

## On simplicity of the lowest eigenvalue of non-commutative harmonic oscillators

Wakayama, Masato  
Faculty of Mathematics, Kyushu University

<https://hdl.handle.net/2324/25576>

---

出版情報 : MI Preprint Series. 2012-12, 2012-12-20. 九州大学大学院数理学研究院  
バージョン :  
権利関係 :



# **MI Preprint Series**

**Kyushu University  
The Global COE Program  
Math-for-Industry Education & Research Hub**

## **On simplicity of the lowest eigenvalue of non-commutative harmonic oscillators**

**Masato Wakayama**

**MI 2012-12**

( Received December 20, 2012 )

Faculty of Mathematics  
Kyushu University  
Fukuoka, JAPAN

# On simplicity of the lowest eigenvalue of non-commutative harmonic oscillators

Masato Wakayama\*

December 12, 2012

## Abstract

The aim of this note is to prove the simplicity of the lowest eigenvalue of the non-commutative harmonic oscillator, which is defined by a  $2 \times 2$  system of ordinary differential operator, for a large class of structure parameters. The proof looks rather simple while uses many basic results obtained so far.

**2010 Mathematics Subject Classification:** *Primary* 34L40, *Secondary* 81Q10, 34M05, 81S05.

**Keywords and phrases:** *non-commutative harmonic oscillator, lowest eigenvalue, multiplicity of eigenvalues, Heun's differential equation.*

## 1 Introduction

Let  $Q$  be a second order parity-preserving ordinary differential operator defined by

$$Q = A \left( -\frac{1}{2} \frac{d^2}{dx^2} + \frac{1}{2} x^2 \right) + B \left( x \frac{d}{dx} + \frac{1}{2} \right),$$

where  $A, B \in \text{Mat}_2(\mathbb{R})$ ,  $A$  is positive definite symmetric, and  $B$  is skew-symmetric. We further assume that the Hermitian matrix  $A+iB$  is positive definite, that is,  $\det(A) > \text{pf}(B)^2$ . The former requirement arises from the formal self-adjointness of the operator  $Q$  relative to the natural inner product on  $L^2(\mathbb{R}, \mathbb{C}^2) (= \mathbb{C}^2 \otimes L^2(\mathbb{R}))$ . The latter guarantees that the eigenvalues of  $Q$  are all positive and form a discrete set with finite multiplicity. We call  $Q$  the *non-commutative harmonic oscillator* (NcHO) [18, 19] (see also [17]). It is shown in [19] that one may always assume that  $Q$  has the following normal form:

$$Q = Q_{(\alpha, \beta)} = \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix} \left( -\frac{1}{2} \frac{d^2}{dx^2} + \frac{1}{2} x^2 \right) + \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \left( x \frac{d}{dx} + \frac{1}{2} \right).$$

One immediately observes the required conditions above can be described as  $\alpha, \beta > 0$  and  $\alpha\beta > 1$ . It should be noted that, when  $\alpha = \beta$ ,  $Q$  is unitarily equivalent to a couple of two quantum harmonic oscillators, whence the eigenvalues are easily calculated as  $\{\sqrt{\alpha^2 - 1}(n + \frac{1}{2}) \mid n \in \mathbb{Z}_{\geq 0}\}$  having multiplicity 2 ([19], I). Actually, there exists a structure corresponding to the tensor product of the 2-dimensional trivial and oscillator representations (see [5]) of the Lie algebra  $\mathfrak{sl}_2$  behind  $Q$ , when  $\alpha = \beta = \sqrt{2}$ . However, the clarification of the spectrum in the general case is considered to be highly non-trivial. Indeed, while the spectrum is well described theoretically by using certain continued fractions (see [18, 19]) and also by Heun's differential equations of special types (see, [12] for odd eigenfunctions and [22] for even ones), and in particular there are some results related to the estimate of upper bound of the lowest eigenvalue [11, 7, 14, 15, 16, 4], almost no information is available in reality for the low-lying eigenvalues when  $\alpha \neq \beta$ . Therefore, in spite of many studies, the spectral description of NcHO is still incomplete.

The eigenvalues of NcHO build a continuous curve with arguments  $\alpha$  and  $\beta$  (see [11]). It comes as an important problem to analyze the behavior of eigenvalue curves, in particular, the characterization of crossing/avoided crossing of eigenvalue curves (see e.g. [21, 2]). Especially, it has been an important problem to determine the structure of the ground state (i.e. the eigenspace of the lowest eigenvalue) when  $\alpha \neq \beta$ :

- i) How much multiplicity does the lowest eigenvalue have?
- ii) Does any odd function contribute the ground state?

