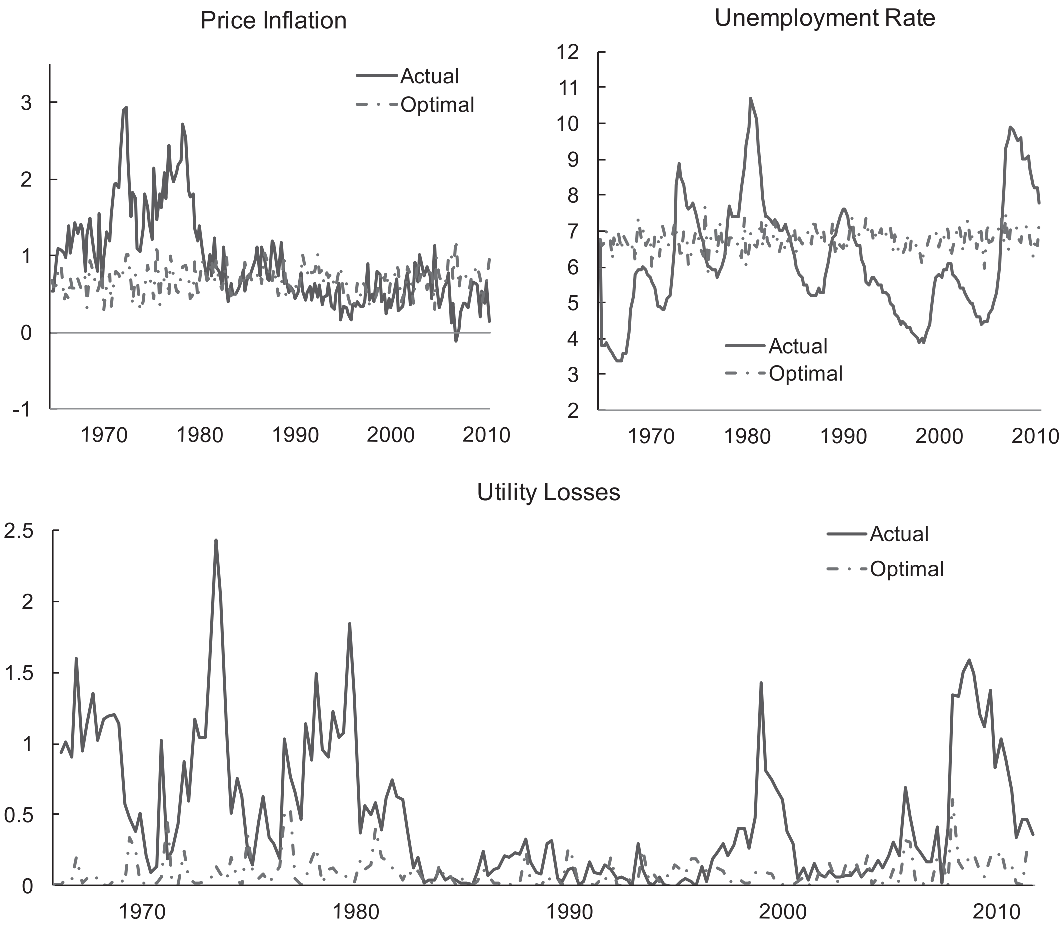

FIG. 3. ACTUAL AND OPTIMAL INFLATION AND THE UNEMPLOYMENT RATE, AND THE UTILITY LOSSES WITH NO WAGE MARKUP SHOCKS

optimal policy with no wage makeup shocks. In other words, the wage makeup shocks are the main source of the policy trade-off. The standard deviation of unemployment in this counterfactual environment is greatly reduced to 0.32, compared to 1.89 previously computed under the optimal policy (that is, under the Ramsey policy with all shocks). Another salient feature of this exercise is that the volatility of price inflation is even reduced under the optimal equilibria without price markup shocks. The standard deviation decreases from 0.19 (optimal, all shocks) to 0.11. The rest of the counterfactual movements depicted in panels also confirms no attainment of the dual mandate. Thus, we can conclude that the key driver behind the policy trade-off is wage markup shocks. Without these exogenous movements in worker's market power, the Ramsey policy show no tension between these two policy objectives.

Figure 3 again shows price inflation and the unemployment rate under the alternative policies with no wage markup shocks, but followed by the corresponding figure that plots the actual and optimal welfare losses. The panel at the bottom shows that the welfare losses during the stagnation are largely reduced under the optimal policy with no markup shocks. What is even more striking, however, is the much diminished pattern of the utility losses after 2000. Taken together with the result of no policy trade-off shown in the two panels above in the same figure, we can reasonably infer that wage markup shocks are not only the main source of the economy stability, but also are the key factor behind the welfare-minimization of the US economy. Our finding is in line with those of Blanchard and Galí (2010) who emphasize the important roles played by wage markup disturbances to achieve the divine coincidence, and is also empirically in line with the work by Galí, Smets and Wouters (2012b) who attribute large portions of fluctuations in unemployment as well as the output gap to those exogenous markup powers in workers. The results are also in line with the works by Justiniano, Primiceri and Tambalotti (2013) and Furlanetto, Gelain and Sanjani (2017) who reach the similar conclusions when the estimated their medium-scale DSGE model without using the two wage series (nominal compensation per hour and average hourly earnings of production no supervisory employees) nor explicit unemployment variable and data. Thus, our major contribution here is that we reach the same conclusions consistent with the existing literature, by providing empirical evidence of the direct assessment of the dual mandate, with its criteria formally measured by the development of the welfare loss function.

