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Formalizing the Definition and Evolution of Models in a Repository using the Relational Graph Expressions

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1 Introduction

The use of repository technology for software developments is regarded as increasingly important by researchers and practitioners alike.

Developing an information system requires a suitable environment designed to support the process of capture, representation, amplification and dissemination of knowledge from a variety of sources, both internal and external. Typically these tools will provide a repository management system for maintaining knowledge specifications using some DBMS; editing facilities to insert new information in the knowledge base; browsing functions, etc.

A repository therefore manages these models for their entire life, from birth to their deletion. During its lifetime a model may undergo many changes. Furthermore, these models represent in essence different views onto a common knowledge. In dealing with the creation and evolution of its models, a repository needs to cater for:

- multiplicity of views on a common concept
- modification of graphical representations and
- modification of stored components.

The majority of models are of a graphical nature. In this paper we focus exclusively on this type of model. We advocate a formal approach to defining the graphical models as well as their changes, both at an external visual level and in their representation in a repository. This formalisation of the definition and evolution of models provides a strong theoretical basis for ensuring consistency of a repository throughout the life time of all its stored models. The formalisation is based on the use of relational calculus whereby both a model and a repository are defined as a graph.

2 Fundamentals on Relational Calculus

In this section, we summerize basic notation and properties of relational calculus. A *relation* α of a set A into another set B is a subset of the cartesian product $A \times B$ and denoted by $\alpha : A \rightharpoonup B$. The *inverse relation* $\alpha^\sharp : B \rightharpoonup A$ of α is a relation such that $(b,a) \in \alpha^\sharp$ if and only if $(a,b) \in \alpha$. The *composite* $\alpha\beta : A \rightharpoonup C$ of $\alpha : A \rightharpoonup B$ followed by $\beta : B \rightharpoonup C$ is a relation such that $(a,c) \in \alpha\beta$ if and only if there exists $b \in B$ with $(a,b) \in \alpha$ and $(b,c) \in \beta$.

As a relation of a set A into a set B is a subset of $A \times B$, the inclusion relation, union, intersection and difference of them are available as usual and denoted by $\subseteq$, $\cup$, $\cap$ and $-$, respectively. The *identity relation* $\mathrm{id}_A : A \rightharpoonup A$ is a relation with $\mathrm{id}_A = \{(a,a) \in A \times A \mid a \in A\}$ (the diagonal set of A).

The followings are the basic properties of relations and indicate that the totality of sets and relations forms a category **Rel** with involution (or shortly I-category).

A *partial function* f of a set A into a set B is a relation $f : A \rightharpoonup B$ with $f^\sharp f \subseteq \mathrm{id}_B$ and it is denoted by $f : A \to B$. A *(total) function* f of a set A into a set B is a relation $f : A \rightharpoonup B$ with $f^\sharp f \subseteq \mathrm{id}_B$ and $\mathrm{id}_A \subseteq f f^\sharp$, and it is also denoted by $f : A \to B$. Clearly a function is a partial function. Note that the

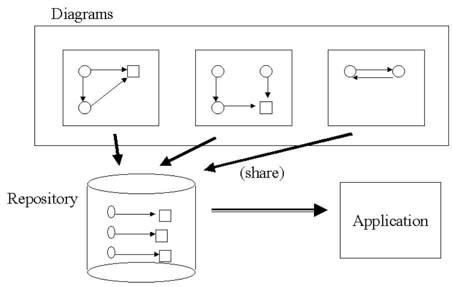

Fig. 1. Common data are stored in a repository

identity relation id_A of a set A is a function. The definitions of partial functions and (total) functions here coincide with ordinary ones. A partial function $f : A \to B$ is injective if and only if $ff^\sharp \subseteq \mathrm{id}_A$ and surjective if and only if $f^\sharp f = \mathrm{id}_B$. For a subset $X \subseteq A$, we denote the inclusion function by $i_X : X \to A$.