Moreover, for recent years, special attention has been paid to studying the spectrum of self-adjoint operators with non-commutative coefficients, like the Rabi model, the Jaynes-Cumming model, etc., not

---

\*Partially supported by Grand-in-Aid for Scientific Research (B) No. 21340011.

only as mathematics [2, 3] but also in experimental physics (see e.g. [1]). Our NcHO may provide one of these Hamiltonians describing such as an interaction between a one-mode photon and a two-level atom. In addition to this direction, in order to get a better understanding to the spectrum (e.g. Weyl's law for the distribution of eigenvalues) and its own interest (e.g. certain modular properties of Ape'ry-like numbers [23]), as an another approach, the spectral zeta function for  $Q$  has been studied extensively from a number theoretic point of view (see e.g. [6, 7, 8, 10, 9]).

In [19], we have constructed the eigenfunctions and eigenvalues in terms of continued fractions determined by a certain three terms recurrence relation, which can be derived from the expansion of eigenfunctions relative to the classical Hermite functions. We call the eigenfunction  $u(x)$  in  $L^2(\mathbb{R}, \mathbb{C}^2)$  is of finite type if  $u(x)$  can be expanded by a finite number of elements of this Hermite basis. The eigenvalue corresponding to the finite type eigenfunction is said to be of finite type. Otherwise, we call the eigenvalues/eigenfunction of infinite type. We denote  $\Sigma_0$  (reps.  $\Sigma_\infty$ ) the set of eigenvalues corresponding to eigenfunctions of finite (resp. infinite) type. Since the operator  $Q$  preserves the parity we define  $\Sigma^\pm$  to be the set of eigenvalues whose eigenfunctions are even/odd, that is those satisfying  $u(-x) = \pm u(x)$ . Then there is a classification of eigenvalues:  $\Sigma_0^\pm = \Sigma_0 \cap \Sigma^\pm$  corresponding to even/odd eigenfunctions of finite type and  $\Sigma_\infty^\pm = \Sigma_\infty \cap \Sigma^\pm$ .

To state our result we recall the facts ([19], II) that

$$\Sigma_0^\pm \subset \Sigma_\infty^\pm \quad \text{and} \quad \Sigma_0^+ \cap \Sigma_0^- = \emptyset. \quad (1.1)$$

The former implies that  $\Sigma_\infty^\pm = \Sigma^\pm$ . Moreover, since the dimension of the eigenspace of  $\Sigma_\infty^\pm$  is at most 2 ([19], II), one notices in particular that the multiplicity of each eigenvalue is at most 3.

What we show is the following result.

**Theorem 1.1.** *Suppose  $\alpha, \beta > \sqrt{2}$  ( $\alpha \neq \beta$ ). Then the lowest eigenvalue  $E$  is simple and  $E \in \Sigma_\infty^+ \setminus \Sigma_0^+$ .*

## 2 The proof of the theorem

As a key of the proof we show the following.

**Proposition 2.1.** *Suppose that the ground state, i.e. the eigenspace of the lowest eigenvalue  $E$  of  $Q$ , consists of even-functions. Then  $E$  is simple.*

*Proof.* By assumption, it follows from (1.1) that the multiplicity of  $E$  is at most 2. Suppose the multiplicity of  $E$  is 2. Then, by the assumption, one observes  $E \in \Sigma_0^+$ . Hence, by Theorem 1.1 ([19], I), there exists a non-negative integer  $n$  such that  $E$  can be expressed as

$$E = 2 \frac{\sqrt{\alpha\beta}\sqrt{\alpha\beta-1}}{\alpha+\beta} \left( n + \frac{1}{2} \right).$$

It follows that  $E \geq \frac{\sqrt{\alpha\beta}\sqrt{\alpha\beta-1}}{\alpha+\beta}$ .

On the other hand, the following important upper bound estimate of  $E$  is known (Theorem 8.2.1 in [17]):

$$\frac{1}{2} \min\{\alpha, \beta\} \sqrt{\frac{\alpha\beta-1}{\alpha\beta}} \leq E \leq \frac{\sqrt{\alpha\beta}\sqrt{\alpha\beta-1}}{\alpha+\beta+|\alpha-\beta|\frac{(\alpha\beta-1)^{1/4}}{\sqrt{\alpha\beta}}\operatorname{Re}\omega}, \quad (2.1)$$

where  $\omega \in \mathbb{C}$  is the solution of  $\omega^2 = \sqrt{\alpha\beta-1} - i$  with  $\operatorname{Re}\omega > 0$ . Thus, in particular, since  $\alpha \neq \beta$  we have

$$E < \frac{\sqrt{\alpha\beta}\sqrt{\alpha\beta-1}}{\alpha+\beta}$$

This contradicts the previous estimate. Hence the proposition follows.  $\square$

In [4], in the proof of the main result (Theorem 1.1), it is shown that there is no odd eigenfunction corresponding the lowest eigenvalue  $E$  when  $\beta > \alpha > \sqrt{2}$ . Hence the theorem follows immediately from the proposition above. This completes the proof of the theorem.

