Topology of Random Geometric Complexes in Thermodynamic Regime

アクシェイ, ゴエル

https://doi.org/10.15017/2534381

出版情報：Kyushu University, 2019, 博士（数理学）, 課程博士
バージョン：

（様式３）

氏　　名：AKSHAY GOEL

論 文 名：Topology of Random Geometric Complexes in Thermodynamic Regime
（熱力学的レジームにおけるランダム幾何的複体のトポロジー）

区　　分：甲

論　文　内　容　の　要　旨

The emerging research area known as random topology comprises theoretical results that characterize the asymptotic behavior of topological properties of random objects. One aspect of this area is the study of random geometric complexes and their topological properties called Betti numbers. Random geometric complexes, regarded as higher-dimensional generalizations of random geometric graphs, are generated from random points under specific deterministic rules. They are built on random points with non-random radius that depends on the number of points and usually goes to zero as the number of points goes to infinity. With the appropriate scaling of the radius, it is known that there are three main regimes: sparse regime, thermodynamic regime and dense regime, in which the limiting behavior of Betti numbers of random geometric complexes is totally different. In the thesis, we mainly concentrate on random Čech complexes and random Vietoris-Rips complexes, the typical types of random geometric complexes, and on the thermodynamic regime, the regime in which fundamental problems such as the law of large numbers and central limit theorem have not been completely understood yet.

We establish the strong law of large numbers for Betti numbers of random Čech complexes (and random Vietoris-Rips complexes) built on R^N-valued binomial point processes and related Poisson point processes in the thermodynamic regime. Here, we consider both the case where the underlying distribution of the point processes is absolutely continuous with respect to the Lebesgue measure on R^N (Euclidean setting) and the case where it is supported on a C^1 compact manifold of dimension strictly less than N (manifold setting). The strong law is proved under the very mild assumption, which only requires that the common probability density function belongs to L^p spaces, for all $1 \leq p < \infty$. The limiting constant in our results is the integral of a function, which is the limit of the Betti numbers of random Čech complexes built on homogenous Poisson point processes on R^N in the thermodynamic regime. Our result in the Euclidean setting is an improvement of an existing result in the sense that the stronger assumptions on the density function in the existing result are replaced by the mild assumption mentioned above. While our result in the manifold setting is new in the sense that the only existing result for Betti numbers in this setting is the linear growth of their expected value.

We also establish the strong law of large numbers for persistent Betti numbers of random Čech filtrations (and random Rips filtrations) built on scaled binomial point processes and related Poison point processes, in both the settings. These results are the generalizations of our results for usual Betti numbers. We also derive the vague convergence of persistence diagrams (in the same settings as for persistent Betti numbers) by considering them as counting measures. The approach used in our results is general enough to apply to other geometric complexes and their filtrations as well.