Table 2 summarizes the standard deviations of price and wage inflations, the unemployment rate, as well as the output gap, under different policy rules and under different counterfactual settings,

TABLE 2. STANDARD DEVIATIONS OF POLICY OBJECTIVES AND WELFARE LOSSES

	$\sigma(\pi_t^p)$		$\sigma(u_t)$		$\sigma(\pi_t^w)$		$\sigma(ygap_t)$		Welfare Loss	
Actual	0.61	(0.00)[a]	1.66	(0.00)	0.78	(0.00)	1.39	(0.00)	0.513	(0.00)
Optimal	0.19	(+69.4)	1.89	(Δ13.8)	0.46	(+40.5)	1.41	(Δ1.9)	0.288	(+44.0)
Optimal										
No preference	0.19	(+69.4)	1.89	(Δ13.8)	0.46	(+40.5)	1.41	(Δ1.9)	0.288	(+44.0)
No technology	0.18	(+70.8)	1.89	(Δ13.7)	0.45	(+41.4)	1.42	(Δ2.3)	0.286	(+44.2)
No labor supply	0.18	(+69.7)	1.89	(Δ13.9)	0.45	(+42.1)	1.41	(Δ1.9)	0.285	(+44.5)
No price markup	0.11	(+81.7)	1.88	(Δ13.1)	0.28	(+64.2)	1.43	(Δ3.3)	0.259	(+49.6)
No wage markup	0.18	(+69.6)	0.32	(+80.6)	0.45	(+41.5)	0.57	(+59.2)	0.090	(+82.4)

[a] Numbers in parenthesis show percentage gains/losses from values in Actual.

TABLE 3. ESTIMATED AND OPTIMIZED POLICY PARAMETERS

	ϕ_r	ϕ_π^p	ϕ_π^w	$\phi_{\Delta y}$	ϕ_u
Taylor	0.700[a]	2.404	—	0.267	—
OSR	0.473	4.999	3.263	-0.196	-0.130

[a] The smoothing parameter for the original Taylor specification is calibrated to the values estimated by Justiniano, Primiceri and Tambalotti (2013).

followed by the corresponding values of the welfare losses. The first row lists the standard deviations and the resulting welfare loss under the actual policy (i.e., the Taylor rule). The second and the following rows list the same values as above under the optimal policy, followed by five alternative values obtained by the counterfactual exercises described above. The numbers in parenthesis report percentage gains or losses of the standard deviations of optimal policies compared to those of the actual policy. Also, those values in welfare losses help to fairly and quantitatively evaluate the alternative policy options, without relying on any arbitrary judgments with no formal theory.

Table 2 confirms, in line with the evidence obtained so far, that the optimal allocation with no wage markup shocks achieves the minimum welfare loss of 0.09. The table identifies that the much reduced volatility in the unemployment rate by optimal policy (from 1.89 to 0.32), as well as that in the output gap (1.41 to 0.57) are the main contributors for this large reduction in the welfare loss, since the other statistics do not significantly differ among policy options. The exogenous wage markup disturbances have nonnegligible impact in achieving the dual mandate to attain the minimal welfare losses.

5.4 What kind of rule is close to the optimal policy?: The Optimized Simple Rule

Although the optimal policy can not attain the joint stabilization of price inflation and unemployment slack, much of the optimal monetary policy literature suggests that the latter is still the best allocation for the central banks to pursue in their conduct of monetary policy. That is, there still remains an substantial incentive for any central banks who seek to maximize the welfare of consumers to design policies that are close to the optimal allocation. In this light, and to pursue that best possible options for the Federal Reserve as our another contribution of this study, this last subsection seeks to provide the policy that can replicate those optimal allocation suggested by Ramsey policy as much as possible. Doing so in a tractable and parsimonious way, we restrict ourselves to the development of the *optimal simple rule* (OSR, hereafter). Conditional on each shock except for the ones in the original Taylor rule in (10), we determined the setting of the coefficients that minimize the unconditional period utility loss suggested by (14). The OSR considered here is particular case of the following specification in the spirit of Taylor:

$$\widehat{R}_t = \phi_r \widehat{R}_{t-1} + (1-\phi_r)[\phi_\pi^p \widehat{\pi}_t^p + \phi_\pi^w \widehat{\pi}_t^w + \phi_{\Delta y}(\widehat{y}_t - \widehat{y}_{t-1}) + \phi_u \widehat{u}_t]. \quad (15)$$

Taking into account the key roles played by the wage markup shocks as well as the nonstabilizing unemployment behaviors, we add these variables to the original Taylor rule specification. This simple yet general specification is of particular help to interpret to what extent that policy reaction rule should react to the variables as its arguments.

Table 3 reports the resulting values for the policy coefficients for the OSR, together with the values estimated for the model economy described in Section 6.2 for comparison purposes. The OSR shows less interest smoothing compared to the (calibrated) value for the original (actual) Taylor rule. The persistence of the policy rates drops from 0.700 to 0.473. Instead, the optimized policy rule responds more strongly to the current state of the economy proxied as the observed variables in price inflation, wage inflation, and the unemployment rate.

Among others, the most salient feature among the optimized coefficients is its strong reaction to development in price inflation—the coefficient goes up to 4.999 from the estimated value of 2.404. This increased value implies that the Federal Reserve should react to the current price inflation more aggressively.