## 3 Note on another proof from a complex analytic viewpoint

Similarly to the result in [12] for the case of odd eigenfunctions of  $Q$ , for the even case, we have shown in [22] that there exists the following linear bijection.

$$\{u \in L^2(\mathbb{R}, \mathbb{C}^2) \mid Qu = \lambda u, u(-x) = u(x)\} \xrightarrow{\sim} \{f \in \mathcal{O}(\Omega) \mid H_\lambda^+ f = 0\}, \quad (3.1)$$

where  $\Omega$  is a simply-connected domain in  $\mathbb{C}$  such that  $0, 1 \in \Omega$  while  $\alpha\beta \notin \Omega$ ,  $\mathcal{O}(\Omega)$  denotes the set of holomorphic functions on  $\Omega$ , and  $H_\lambda^+ = H_\lambda^+(w, \partial_w)$  is the Heun ordinary differential operator (see e.g. [21, 20]) given by

$$H_\lambda^+(w, \partial_w) := \frac{d^2}{dw^2} + \left( \frac{\frac{1}{2} - n}{w} + \frac{-\frac{1}{2} - n}{w - 1} + \frac{n + 1}{w - \alpha\beta} \right) \frac{d}{dw} + \frac{-\frac{1}{2}(n + \frac{1}{2})w - q}{w(w - 1)(w - \alpha\beta)}. \quad (3.2)$$

Here the numbers  $n$  and  $\nu$  are defined through the following relation:

$$n = \frac{2\nu - 3}{4}, \quad \lambda = 2\nu \frac{\sqrt{\alpha\beta(\alpha\beta - 1)}}{\alpha + \beta}. \quad (3.3)$$

The accessory parameter  $q$  in (3.2) can be expressed by the parameters  $\alpha, \beta$  and the eigenvalue  $\lambda$ . We note that the Heun operator  $H_\lambda^+(w, \partial_w)$  has four regular singular points,  $w = 0, 1, \alpha\beta$  and  $\infty$ . The Riemann scheme of the operator  $H_\lambda^+(w, \partial_w)$  is

$$\begin{pmatrix} 0 & 1 & \alpha\beta & \infty & ; w \\ 0 & 0 & 0 & \frac{1}{2} \\ n + \frac{1}{2} & n + \frac{3}{2} & -n & -(n + \frac{1}{2}) \end{pmatrix},$$

where each element of the first row indicates a regular singular point of  $H_\lambda^+$  and those in the second and third rows are expressing the corresponding exponents. Since it is irrelevant with the present discussion, we omit the accessory parameter  $q$  in the picture.

Let us now give another proof of Proposition 2.1. We first note that, by the linear bijection (3.1), there is a one to one correspondence between the finite type even eigenfunction of  $Q$  and a polynomial solution of the Heun differential equation  $H_\lambda^+ f = 0$  [22] (The proof is done by the same way in [12]). Let us assume that the multiplicity of the lowest eigenvalue  $E$  is equal to 2. This implies that there is a polynomial solution of the Heun differential equation  $H_E^+ f = 0$ . Then, by the Riemann scheme above one concludes that  $n + \frac{1}{2} = \frac{2\nu - 1}{4}$  is a nonnegative integer (see [20]). This implies  $\nu \in 2\mathbb{Z}_{\geq 0} + \frac{1}{2}$ . It follows from (3.3) that  $E = 2\nu \frac{\sqrt{\alpha\beta(\alpha\beta - 1)}}{\alpha + \beta} \geq \frac{\sqrt{\alpha\beta(\alpha\beta - 1)}}{\alpha + \beta}$ . Hence, by the same reasoning as before, the contradiction appears. This proves again Proposition 2.1.

*Remark 3.1.* The fact  $\Sigma_0^+ \subset \Sigma_\infty^+$  which we have used in the proof is reproved in [22]. That proof is an imitation of that of the odd case  $\Sigma_0^- \subset \Sigma_\infty^-$ , which has been reproved in [13].