Given a relation $\alpha : A \rightharpoondown B$, the *domain* is defined by the set $\mathrm{dom}(\alpha) = \{a \in A | (a,a) \in \alpha\alpha^\sharp\}$, and *domain relation* $d(\alpha) : A \rightharpoondown A$ of α is a relation defined by $d(\alpha) = \alpha\alpha^\sharp \cap \mathrm{id}_A$. The domain relation $d(\alpha^\sharp) : B \rightharpoondown B$ of $\alpha^\sharp$ corresponds with the image of α. A partial function $f : A \to B$ is a function if and only if $d(f) = \mathrm{id}_A$.

We denote the category of sets and functions by **Set** and the category of sets and partial functions by **Pfn**. Both of **Set** and **Pfn** have all small limits and colimits, so in particular, they have pushouts. Note that **Pfn** is equivalent to the category of sets with a base point (a selected element) and base point preserving functions. We assume that the readers are familiar with pushout constructions in **Pfn**. A singleton set $\{*\}$ is denoted by 1 and the maximum relation from a set A into 1 by $\Omega_A : A \rightharpoondown 1$, that is, $\Omega_A = \{(a,*) | a \in A\}$. We define a relation $\Theta_A = \Omega_A \Omega_A^\sharp$.

3 Models in a Repository

In this section, we formalize models, repositories, and those modifications and changes using graphs and relational expressions. We consider a model and a repository as graphs (cf. Figure 1).

Common data in a model is stored in a repositry, this stuation is expressed by a partial function between graphs. At the case of editing some model, the modification of the model is expressed also by a partial function between graphs. A modification of a model which common data is stored in a repository induce some change of the repository. The change of the repository is defined by a graph using relational expressions (cf. Figure 2). A change of a repository induce modifications of other models which common data are stored in the repository. The modification of the other model also defined by a graph using relational expressions. We show some properties about those modifications and changes. Especially, we prove that we obtain the same graph by doing step-by-step the sequence of modifications and doing all-step-together of them. We also have same results about step-by-step changing and all-step-together changing.

We fix a set Σ which represent labels of nodes which are stored in repositories. Let M is an integer greater than 1 and $\Sigma = \{\cdot, \wedge, \vee, G_1, G_2, \cdots, G_M\}$. It is not essential to fix elements in a Σ in our arguments. In our expression of a model using a graph, we use a edge connecting to a node in Σ for a label of a node (cf. Figure 3).

A model is considered as a simple graph. A repository which contains common data of models is also expressed by a simple graph. Since a repository stores only informations of labels of nodes, a graph which expresses a repository does not have any edge between nodes not in Σ. This condition is denoted by a relational expression $\Theta_{R-\Sigma} \cap \xi = \phi$. Common data of a model stored in a repository are expressed by a partial injective morphism. We show those things formally as follows.

Definition 1. A *model* $\langle A, \alpha \rangle$ is a pair of a set A and a relation $\alpha : A \rightharpoondown A$. A *repository* $\langle R, \xi \rangle$ is a pair of a set R and a relation $\xi : R \rightharpoondown R$ where $\Theta_{R-\Sigma} \cap \xi = \phi$. A *model in a repository* $\langle \langle A, \alpha \rangle, i_A, \langle R, \xi \rangle \rangle$ is a triple of a diagram $\langle A, \alpha \rangle$, an injective partial morphism $i_A : A \to R$, and a repository $\langle R, \xi \rangle$.