The optimized rule also implies the strong policy reaction to inflation in wages, implying that inflations in price and wages have nonnegligible impact in replicating the optimal (Ramsey) allocation, and the original Taylor-type rule could perform better by incorporating the latter as its argument.

Surprisingly, the OSR changes the sign of the policy coefficient to the output growth. It alters from moderate and positive reaction of 0.267 to the negative and slightly less aggressive value of -0.196. This finding is consistent with the similar optimized rule comparisons by Galí (2011), who reports that some specifications of the OSR result in the negative value for the output (gap) once wage inflations and the unemployment rate are explicitly incorporated.

Finally, and as expected, the optimized rule shows that the latter takes countermeasure to the unemployment fluctuations—the negative coefficient of -0.130 to the latter suggests that when Federal Reserve observes the rise in the unemployment rate, it should decrease the nominal interest rate to stimulate aggregate demand to maintain (maximum) employment. Thus, and with our OSR with the coefficients describe so far in hand, we are now ready to explore to what extent that optimized rule can be close (or replicate) the Ramsey allocation.

Figure 4 compares impulse responses of price inflation and the unemployment rate, followed by two additional variables relevant to the stabilization policy in investigation—the output gap and the wage inflation. Each column lists the impulse responses to each shocks, namely and from the first to the fifth, the intertemporal shock, the intratemporal (labor supply) shock, the natural technology shock, the price markup shock and the wage markup shock. In order to ease the comparison, each of the left axes, which represents the percentage deviations from zero, is identical across the same variable—horizontally we plot the impulse responses of the same variable but to the different shocks, with the same scale of the left axis. This alignment helps to compare the magnitude of each shock—transitory as well as persistent impacts of shocks in consideration.

Overall, the impulse responses confirm the nonnegligible policy trade-off between price inflation and unemployment stemming from the wage markup shocks. Although the response of price inflation to those shocks can be effectively reduced to zero, unemployment shows large reactions at impact. That impact persists well into the long-run, showing that the impulse does not die out after twenty quarters. This large and persistent response does not depend on the policy rules in comparison—although the optimal policy can reduce the reactions of the unemployment rate by about one percent compared to the other policies (actual and OSR), but it still shows greater responses compared to the other shocks. The rest of the shocks—intertemporal, intratemporal, technology, and price markup—show some transitory impact on these four variables, but their eventual effects, especially under the optimal policy and OSR, almost disappear within 10 quarters. Thus, and overall, the impulse response analysis confirms our previous finding—wage markup shocks are the source of the tension between the price stability and that of unemployment.

6 Conclusions

Is the dual mandate—joint stabilization of inflation and unemployment—achievable? No, is the answer we find in an estimated DSGE model of involuntary unemployment for the US economy in which optimal policy shows the tension between stabilizations of inflation and unemployment due to the exogenous movements in worker's market power.

Our counterfactual experiments also show in the absence of the movements in workers' exogenous market powers emerges the joint stabilization of the two policy objectives, with effectively minimal historical welfare losses. We reach this and the similar conclusion with the existing literature on optimal policy of the *divine coincidence* (Blanchard and Galí (2010)) and the *trinity* (Justiniano, Primiceri and Tambalotti (2013) and Furlanetto, Gelain and Sanjani (2017)), but we do so by directly judging the attainment of the dual mandate—by combining the explicit measure of unemployment in the spirit of Galí (2011), and the formal development of the welfare loss function in the spirit of Woodford (2011) and Clarida, Galí and Gertler (1998).

Finally, our optimized and simple Taylor-type rule is also shown to replicate the responses of key policy objectives to those of the optimal allocation well. The impact of shocks other than in wage