**Acknowledgement:** I thank Alberto Parmeggiani for many enjoyable discussions about the spectral problems of NcHO.

## References

- [1] S. Haroche and J. M. Raimond, *Exploring Quantm. Atoms, Cavities, and Photons*, Oxford University Press, 2008.
- [2] M. Hirokawa, *The Dicke-type crossing among eigenvalues of differential operators in a class of non-commutative oscillators*, *Indiana Univ. Math. J.* **58** (2009), 1493–1536.
- [3] M. Hirokawa and F. Hiroshima, *Absence of energy level crossing for the ground state energy of the Rabi model*, preprint 2012.
- [4] F. Hiroshima and I. Sasaki, *Multiplicity of the lowest eigenvalue of non-commutative harmonic oscillators*, *Kyushu J. Math.* (to appear).
- [5] R. Howe and E. C. Tan, *Non-Abelian Harmonic Analysis. Applications of  $SL(2, \mathbb{R})$* . Springer, 1992.
- [6] T. Ichinose and M. Wakayama, *Zeta functions for the spectrum of the non-commutative harmonic oscillators*, *Comm. Math. Phys.* **258** (2005), 697–739.
- [7] T. Ichinose and M. Wakayama, *On the spectral zeta function for the noncommutative harmonic oscillator*, *Rep. Math. Phys.* **59** (2007), 421–432.
- [8] K. Kimoto and M. Wakayama, *Elliptic curves arising from the spectral zeta function for non-commutative harmonic oscillators and  $\Gamma_0(4)$ -modular forms*, in “Proceedings Conf.  $L$ -functions” (eds. L. Weng, M. Kaneko), 201–218, World Scientific, 2007.
- [9] K. Kimoto and M. Wakayama, *Residual modular forms and Eichler cohomology groups arising from non-commutative harmonic oscillators*, in “Noncommutative Geometry and Physics” (eds. G. Dito, H. Moriyoshi, T. Natsume, S. Watamura), World Scientific (to appear).

- [10] K. Kimoto and Y. Yamasaki, *A partial alternating multiple zeta value*. Proc. Amer. Math. Soc. **137** (2009), 2503–2515.
- [11] K. Nagatou, M. T. Nakao and M. Wakayama. *Verified numerical computations for eigenvalues of non-commutative harmonic oscillators*, Numer. Funct. Anal. Optim. **23** (2002), 633–650.
- [12] H. Ochiai, *Non-commutative harmonic oscillators and Fuchsian ordinary differential operators*, Comm. Math. Phys. **217** (2001), 357–373.
- [13] H. Ochiai, *Non-commutative harmonic oscillators and the connection problem for the Heun differential equation*, Lett. Math. Phys. **70** (2004), 133–139.
- [14] A. Parmeggiani, *On the spectrum and the lowest eigenvalue of certain non-commutative harmonic oscillators*, Kyushu J. Math. **58** (2004), 277–322.
- [15] A. Parmeggiani, *On the spectrum of certain noncommutative harmonic oscillators*, Annali dell’Università di Ferrara **52** (2006), 431–456.
- [16] A. Parmeggiani, *On the spectrum of certain non-commutative harmonic oscillators and semiclassical analysis*, Comm. Math. Phys. **279** (2008), 285–308.
- [17] A. Parmeggiani, *Spectral Theory of Non-commutative Harmonic Oscillators: An Introduction*. Lecture Notes in Math. **1992**, Springer, 2010.
- [18] A. Parmeggiani and M. Wakayama, *Oscillator representations and systems of ordinary differential equations*, Proc. Natl. Acad. Sci. USA **98** (2001), 26–30.
- [19] A. Parmeggiani and M. Wakayama, *Non-commutative harmonic oscillators-I, II, Corrigenda and remarks to I*, Forum. Math. **14** (2002), 539–604, 669–690, *ibid* **15** (2003), 955–963.
- [20] A. Ronveaux (eds.), *Heun’s Differential Equations*, Oxford University Press, 1995.
- [21] S. Y. Slavyanov and W. Lay, *A Unified Theory Based on Singularities*, Oxford Mathematical Monographs, 2000.
- [22] M. Wakayama, *Correspondence between eigenfunctions of non-commutative harmonic oscillators and holomorphic solutions of Heun’s differential equations*, in preparation.
- [23] D. Zagier, *Integral solutions of Apéry-like recurrence equations*, Groups and symmetries, 349–366, CRM Proc. Lecture Notes **47**, Amer. Math. Soc., Providence, RI, 2009.