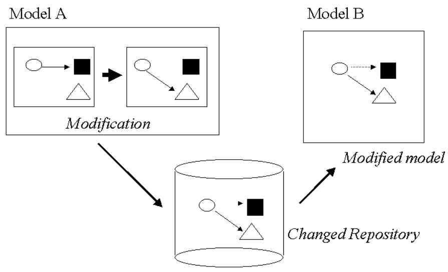

Fig. 2. Modifications of models and a change of a repository

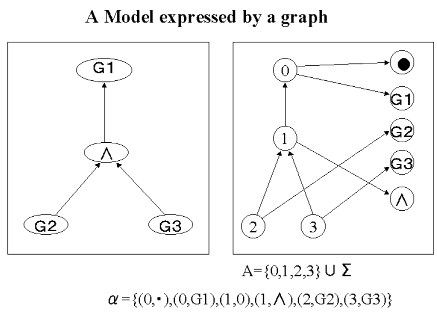

Fig. 3. A model expressed by a graph

Example 1. Figure 4 shows a model in a repository $\langle\langle A,\alpha\rangle, i_A, \langle R,\xi\rangle\rangle$. A model $\langle A,\alpha\rangle$ consists of $A = \{0,1,2,3\} \cup \Sigma$ and $\alpha = \{(0,\cdot),(0,G_1),(1,0),(1,\wedge),(2,G_2),(3,G_3)\}$. A repository $\langle R,\xi\rangle$ consists of $R = \{0,2,3\} \cup \Sigma$ and $\xi = \{(0,G_1),(2,G_2),(3,G_3)\}$. And $i_A = \{(0,0),(2,2),(3,3)\}$.

To modify a model using a editing tool are expressed by a graph transformation from a graph to an edited graph. The transformation is denoted by a partial function f between two graphs. A transformation preserve labels of nodes, so we restrict f by a relational expression $\mathrm{id}_\Sigma \cap f = \Theta_\Sigma \cap f$. We assume f a partial injective morphism to be simplify following arguments. Further we do not allow a transformation which merge two nodes to one node. Many practical editings are deletion and insertion of nodes, so it is enough to discuss with those restrictions to f.

Definition 2. A *modification* of a model $\langle\langle A,\alpha\rangle, f, \langle B,\beta\rangle\rangle$ is a triple of models $\langle A,\alpha\rangle$ and $\langle B,\beta\rangle$ and an injective partial morphism $f : A \to B$ where $\mathrm{id}_\Sigma \cap f = \Theta_\Sigma \cap f$.

Example 2. The Figure 5 is an example of a modification of a model, where $f = \{(0,4),(2,6)\} \cup \mathrm{id}_\Sigma$. In other words, the modification remove a goal G_3, insert a goal G_4 and change $\wedge$ to $\vee$.

At the case of modifying a model in a repository, common data stored in a repository are influenced by the modification. The definition of the influenced modifications of a repository, which we call *a change of a repository*, is not straightforward. There are many possibility of defining. In this paper, we define it by a set of nodes constructed using a property of pushout and a set of edges defined by a relational mathematical expression (Mizoguchi and Kawahara 1995 [3]).

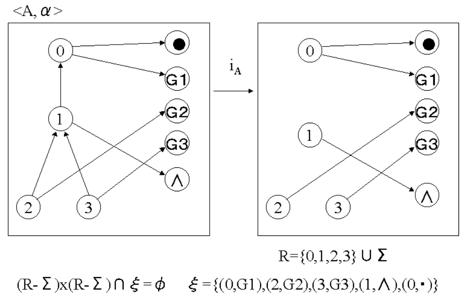

Fig. 4. A model in a repository

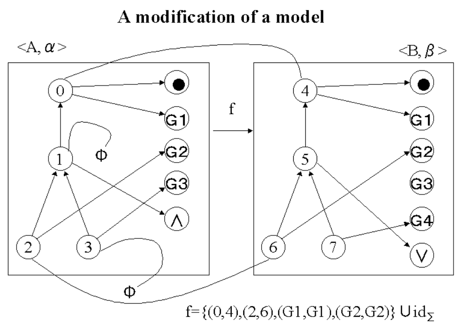

Fig. 5. A modification of a model

Definition 3. Let $\langle\langle A,\alpha\rangle, i_A, \langle R,\xi\rangle\rangle$ be a model in a repository, and $\langle\langle A,\alpha\rangle, f, \langle B,\beta\rangle\rangle$ a modification. Construct a pushout