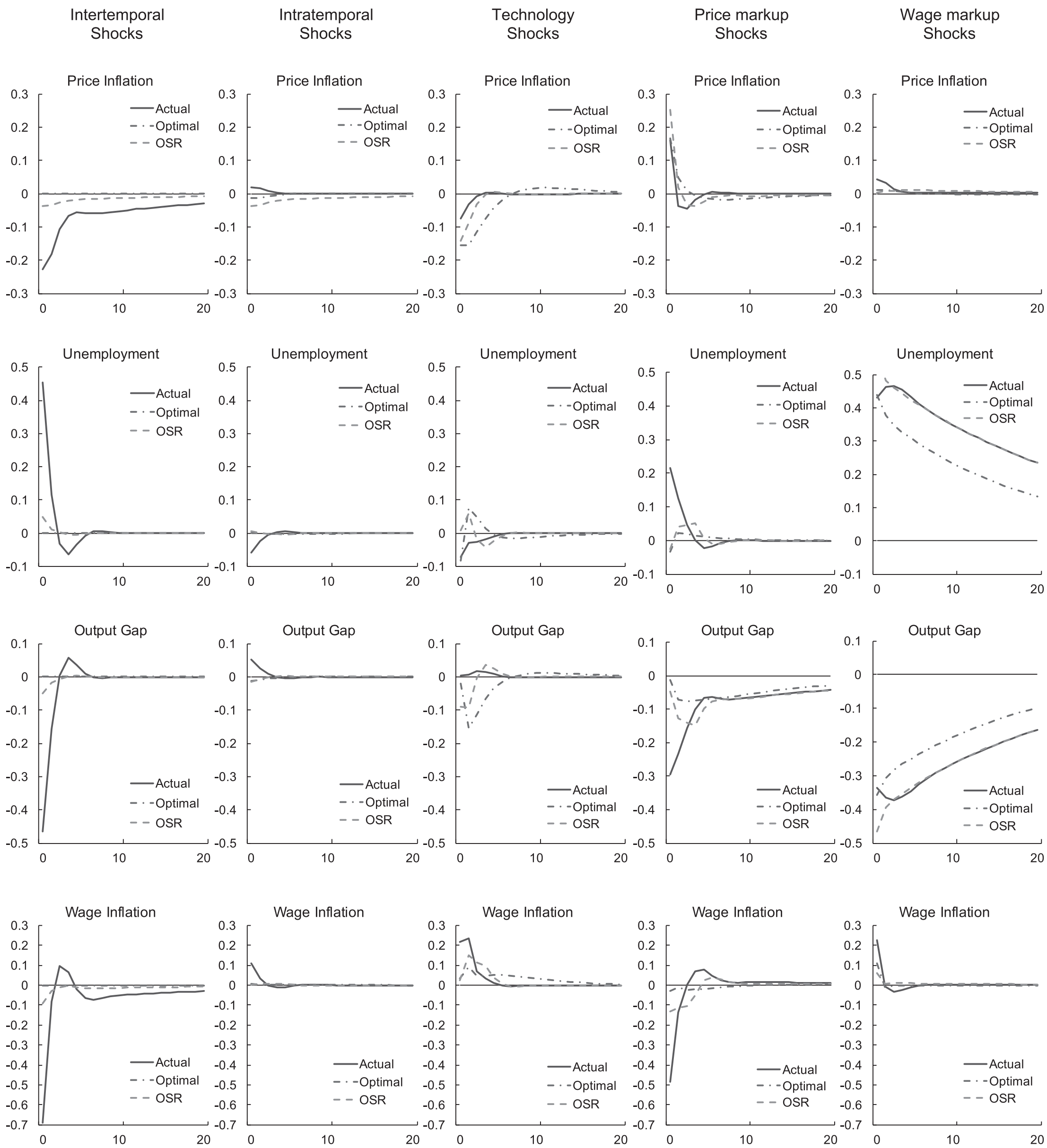

FIG. 4. IMPULSE RESPONSES

markups—the impact of shocks in demand (intertemporal), supply (intratemporal and technology) and another market competitiveness (price markup) to the inflations in price and wages, unemployment and the output gaps are shown to be effectively eliminated under the Ramsey as well as the optimized simple rule.

One limitation of our approach—and the one that much of the optimal policy literature also follows—is to abstract capital for the analysis of social welfare (e.g., Woodford (2011), Clarida, Galí and

Gertler (1998), Galí (2011), Galí (2015), Galí (2019), Erceg, Henderson and Levin (2000)). This allows us to quantify the exact welfare losses experienced by consumers in any given historical episodes (e.g., stagflation), and formally assess the attainment of the dual mandate. In an environment in which the agents' decisions on capital accumulation is considered, however, the output gap and markup stabilization are not equivalent, due to the presence of investment that shares the aggregate income with consumption. As a result the policy trade-off is not only between output and inflation stabilization, but also involves the composition of demand between consumption and investment. Although the presence of capital accumulation makes it difficult to obtain the explicit expression for the utility loss function, the future research will be fruitful if one explicitly considers the investment and the capital formation.

7 Appendix

7.1 Deriving The Welfare Loss Function

This appendix derives a second-order approximation to the utility of the average representative household (14) consistent with the assumptions made in the main text. That is, the presence of uncorrected real distortions such as the firms' market power in goods market generate a permanent gap between the observed macro variables and the unobserved efficient ones, reflected in an inefficient steady state. The size of the steady state distortion is measured by a parameter Φ representing the wedge between the marginal product of labor and the marginal rate of substitution between consumption and hours, both evaluated at the steady state. Formally, and in consistent with the notation presented in (12), Φ is defined by

$$MRS=(1-\Phi)MPN, \tag{16}$$

where $\Phi\equiv 1-\dfrac{1}{\mathcal{M}_p\mathcal{M}_w}>0$ implies that the steady state levels of output and employment are below their respective efficient levels.

A second-order approximation of utility is derived around a given steady state allocation. A frequent use is made of the following second-order approximation of relative deviations in terms of log deviations:

$$\frac{X_t-X}{X}\simeq \hat{x}_t+\frac{1}{2}\hat{x}_t^2,$$

where $\hat{x}_t\equiv x_t-x\equiv\ln(X_t/X)$ is the log deviation from steady state for a generic variable X_t.

The second-order Taylor expansion to the utility U_t around a steady state U is given by

$$U_t=\sum_{k=0}^{2}\frac{1}{k!}\left\{(C_t-C)\frac{\partial}{\partial C_t}+\int_0^1(N_t(h)-N)\frac{\partial}{\partial N_t(h)}dh+(\Theta_t-\Theta)\frac{\partial}{\partial\Theta_t}+(\varepsilon_t^b-\varepsilon^b)\frac{\partial}{\partial\varepsilon_t^b}+(\varepsilon_t^l-\varepsilon^l)\frac{\partial}{\partial\varepsilon_t^l}\right\}^k U$$
$$+\mathbb{O}\left(\|\xi\|^3\right),$$

where $\mathbb{O}(\|\xi\|^3)$ are the terms of order three or higher. Noting that some terms (e.g., exogenous disturbances) are independent of monetary policy, the second-order approximation above yields