Masato Wakayama  
Institute of Mathematics for Industry,  
Kyushu University  
744 Motooka, Nishi-ku, Fukuoka 819-0395 JAPAN  
wakayama@imi.kyushu-u.ac.jp

**List of MI Preprint Series, Kyushu University**  
**The Global COE Program**  
**Math-for-Industry Education & Research Hub**

MI

- MI2008-1 Takahiro ITO, Shuichi INOKUCHI & Yoshihiro MIZOGUCHI  
Abstract collision systems simulated by cellular automata
- MI2008-2 Eiji ONODERA  
The initial value problem for a third-order dispersive flow into compact almost Hermitian manifolds
- MI2008-3 Hiroaki KIDO  
On isosceles sets in the 4-dimensional Euclidean space
- MI2008-4 Hirofumi NOTSU  
Numerical computations of cavity flow problems by a pressure stabilized characteristic-curve finite element scheme
- MI2008-5 Yoshiyasu OZEKI  
Torsion points of abelian varieties with values in infinite extensions over a p-adic field
- MI2008-6 Yoshiyuki TOMIYAMA  
Lifting Galois representations over arbitrary number fields
- MI2008-7 Takehiro HIROTSU & Setsuo TANIGUCHI  
The random walk model revisited
- MI2008-8 Silvia GANDY, Masaaki KANNO, Hirokazu ANAI & Kazuhiro YOKOYAMA  
Optimizing a particular real root of a polynomial by a special cylindrical algebraic decomposition
- MI2008-9 Kazufumi KIMOTO, Sho MATSUMOTO & Masato WAKAYAMA  
Alpha-determinant cyclic modules and Jacobi polynomials
- MI2008-10 Sangyeol LEE & Hiroki MASUDA  
Jarque-Bera Normality Test for the Driven Lévy Process of a Discretely Observed Univariate SDE
- MI2008-11 Hiroyuki CHIHARA & Eiji ONODERA  
A third order dispersive flow for closed curves into almost Hermitian manifolds
- MI2008-12 Takehiko KINOSHITA, Kouji HASHIMOTO and Mitsuhiro T. NAKAO  
On the  $L^2$  a priori error estimates to the finite element solution of elliptic problems with singular adjoint operator
- MI2008-13 Jacques FARAUT and Masato WAKAYAMA  
Hermitian symmetric spaces of tube type and multivariate Meixner-Pollaczek polynomials

- MI2008-14 Takashi NAKAMURA  
Riemann zeta-values, Euler polynomials and the best constant of Sobolev inequality
- MI2008-15 Takashi NAKAMURA  
Some topics related to Hurwitz-Lerch zeta functions
- MI2009-1 Yasuhide FUKUMOTO  
Global time evolution of viscous vortex rings
- MI2009-2 Hidetoshi MATSUI & Sadanori KONISHI  
Regularized functional regression modeling for functional response and predictors
- MI2009-3 Hidetoshi MATSUI & Sadanori KONISHI  
Variable selection for functional regression model via the  $L_1$  regularization
- MI2009-4 Shuichi KAWANO & Sadanori KONISHI  
Nonlinear logistic discrimination via regularized Gaussian basis expansions
- MI2009-5 Toshiro HIRANOUCI & Yuichiro TAGUCHI  
Flat modules and Groebner bases over truncated discrete valuation rings
- MI2009-6 Kenji KAJIWARA & Yasuhiro OHTA  
Bilinearization and Casorati determinant solutions to non-autonomous 1+1 dimensional discrete soliton equations
- MI2009-7 Yoshiyuki KAGEI  
Asymptotic behavior of solutions of the compressible Navier-Stokes equation around the plane Couette flow
- MI2009-8 Shohei TATEISHI, Hidetoshi MATSUI & Sadanori KONISHI  
Nonlinear regression modeling via the lasso-type regularization
- MI2009-9 Takeshi TAKAISHI & Masato KIMURA  
Phase field model for mode III crack growth in two dimensional elasticity
- MI2009-10 Shingo SAITO  
Generalisation of Mack's formula for claims reserving with arbitrary exponents for the variance assumption
- MI2009-11 Kenji KAJIWARA, Masanobu KANEKO, Atsushi NOBE & Teruhisa TSUDA  
Ultradiscretization of a solvable two-dimensional chaotic map associated with the Hesse cubic curve
- MI2009-12 Tetsu MASUDA  
Hypergeometric  $q$ -functions of the  $q$ -Painlevé system of type  $E_8^{(1)}$
- MI2009-13 Hidenao IWANE, Hitoshi YANAMI, Hirokazu ANAI & Kazuhiro YOKOYAMA  
A Practical Implementation of a Symbolic-Numeric Cylindrical Algebraic Decomposition for Quantifier Elimination
- MI2009-14 Yasunori MAEKAWA  
On Gaussian decay estimates of solutions to some linear elliptic equations and its applications