$$\begin{array}{ccc} A & \xrightarrow{f} & B \\ {\scriptstyle i_A}\downarrow & & \downarrow{\scriptstyle i_B} \\ R & \xrightarrow[f_+]{} & S \end{array}$$

in **Pfn** and define $\sigma = i_B^\sharp \beta i_B \cup f_+^\sharp(\xi - i_A^\sharp \Theta_A i_A) f_+$. Then we have a repository $\langle S,\sigma\rangle$ and a model in a repository $\langle\langle B,\beta\rangle, i_B, \langle S,\sigma\rangle\rangle$. We call $\langle S,\sigma\rangle$ a *changed repository* induced by the modification $\langle\langle A,\alpha\rangle, f, \langle B,\beta\rangle\rangle$ and the modification $\langle\langle A,\alpha\rangle, f, \langle B,\beta\rangle\rangle$. And simply we write the situation with a diagram

$$\begin{array}{ccc} \langle A,\alpha\rangle & \xrightarrow{f} & \langle B,\beta\rangle \\ {\scriptstyle i_A}\downarrow & & \downarrow{\scriptstyle i_B} \\ \langle R,\xi\rangle & \xrightarrow[f_+]{} & \langle S,\sigma\rangle \end{array}$$

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The set S defined in above definition is roughly considered as a set $B \cup R$ in which common data in A are identified. The relational expressin for σ means that a set of edge contains all edges in β and ξ removing edges in ξ related to nodes in A.

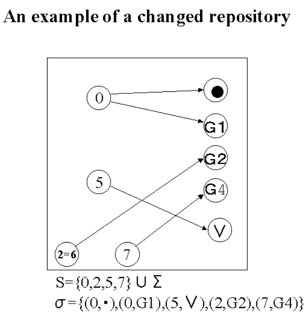

Fig. 6. An example of a changed repository

Example 3. Figure 6 shows a changed repository $\langle S,\sigma\rangle$ induced from the model in Figure 4 by the modification in Figure 5. where $S = \{0, 6, 7\} \cup \Sigma$ and $\sigma = \{(0, G_1), (6, G_2), (7, G_4)\}$.

We defined a change of a repository induced by a modification of a model by our own way. To confirm our definition's correctness, we prove the following property. Let $\langle\langle A,\alpha\rangle, i_A, \langle R,\xi\rangle\rangle$ be a model in a repository. When we modify a model twice from A to B to C, we can change the repository step by step from R to S to T. We call this changing *step-by-step change*. When we consider those two modification as a single modification from A to C, we have another changed repository T' from R by our definition. We call the last changing *all-step-together change*. A correct and useful definition of a changed repository should garantee that those two change step-by-step change and all-step-toghether change are same. In next proposition, we prove this property using simple but formal relational caluculations.

Proposition 1. *Let*

$$\underline{\text{step-by-step:}}\quad \begin{array}{ccccc} \langle A,\alpha\rangle & \xrightarrow{f} & \langle B,\beta\rangle & \xrightarrow{g} & \langle C,\gamma\rangle \\ {\scriptstyle i_A}\downarrow & & \downarrow{\scriptstyle i_B} & & \downarrow{\scriptstyle i_C,} \\ \langle R,\xi\rangle & \xrightarrow[f_+]{} & \langle S,\sigma\rangle & \xrightarrow[g_+]{} & \langle T,\tau\rangle \end{array}$$

and

$$\underline{\text{all-step-together:}}\quad \begin{array}{ccc} \langle A,\alpha\rangle & \xrightarrow{fg} & \langle C,\beta\rangle \\ {\scriptstyle i_A}\downarrow & & \downarrow{\scriptstyle i_C} \\ \langle R,\xi\rangle & \xrightarrow[(fg)_+]{} & \langle T',\tau'\rangle \end{array} \quad \begin{array}{l} \text{(Model modification)} \\ \\ \text{(Changed repository)} \end{array}$$

are changes of repositories. Then $\langle T,\tau\rangle$ and $\langle T',\tau\rangle$ are same.