$$\begin{aligned}U_t - U = & U_C C\left(\frac{C_t - C}{C}\right) + U_N N \int_0^1 \frac{N_t(h) - N}{N} dh \\ & + \frac{1}{2} U_{CC} C^2 \left(\frac{C_t - C}{C}\right)^2 + \frac{1}{2} U_{NN} N^2 \int_0^1 \left(\frac{N_t(h) - N}{N}\right)^2 dh \\ & + U_{C\varepsilon^b} C \varepsilon^b \left(\frac{C_t - C}{C}\right)\left(\frac{\varepsilon_t^b - \varepsilon^b}{\varepsilon^b}\right) + U_{C\varepsilon^l} C \varepsilon^l \left(\frac{C_t - C}{C}\right)\left(\frac{\varepsilon_t^l - \varepsilon^l}{\varepsilon^l}\right) \\ & + U_{N\varepsilon^b} N \varepsilon^b \int_0^1 \frac{N_t(h) - N}{N} dh \left(\frac{\varepsilon_t^b - \varepsilon^b}{\varepsilon^b}\right) \\ & + U_{N\varepsilon^l} N \varepsilon^l \int_0^1 \frac{N_t(h) - N}{N} dh \left(\frac{\varepsilon_t^l - \varepsilon^l}{\varepsilon^l}\right) \\ & + t.i.p. + \mathbb{O}(\|\xi\|^3),\end{aligned}$$

where U_X stands for the partial derivative of U_t with respect to X_t evaluated at the steady state, and *t.i.p.* stands for *terms independent of policy*. The use has been made with $U_{CN}=0$ as implied by the separability of households' utility function (Galí (2011)).

The approximation above is further rewritten in terms of log deviations

$$\begin{aligned}U_t - U = & U_C C\{(1 + \varepsilon_t^b + \varepsilon_t^l)\hat{y}_t\} \\ & + U_N N\left\{(1 + \varepsilon_t^b + \varepsilon_t^l)\int_0^1 \hat{n}_t(h) dh + \frac{1+\varphi}{2}\int_0^1 \hat{n}_t(h)^2 dh\right\} \\ & + t.i.p. + \mathbb{O}(\|\xi\|^3),\end{aligned}$$

where $\sigma \equiv -\frac{U_{CC}C}{U_C} = 1$ (log-utility) and $\varphi \equiv \frac{U_{NN}N}{U_N}$, and where use of the market clearing condition $\hat{c}_t = \hat{y}_t$ has been made.

Notice that the second order approximation to the wage index (2.3) can be expressed as log deviation: $1 = \int_0^1 \hat{w}_t(h)^{1-\epsilon_{w,t}} dh$, and further can be rewritten in terms of some exceptional terms (i.e., mean and variance) as follows:

$$\begin{aligned}1 & \simeq 1 + (1 - \epsilon_w)\int_0^1 \hat{w}_t(h) dh + \frac{(1-\epsilon_w)^2}{2}\int_0^1 \hat{w}_t(h)^2 dh \\ \Leftrightarrow \quad E_t\{\hat{w}_t(h)\} & \simeq \frac{\epsilon_w - 1}{2} E_t\{\hat{w}_t(h)^2\} \\ \Leftrightarrow \quad E_t\{\hat{w}_t(h)\} & \simeq \frac{\epsilon_w - 1}{2} var_h\{\hat{w}_t(h)\}.\end{aligned}$$

Also notice that, consistent with the assumptions made in the main text, aggregate employment can be defined as $N_t \equiv \int_0^1 N_t(h) dh$, or, in terms of log deviations from steady state and up to a second-order approximation

$$\widehat{n}_t+\frac{1}{2}\widehat{n}_t^2\simeq\int_0^1\widehat{n}_t(h)dh+\frac{1}{2}\int_0^1\widehat{n}_t(h)^2dh.$$

The last term can be further expressed as

$$\begin{aligned}\int_0^1\widehat{n}_t(h)^2dh&=\int_0^1(\widehat{n}_t(h)-\widehat{n}_t+\widehat{n}_t)^2dh\\&=\widehat{n}_t^2-2\widehat{n}_t\epsilon_w\int_0^1\widehat{w}_t(h)dh+\epsilon_w^2\int_0^1\widehat{w}_t(h)^2dh\\&=\widehat{n}_t^2+\epsilon_w^2var_h\{\widehat{w}_t(h)\},\end{aligned}$$

where the second lines makes use of the log-linear approximation of the labor demand equation (4): $\widehat{n}_t(h)-\widehat{n}_t=-\epsilon_w\widehat{w}_t(h)$, whereas the last line makes use of the result in (17). Combining these results yields

$$\begin{aligned}U_t-U=&U_CC\{(1+\varepsilon_t^b+\varepsilon_t^l)\widehat{y}_t\}\\&+U_NN\left[(1+\varepsilon_t^b+\varepsilon_t^l)\widehat{n}_t+\frac{1+\varphi}{2}\widehat{n}_t^2+\frac{\epsilon_w^2\varphi}{2}var_h\{\widehat{w}_t(h)\}\right]\\&+t.i.p.+\mathbb{O}(\|\xi\|^3).\end{aligned}$$