- MI2009-15 Yuya ISHIHARA & Yoshiyuki KAGEI  
Large time behavior of the semigroup on  $L^p$  spaces associated with the linearized compressible Navier-Stokes equation in a cylindrical domain
- MI2009-16 Chikashi ARITA, Atsuo KUNIBA, Kazumitsu SAKAI & Tsuyoshi SAWABE  
Spectrum in multi-species asymmetric simple exclusion process on a ring
- MI2009-17 Masato WAKAYAMA & Keitaro YAMAMOTO  
Non-linear algebraic differential equations satisfied by certain family of elliptic functions
- MI2009-18 Me Me NAING & Yasuhide FUKUMOTO  
Local Instability of an Elliptical Flow Subjected to a Coriolis Force
- MI2009-19 Mitsunori KAYANO & Sadanori KONISHI  
Sparse functional principal component analysis via regularized basis expansions and its application
- MI2009-20 Shuichi KAWANO & Sadanori KONISHI  
Semi-supervised logistic discrimination via regularized Gaussian basis expansions
- MI2009-21 Hiroshi YOSHIDA, Yoshihiro MIWA & Masanobu KANEKO  
Elliptic curves and Fibonacci numbers arising from Lindenmayer system with symbolic computations
- MI2009-22 Eiji ONODERA  
A remark on the global existence of a third order dispersive flow into locally Hermitian symmetric spaces
- MI2009-23 Stjepan LUGOMER & Yasuhide FUKUMOTO  
Generation of ribbons, helicoids and complex scherk surface in laser-matter Interactions
- MI2009-24 Yu KAWAKAMI  
Recent progress in value distribution of the hyperbolic Gauss map
- MI2009-25 Takehiko KINOSHITA & Mitsuhiro T. NAKAO  
On very accurate enclosure of the optimal constant in the a priori error estimates for  $H_0^2$ -projection
- MI2009-26 Manabu YOSHIDA  
Ramification of local fields and Fontaine's property (Pm)
- MI2009-27 Yu KAWAKAMI  
Value distribution of the hyperbolic Gauss maps for flat fronts in hyperbolic three-space
- MI2009-28 Masahisa TABATA  
Numerical simulation of fluid movement in an hourglass by an energy-stable finite element scheme
- MI2009-29 Yoshiyuki KAGEI & Yasunori MAEKAWA  
Asymptotic behaviors of solutions to evolution equations in the presence of translation and scaling invariance

- MI2009-30 Yoshiyuki KAGEI & Yasunori MAEKAWA  
On asymptotic behaviors of solutions to parabolic systems modelling chemotaxis
- MI2009-31 Masato WAKAYAMA & Yoshinori YAMASAKI  
Hecke's zeros and higher depth determinants
- MI2009-32 Olivier PIRONNEAU & Masahisa TABATA  
Stability and convergence of a Galerkin-characteristics finite element scheme of lumped mass type
- MI2009-33 Chikashi ARITA  
Queueing process with excluded-volume effect
- MI2009-34 Kenji KAJIWARA, Nobutaka NAKAZONO & Teruhisa TSUDA  
Projective reduction of the discrete Painlevé system of type  $(A_2 + A_1)^{(1)}$
- MI2009-35 Yosuke MIZUYAMA, Takamasa SHINDE, Masahisa TABATA & Daisuke TAGAMI  
Finite element computation for scattering problems of micro-hologram using DtN map
- MI2009-36 Reiichiro KAWAI & Hiroki MASUDA  
Exact simulation of finite variation tempered stable Ornstein-Uhlenbeck processes
- MI2009-37 Hiroki MASUDA  
On statistical aspects in calibrating a geometric skewed stable asset price model
- MI2010-1 Hiroki MASUDA  
Approximate self-weighted LAD estimation of discretely observed ergodic Ornstein-Uhlenbeck processes
- MI2010-2 Reiichiro KAWAI & Hiroki MASUDA  
Infinite variation tempered stable Ornstein-Uhlenbeck processes with discrete observations
- MI2010-3 Kei HIROSE, Shuichi KAWANO, Daisuke MIIKE & Sadanori KONISHI  
Hyper-parameter selection in Bayesian structural equation models
- MI2010-4 Nobuyuki IKEDA & Setsuo TANIGUCHI  
The Itô-Nisio theorem, quadratic Wiener functionals, and 1-solitons
- MI2010-5 Shohei TATEISHI & Sadanori KONISHI  
Nonlinear regression modeling and detecting change point via the relevance vector machine
- MI2010-6 Shuichi KAWANO, Toshihiro MISUMI & Sadanori KONISHI  
Semi-supervised logistic discrimination via graph-based regularization
- MI2010-7 Teruhisa TSUDA  
UC hierarchy and monodromy preserving deformation
- MI2010-8 Takahiro ITO  
Abstract collision systems on groups