Proof. Since T and T' are defined by pushouts, we can prove $T = T'$ using simple pushout properties. Next we show $\tau = \tau'$ using relational calculuses.

$$\begin{aligned}
\tau &= i_C^\sharp \gamma i_C \cup g_+^\sharp(\sigma - i_B^\sharp \Theta_B i_B) g_+ \\
&= i_C^\sharp \gamma i_C \cup g_+^\sharp((i_B^\sharp \beta i_B \cup f_+^\sharp(\xi - i_A^\sharp \Theta_A i_A) f_+) - i_B^\sharp \Theta_B i_B) g_+ \\
&= i_C^\sharp \gamma i_C \cup g_+^\sharp((f_+^\sharp \xi f_+ - f_+^\sharp i_A^\sharp \Theta_A i_A f_+) - i_B^\sharp \Theta_B i_B) g_+ \\
&= i_C^\sharp \gamma i_C \cup g_+^\sharp((f_+^\sharp \xi f_+ - i_B^\sharp \Theta_B i_B) - f_+^\sharp i_A^\sharp \Theta_A i_A f_+) g_+ \\
&= i_C^\sharp \gamma i_C \cup g_+^\sharp((f_+^\sharp \xi f_+ - (f_+^\sharp \xi f_+ \cap i_B^\sharp \Theta_B i_B)) - f_+^\sharp i_A^\sharp \Theta_A i_A f_+) g_+
\end{aligned}$$

Since $f_+^\sharp \xi f_+ \cap i_B^\sharp \Theta_B i_B \subseteq f_+^\sharp \Theta_R f_+ \cap i_B \Theta_B i_B \subseteq f_+^\sharp i_A^\sharp \Theta_A i_A f_+$, We have

$$\begin{aligned}
\tau &= i_C^\sharp \gamma i_C \cup g_+^\sharp(f_+^\sharp \xi f_+ - f_+^\sharp i_A \Theta_A i_A f_+) g_+ \\
&= i_C^\sharp \gamma i_C \cup g_+^\sharp f_+^\sharp(\xi - i_A^\sharp \Theta_A) g_+ \\
&= i_C^\sharp \gamma i_C (f_+ g_+)^\sharp(\xi - i_A^\sharp \Theta_A i_A)(f_+ g_+) \\
&= \tau'.
\end{aligned}$$

□

Next, we consider modifications of other models effected by a change of a repository which the models have common data in. There also exist many possibilities of definitions of an effected modified model. We introduce a definition of a modified model using relational expressions. Let $\langle\langle X,\chi\rangle, i_X, \langle R,\xi\rangle\rangle$ be a model in a repository, $\langle S,\sigma\rangle$ a changed repository, $f_+ : \langle X,\chi\rangle \to \langle S,\sigma\rangle$ a relation between repositories. A set of edges of modified model are expressed by the expression $\chi' = (\chi - i_X i_X^\sharp \chi i_X i_X^\sharp) \cup i_X f_+ \sigma f_+^\sharp i_X^\sharp$. This means to remove edges related to edges in R from ξ and to insert all edges in σ.

Definition 4. Let

$$\begin{array}{ccc}
\langle A,\alpha\rangle & \xrightarrow{f} & \langle B,\beta\rangle \\
{\scriptstyle i_A}\downarrow & & \downarrow{\scriptstyle i_B} \\
\langle R,\xi\rangle & \xrightarrow[f_+]{} & \langle S,\sigma\rangle
\end{array}$$

be a changed repository and $\langle\langle X,\chi\rangle, i_X, \langle R,\xi\rangle\rangle$ a model in a repository. A *modified model* induced by a changed repository is a model in a repository $\langle\langle X,\chi'\rangle, i_X f_+, \langle S,\sigma\rangle\rangle$ where $\chi' = (\chi - i_X i_X^\sharp \chi i_X i_X^\sharp) \cup i_X f_+ \sigma f_+^\sharp i_X^\sharp$.