Next, we derive a relationship between aggregate employment and output, as shown in (13) in Section 4.2. The log aggregate output and log aggregate employment are related, up to a second-order

$$(1-\alpha)\widehat{n}_t=\widehat{y}_t+\delta_t^w+\delta_t^p,$$

where $\delta_{w,t}\equiv(1-\alpha)\int_0^1\{w_t(h)\}^{-\epsilon_{w,t}}dh$ and $\delta_{p,t}\equiv(1-\alpha)\int_0^1\{p_t(f)\}^{\frac{-\epsilon_{p,t}}{1-\alpha}}df$. Using analogous approximations to (17), these two dispersion terms can be expressed as

$$\begin{aligned}\delta_{w,t}&\simeq\frac{\epsilon_w(1-\alpha)}{2}var_h\{\widehat{w}_t(h)\}\\\delta_{p,t}&\simeq\frac{\epsilon_p}{2\Theta}var_f\{\widehat{p}_t(h)\},\end{aligned}$$

where $\Theta\equiv\dfrac{1-\alpha}{1-\alpha-\alpha\epsilon_p}\in(0,\ 1]$

Hence, and by using the equations obtained so far, deviations of period utility from steady state can be expressed as

$$\begin{aligned}U_t-U=&U_CC\{(1+\varepsilon_t^b+\varepsilon_t^l)\widehat{y}_t\}\\&-\frac{U_NN}{1-\alpha}\left[(1+\varepsilon_t^b+\varepsilon_t^l)\widehat{y}_t+\frac{\epsilon_p}{2\Theta}var_f\{\widehat{p}_t(f)\}+\frac{\Gamma}{2}var_h\{\widehat{w}_t(h)\}+\frac{1+\varphi}{2(1-\alpha)}\widehat{y}_t^2\right]\\&+t.i.p.+\mathbb{O}(\|\xi\|^3),\end{aligned}$$

where $\Gamma\equiv\varepsilon_w(1-\alpha)(1+\epsilon_w\varphi)$.

Using the fact that $MPN=(1-\alpha)(Y/N)$ and $Y=C$, and recalling that Φ denotes the size of the steady state distortion defined in (16), the previous equation can be rewritten as

$$\frac{U_t-U}{U_C C}=(1+\varepsilon_t^b+\varepsilon_t^l)\hat{y}_t$$
$$-(1-\Phi)\left[(1+\varepsilon_t^b+\varepsilon_t^l)\hat{y}_t+\frac{\epsilon_p}{2\Theta}var_f\{\hat{p}_t(f)\}+\frac{\Gamma}{2}var_h\{\hat{w}_t(h)\}+\frac{1+\varphi}{2(1-\alpha)}\hat{y}_t^2\right]$$
$$+t.i.p.+\mathbb{O}(\|\xi\|^3).$$

The assumption of "small distortion" implies that the product of Φ with second-order terms is taken to be negligible

$$\frac{U_t-U}{U_C C}=\Phi\hat{y}_t-\frac{1}{2}\left[\frac{\epsilon_p}{\Theta}var_f\{\hat{p}_t(f)\}+\Gamma var_h\{\hat{w}_t(h)\}+\frac{1+\varphi}{(1-\alpha)}\hat{y}_t^2\right]$$
$$+t.i.p.+\mathbb{O}(\|\xi\|^3)$$
$$=\Phi\widehat{ygap}_t-\frac{1}{2}\left[\frac{\epsilon_p}{\Theta}var_f\{\hat{p}_t(f)\}+\Gamma var_h\{\hat{w}_t(h)\}+\frac{1+\varphi}{(1-\alpha)}\widehat{ygap}_t^2\right]$$
$$+t.i.p.+\mathbb{O}(\|\xi\|^3),$$

where the use has been made with the log deviation of the potential output given by $\hat{y}_t^n=0$, thus $\widehat{ygap}_t\equiv\hat{y}_t-\hat{y}_t^n=\hat{y}_t$, in an environment with no habit formation but with the (stochastic) trend, which requires the model's real quantities are expressed in terms of normalized (stationary) quantities.

Expressed as a fraction of steady state consumption, the second-order approximation to the consumer's welfare losses can be written as follows:

$$\mathbb{W}=-E_0\sum_{t=0}^{\infty}\beta^t\left(\frac{U_t-U}{U_C C}\right)+t.i.p.+\mathbb{O}(\|\xi\|^3)$$
$$=-E_0\sum_{t=0}^{\infty}\beta^t\left(\Phi\hat{y}_t-\frac{1}{2}\left[\frac{\epsilon_p}{\Theta}var_f\{\hat{p}_t(f)\}+\Gamma var_h\{\hat{w}_t(h)\}+\frac{1+\varphi}{(1-\alpha)}\hat{y}_t^2\right]\right)$$
$$+t.i.p.+\mathbb{O}(\|\xi\|^3).$$

The final step involves rewriting both price and wage dispersions as functions of inflations in price and wages. We do so by resorting to the following lemma shown by Woodford (2011) (chapter 6):

$$\sum_{t=0}^{\infty}\beta^t var_f\{\hat{p}_t(f)\}=\frac{\theta_p}{(1-\theta_p)(1-\theta_p\beta)}\sum_{t=0}^{\infty}\beta^t(\hat{\pi}_t^p-\iota_p\hat{\pi}_{t-1}^p)$$
$$\sum_{t=0}^{\infty}\beta^t var_h\{\hat{w}_t(h)\}=\frac{\theta_w}{(1-\theta_w)(1-\theta_w\beta)}\sum_{t=0}^{\infty}\beta^t(\hat{\pi}_t^w-\iota_w\hat{\pi}_{t-1}^p).$$