- MI2010-9 Hiroshi YOSHIDA, Kinji KIMURA, Naoki YOSHIDA, Junko TANAKA & Yoshihiro MIWA  
An algebraic approach to underdetermined experiments
- MI2010-10 Kei HIROSE & Sadanori KONISHI  
Variable selection via the grouped weighted lasso for factor analysis models
- MI2010-11 Katsusuke NABESHIMA & Hiroshi YOSHIDA  
Derivation of specific conditions with Comprehensive Groebner Systems
- MI2010-12 Yoshiyuki KAGEI, Yu NAGAFUCHI & Takeshi SUDO  
Decay estimates on solutions of the linearized compressible Navier-Stokes equation around a Poiseuille type flow
- MI2010-13 Reiiichiro KAWAI & Hiroki MASUDA  
On simulation of tempered stable random variates
- MI2010-14 Yoshiyasu OZEKI  
Non-existence of certain Galois representations with a uniform tame inertia weight
- MI2010-15 Me Me NAING & Yasuhide FUKUMOTO  
Local Instability of a Rotating Flow Driven by Precession of Arbitrary Frequency
- MI2010-16 Yu KAWAKAMI & Daisuke NAKAJO  
The value distribution of the Gauss map of improper affine spheres
- MI2010-17 Kazunori YASUTAKE  
On the classification of rank 2 almost Fano bundles on projective space
- MI2010-18 Toshimitsu TAKAESU  
Scaling limits for the system of semi-relativistic particles coupled to a scalar bose field
- MI2010-19 Reiiichiro KAWAI & Hiroki MASUDA  
Local asymptotic normality for normal inverse Gaussian Lévy processes with high-frequency sampling
- MI2010-20 Yasuhide FUKUMOTO, Makoto HIROTA & Youichi MIE  
Lagrangian approach to weakly nonlinear stability of an elliptical flow
- MI2010-21 Hiroki MASUDA  
Approximate quadratic estimating function for discretely observed Lévy driven SDEs with application to a noise normality test
- MI2010-22 Toshimitsu TAKAESU  
A Generalized Scaling Limit and its Application to the Semi-Relativistic Particles System Coupled to a Bose Field with Removing Ultraviolet Cutoffs
- MI2010-23 Takahiro ITO, Mitsuhiro FUJIO, Shuichi INOKUCHI & Yoshihiro MIZOGUCHI  
Composition, union and division of cellular automata on groups
- MI2010-24 Toshimitsu TAKAESU  
A Hardy's Uncertainty Principle Lemma in Weak Commutation Relations of Heisenberg-Lie Algebra

- MI2010-25 Toshimitsu TAKAESU  
On the Essential Self-Adjointness of Anti-Commutative Operators
- MI2010-26 Reiichiro KAWAI & Hiroki MASUDA  
On the local asymptotic behavior of the likelihood function for Meixner Lévy processes under high-frequency sampling
- MI2010-27 Chikashi ARITA & Daichi YANAGISAWA  
Exclusive Queueing Process with Discrete Time
- MI2010-28 Jun-ichi INOBUCHI, Kenji KAJIWARA, Nozomu MATSUURA & Yasuhiro OHTA  
Motion and Bäcklund transformations of discrete plane curves
- MI2010-29 Takanori YASUDA, Masaya YASUDA, Takeshi SHIMOYAMA & Jun KOGURE  
On the Number of the Pairing-friendly Curves
- MI2010-30 Chikashi ARITA & Kohei MOTEGI  
Spin-spin correlation functions of the  $q$ -VBS state of an integer spin model
- MI2010-31 Shohei TATEISHI & Sadanori KONISHI  
Nonlinear regression modeling and spike detection via Gaussian basis expansions
- MI2010-32 Nobutaka NAKAZONO  
Hypergeometric  $\tau$  functions of the  $q$ -Painlevé systems of type  $(A_2 + A_1)^{(1)}$
- MI2010-33 Yoshiyuki KAGEI  
Global existence of solutions to the compressible Navier-Stokes equation around parallel flows
- MI2010-34 Nobushige KUROKAWA, Masato WAKAYAMA & Yoshinori YAMASAKI  
Milnor-Selberg zeta functions and zeta regularizations
- MI2010-35 Kissani PERERA & Yoshihiro MIZOGUCHI  
Laplacian energy of directed graphs and minimizing maximum outdegree algorithms
- MI2010-36 Takanori YASUDA  
CAP representations of inner forms of  $Sp(4)$  with respect to Klingen parabolic subgroup
- MI2010-37 Chikashi ARITA & Andreas SCHADSCHNEIDER  
Dynamical analysis of the exclusive queueing process
- MI2011-1 Yasuhide FUKUMOTO & Alexander B. SAMOKHIN  
Singular electromagnetic modes in an anisotropic medium
- MI2011-2 Hiroki KONDO, Shingo SAITO & Setsuo TANIGUCHI  
Asymptotic tail dependence of the normal copula
- MI2011-3 Takehiro HIROTSU, Hiroki KONDO, Shingo SAITO, Takuya SATO, Tatsushi TANAKA & Setsuo TANIGUCHI  
Anderson-Darling test and the Malliavin calculus
- MI2011-4 Hiroshi INOUE, Shohei TATEISHI & Sadanori KONISHI  
Nonlinear regression modeling via Compressed Sensing