We prove a similar property of a modified model as Proposition 1 of a changed repository. We show that a *step-by-step* modification induce a same result as an *all-step-together* modification.

Proposition 2. *Let*

$$\begin{array}{cccccc}
 & \langle A,\alpha\rangle & \xrightarrow{f} & \langle B,\beta\rangle & \xrightarrow{g} & \langle C,\gamma\rangle \\
 & {\scriptstyle i_A}\downarrow & & \downarrow{\scriptstyle i_B} & & \downarrow{\scriptstyle i_C} \\
 & \langle R,\xi\rangle & \xrightarrow[f_+]{} & \langle S,\sigma\rangle & \xrightarrow[g_+]{} & \langle T,\tau\rangle \\
\underline{\text{step-by-step:}} & {\scriptstyle i_X}\uparrow & & \uparrow{\scriptstyle i_X f_+} & & \uparrow{\scriptstyle i_X f_+ g_+} \\
 & \langle X,\chi\rangle & & \langle X,\chi'\rangle & & \langle X,\chi''\rangle
\end{array}$$

and

$$\begin{array}{cccc}
 & \langle A,\alpha\rangle & \xrightarrow{fg} & \langle C,\beta\rangle \\
 & {\scriptstyle i_A}\downarrow & & \downarrow{\scriptstyle i_B} \\
 & \langle R,\xi\rangle & \xrightarrow[(fg)_+]{} & \langle T,\tau\rangle \\
\underline{\text{all-step-together:}} & {\scriptstyle i_X}\uparrow & & \uparrow{\scriptstyle i_X (fg)_+} \\
 & \langle X,\chi\rangle & & \langle X,\chi^*\rangle
\end{array}$$

are modified models. Then $\langle X,\chi''\rangle$ and $\langle X,\chi^*\rangle$ are same, that $\chi'' = \chi^*$.

Proof.

$$\begin{aligned}\chi'' &= (\chi' - i_{X'} i^\sharp_{X'} \chi' i_{X'} i^\sharp_{X'}) \cup i_{X'} g_+ \tau g^\sharp_+ i^\sharp_{X'} \\ &= (((\chi - i_X i^\sharp_X \chi i_X i^\sharp_X) \cup i_X f_+ \sigma f^\sharp_+ i^\sharp_X) - i_{X'} i^\sharp_{X'} \chi' i_{X'} i^\sharp_{X'}) \cup i_X f_+ g_+ \tau g^\sharp_+ f^\sharp_+ i^\sharp_X)\end{aligned}$$

Since $i_X f_+ \sigma \chi' f^\sharp_+ i^\sharp_X \subseteq i_{X'} i^\sharp_{X'} \chi' i_{X'} i^\sharp_{X'}$ and $i_{X'} i^\sharp_{X'} \chi' i_{X'} i^\sharp_{X'} \cap \chi \subseteq i_X i^\sharp_X \chi i_X i^\sharp_X$, we have

$$\begin{aligned}\chi'' &= ((\chi - i_X i^\sharp_X \chi i_X i^\sharp_X) - i_{X'} i^\sharp_{X'} \chi' i_{X'} i^\sharp_{X'}) \cup i_X f_+ g_+ \tau g^\sharp_+ f^\sharp_+ i^\sharp_X \\ &= (\chi - i_X i^\sharp_X \chi i_X i^\sharp_X) \cup i_X f_+ g_+ \tau g^\sharp_+ f^\sharp_+ i^\sharp_X \\ &= \chi'\end{aligned}$$

□

4 Conclusion

In this paper, we introduce a formalization of definitions and evolutions of models in a repository using graph structures and relational expressions. We analysed some properties by formal ways. We proved a property of the step-by-step modification and the all-step-together modification in our formalizing. We are going to investigate further related properties with our introduced mechanisms and find out some useful insights to develop model modification tools.

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