Combining the previous lemma allows us to finally obtain the welfare loss function (14)

$$\mathbb{W}=E_0\sum_{t=0}^{\infty}\left[\frac{1}{2}\left\{\frac{\epsilon_p}{\lambda_p}(\hat{\pi}_t^p-\iota_p\hat{\pi}_{t-1}^p)^2+\frac{\epsilon_w(1-\alpha)}{\lambda_w}(\hat{\pi}_t^w-\iota_w\hat{\pi}_{t-1}^p)^2+\left(\frac{1+\varphi}{1-\alpha}\right)(\widehat{ygap}_t)^2\right\}-\Phi\hat{y}_t\right]$$
$$+t.i.p.+\mathbb{O}(\|\xi\|^3),$$

where $\lambda_p\equiv\frac{(1-\theta_p)(1-\theta_p\beta)}{\theta_p}\Theta$ and $\lambda_w\equiv\frac{(1-\theta_w)(1-\theta_w\beta)}{\theta_w(1+\epsilon_w\varphi)}$.

7.2 Source of Data

Six US quarterly times series beginning from 1967Q1 to 2012Q4 are employed for estimation. We follow Lindé, Smets and Wouters (2016), Galí, Smets and Wouters (2012a), and Galí, Smets and Wouters (2012b) for the construction of our data set. The real GDP is constructed by diving the nominal GDP (expressed in billions of chained 2005 dollar) by Implicit Price Deflator (100 in 2005), and both series are obtained from the US Department of Commerce, Bureau of Economic Analysis. The real wage is given by the BLS measure of compensation per hour for the non-farm business sector (FRED mnemonic "COMPNFB" / BLS series "PRS85006103") divided by Implicit Price Deflator (100 in 2005). Hours worked is obtained from the index of average weekly non-farm business hours (FRED mnemonic / BLS series "PRS85006023"), divided by the number of employed civilians (FRED mnemonic "CE16OV"), and also is normalized (1992Q3 value is set to 1). The federal funds rate is the effective federal funds rate in percent provided by the Bond of Governors of the Federal Reserve System. The unemployment rate is obtained from FRED (LNS14000000). All the series except for the federal funds rate and the unemployment rate are seasonally adjusted.

References

Bernanke, Ben, Mark Gertler, and Simon Gilchrist, "Chapter 21 The financial accelerator in a quantitative business cycle framework," in "in," Vol. 1 of *Handbook of Macroeconomics*, Elsevier, 1999, pp. 1341-1393.

Blanchard, Olivier and Charles M Kahn, "The Solution of Linear Difference Models under Rational Expectations," *Econometrica*, July 1980, *48* (5), 1305-1311.

__ and Jordi Galí, "Labor Markets and Monetary Policy: A New Keynesian Model with Unemployment," *American Economic Journal: Macroeconomics*, April 2010, *2* (2), 1-30.

Boivin, Jean and Marc Giannoni, "DSGE Models in a Data-Rich Environment," NBER Working Papers 12772, National Bureau of Economic Research, Inc December 2006.

Calvo, Guillermo A., "Staggered prices in a utility-maximizing framework," *Journal of Monetary Economics*, 1983, *12* (3), 383-398.

Chari, V. V., Patrick J. Kehoe, and Ellen R. McGrattan, "New Keynesian Models: Not Yet Useful for Policy Analysis," *American Economic Journal: Macroeconomics*, 2009, *1* (1), 242-266.

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans, "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, February 2005, *113* (1), 1-45.

__, Mathias Trabandt, and Karl Walentin, "Involuntary Unemployment and the Business Cycle," Working Paper 15801, National Bureau of Economic Research March 2010.

__, __, and __, "Introducing financial frictions and unemployment into a small open economy model," *Journal of Economic Dynamics and Control*, 2011, *35* (12), 1999 - 2041. Frontiers in Structural Macroeconomic Modeling.

__, Roberto Motto, and Massimo Rostagno, "Risk Shocks," *American Economic Review*, January 2014, *104* (1), 27-65.

Clarida, Richard, Jordi Galí, and Mark Gertler, "Monetary policy rules in practice Some international evidence," *European Economic Review*, June 1998, *42* (6), 1033-1067.

__, __, and __, "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory," *The Quarterly Journal of Economics*, 2000, *115* (1), 147-180.

Cogley, Timothy and Thomas J. Sargent, "The conquest of US inflation: Learning and robustness to model uncertainty," *Review of Economic Dynamics*, April 2005, *8* (2), 528-563.

Eggertsson, Gauti B. and Michael Woodford, "The Zero Bound on Interest Rates and Optimal Monetary Policy," *Brookings Papers on Economic Activity*, 2003, *34* (1), 139-235.

Erceg, Christopher J., Dale W. Henderson, and Andrew T. Levin, "Optimal monetary policy with staggered wage and price contracts," *Journal of Monetary Economics*, October 2000, *46* (2), 281-313.

Furlanetto, Francesco, Paolo Gelain, and Marzie Taheri Sanjani, "Output gap, monetary policy trade-offs and financial frictions," Working Paper 2017/8, Norges Bank April 2017.

Galí, Jordi, *Unemployment Fluctuations and Stabilization Policies: A New Keynesian Perspective*, MIT Press, 2011.

__, *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and Its Applications Second edition*, 2 ed., Princeton University Press, 2015.

__, "The effects of a money-financed fiscal stimulus," *Journal of Monetary Economics (forthcoming)*, 2019.

__, Frank Smets, and Rafael Wouters, "Slow Recoveries: A Structural Interpretation," *Journal of Money, Credit and Banking*, 2012, *44* (s2), 9-30.