- MI2011-5 Hiroshi INOUE  
Implications in Compressed Sensing and the Restricted Isometry Property
- MI2011-6 Daeju KIM & Sadanori KONISHI  
Predictive information criterion for nonlinear regression model based on basis expansion methods
- MI2011-7 Shohei TATEISHI, Chiaki KINJYO & Sadanori KONISHI  
Group variable selection via relevance vector machine
- MI2011-8 Jan BREZINA & Yoshiyuki KAGEI  
Decay properties of solutions to the linearized compressible Navier-Stokes equation around time-periodic parallel flow  
Group variable selection via relevance vector machine
- MI2011-9 Chikashi ARITA, Arvind AYYER, Kirone MALLICK & Sylvain PROLHAC  
Recursive structures in the multispecies TASEP
- MI2011-10 Kazunori YASUTAKE  
On projective space bundle with nef normalized tautological line bundle
- MI2011-11 Hisashi ANDO, Mike HAY, Kenji KAJIWARA & Tetsu MASUDA  
An explicit formula for the discrete power function associated with circle patterns of Schramm type
- MI2011-12 Yoshiyuki KAGEI  
Asymptotic behavior of solutions to the compressible Navier-Stokes equation around a parallel flow
- MI2011-13 Vladimír CHALUPECKÝ & Adrian MUNTEAN  
Semi-discrete finite difference multiscale scheme for a concrete corrosion model: approximation estimates and convergence
- MI2011-14 Jun-ichi INOBUCHI, Kenji KAJIWARA, Nozomu MATSUURA & Yasuhiro OHTA  
Explicit solutions to the semi-discrete modified KdV equation and motion of discrete plane curves
- MI2011-15 Hiroshi INOUE  
A generalization of restricted isometry property and applications to compressed sensing
- MI2011-16 Yu KAWAKAMI  
A ramification theorem for the ratio of canonical forms of flat surfaces in hyperbolic three-space
- MI2011-17 Naoyuki KAMIYAMA  
Matroid intersection with priority constraints
- MI2012-1 Kazufumi KIMOTO & Masato WAKAYAMA  
Spectrum of non-commutative harmonic oscillators and residual modular forms
- MI2012-2 Hiroki MASUDA  
Mighty convergence of the Gaussian quasi-likelihood random fields for ergodic Levy driven SDE observed at high frequency

- MI2012-3 Hiroshi INOUE  
A Weak RIP of theory of compressed sensing and LASSO
- MI2012-4 Yasuhide FUKUMOTO & Youich MIE  
Hamiltonian bifurcation theory for a rotating flow subject to elliptic straining field
- MI2012-5 Yu KAWAKAMI  
On the maximal number of exceptional values of Gauss maps for various classes of surfaces
- MI2012-6 Marcio GAMEIRO, Yasuaki HIRAOKA, Shunsuke IZUMI, Miroslav KRAMAR, Konstantin MISCHAIKOW & Vidit NANDA  
Topological Measurement of Protein Compressibility via Persistence Diagrams
- MI2012-7 Nobutaka NAKAZONO & Seiji NISHIOKA  
Solutions to a  $q$ -analog of Painlevé III equation of type  $D_7^{(1)}$
- MI2012-8 Naoyuki KAMIYAMA  
A new approach to the Pareto stable matching problem
- MI2012-9 Jan BREZINA & Yoshiyuki KAGEI  
Spectral properties of the linearized compressible Navier-Stokes equation around time-periodic parallel flow
- MI2012-10 Jan BREZINA  
Asymptotic behavior of solutions to the compressible Navier-Stokes equation around a time-periodic parallel flow
- MI2012-11 Daeju KIM, Shuichi KAWANO & Yoshiyuki NINOMIYA  
Adaptive basis expansion via the extended fused lasso
- MI2012-12 Masato WAKAYAMA  
On simplicity of the lowest eigenvalue of non-commutative harmonic oscillators