__, __, and __, "Unemployment in an Estimated New Keynesian Model," *NBER Macroeconomics Annual*, 2012, *26* (1), 329-360.

Goodfriend, Marvin and Robert King, "The New Neoclassical Synthesis and the Role of Monetary Policy," in "NBER Macroeconomics Annual 1997, Volume 12," National Bureau of Economic Research, Inc, 1997, pp. 231-296.

Ireland, Peter N., "Changes in the Federal Reserve's Inflation Target: Causes and Consequences," *Journal of Money, Credit and Banking*, December 2007, *39* (8), 1851-1882.

Judd, John P. and Glenn D. Rudebusch, "Taylor's rule and the Fed, 1970-1997," *Economic Review*, 1998, pp. 3-16.

Jung, Taehun, Yuki Teranishi, and Tsutomu Watanabe, "Optimal Monetary Policy at the Zero-Interest-Rate Bound," *Journal of Money, Credit and Banking*, 2005, *37* (5), 813-35.

Justiniano, Alejandro, Giorgio E. Primiceri, and Andrea Tambalotti, "Is There a Trade-Off between Inflation and Output Stabilization?," *American Economic Journal: Macroeconomics*, April 2013, *5* (2), 1-31.

Koenig, Evan F., Robert Leeson, and George A. Kahn, eds, *The Taylor Rule and the Trans- formation of Monetary Policy* number 4. In 'Books.', Hoover Institution, Stanford University, October 2012.

Levin, Andrew, Alexei Onatski, John Williams, and Noah Williams, "Monetary Policy Under Uncertainty in Micro-Founded Macroeconometric Models," in "NBER Macroeconomics Annual 2005, Volume 20," National Bureau of Economic Research, Inc, 2006, pp. 229-312.

Lindé, Jespter., F. Smets, and R. Wouters, "Chapter 28 - Challenges for Central Banks' Macro Models," in John B. Taylor and Harald Uhlig, eds., *John B. Taylor and Harald Uhlig, eds.*, Vol. 2 of *Handbook of Macroeconomics*, Elsevier, 2016, pp. 2185-2262.

Lubik, Thomas A. and Frank Schorfheide, "Testing for Indeterminacy: An Application to U.S. Monetary Policy," *American Economic Review*, March 2004, *94* (1), 190-217.

Merz, Monika, "Search in the labor market and the real business cycle," *Journal of Monetary Economics*, 1995, *36* (2), 269-300.

Orphanides, Athanasios, "Historical monetary policy analysis and the Taylor rule," *Journal of Monetary Economics*, July 2003, *50* (5), 983-1022.

Phillips, A. W., "The Relation Between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861ÃćÂĂÂŞ19571," *Economica*, 1958, *25* (100), 283-299.

Primiceri, Giorgio E., "Why Inflation Rose and Fell: Policy-Makers' Beliefs and U. S. Postwar Stabilization Policy," *The Quarterly Journal of Economics*, 2006, *121* (3), 867-901.

Rotemberg, Julio and Michael Woodford, "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy," in "NBER Macroeconomics Annual 1997, Volume 12," National Bureau of Economic Research, Inc, 1997, pp. 297-361.

Sala, Luca, Ulf Söderström, and Antonella Trigari, "The Output Gap, the Labor Wedge, and the Dynamic Behavior of Hours," *IGIER (Innocenzo Gasparini Institute for Economic Research), Bocconi University, Working Papers*, 01 2010.

Schmitt-Grohé, Stephanie and Martín Uribe, "Optimal Operational Monetary Policy in the Christiano-Eichenbaum-Evans Model of the U.S. Business Cycle," NBER Working Papers 10724, National Bureau of Economic Research, Inc September 2004.

__ and ,__ "Optimal Inflation Stabilization in a Medium-Scale Macroeconomic Model," in Frederic S. Miskin, Klaus Schmidt-Hebbel, Norman Loayza (Series Editor), and Klaus Schmidt-Hebbel (Se, eds., *Monetary Policy under Inflation Targeting*, Vol. 11 of *Central Banking, Analysis, and Economic Policies Book Series*, Central Bank of Chile, August 2007, chapter 5, pp. 125-186.

__ and __, "The Optimal Rate of Inflation," in Benjamin M. Friedman and Michael Woodford, eds., *Handbook of Monetary Economics*, 1 ed., Vol. 3, Elsevier, 2010, chapter 13, pp. 653-722.

Sims, Christopher A, "Solving Linear Rational Expectations Models," *Computational Economics*, 2002, *20* (1-2), 1-20.

Smets, Frank and Rafael Wouters, "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area," *Journal of the European Economic Association*, 2003, *1* (5), 1123-1175.

__ and __, "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, June 2007, *97* (3), 586-606.

Svensson, Lars E. O., "Inflation forecast targeting: Implementing and monitoring inflation targets," *European Economic Review*, June 1997, *41* (6), 1111-1146.

Taylor, John B., "Discretion versus policy rules in practice," *Carnegie-Rochester Conference Series on Public Policy*, 1993, *39*, 195-214.

__, "An Historical Analysis of Monetary Policy Rules," NBER Working Papers 6768, National Bureau of Economic Research, Inc October 1999.

__, *Monetary Policy Rules*, National Bureau of Economic Research, Inc, 1999.

Woodford, M., *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press, 2011.

Yun, Tack, "Nominal price rigidity, money supply endogeneity, and business cycles," *Journal of Monetary Economics*, April 1996, *37* (2-3), 345-370.