Dynamic Asset Allocation with Value-at-Risk Regulation described by Variables having Jump Diffusion Processes for Multiple Assets

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Dynamic Asset Allocation with Value-at-Risk Regulation described by Variables having Jump Diffusion Processes for Multiple Assets

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1 Introduction

One of the inherent hazards of investing in financial market is the risk incurred by the sudden large shock in security prices and volatilities [1]-[8]. So far Value at Risk (VaR) has gained increasingly popularity in recent years as a risk measure to understand the maximum loss of a portfolio. In the investment horizon, the problem of intertemporal optimization problem under VaR constraints is resolved [1]-[8]. Conventional works showed several VaR regulation scheme, however, they are restricted to cases having asset with ordinary Brownian motions without risk incurred by sudden large shocks (jumps). Also in previous works, we demonstrated VaR regulation method with asset having jump diffusion processes, but the solution is restricted to two-asset case [8]. It is also necessary to estimate diffusion processes from time series. In this paper, we show the extension of VaR regulation. This paper deals with the dynamic asset allocation with Value-at-Risk regulation described by variables having jump diffusion processes for multiple assets.

At first, we assume in the model the security price follows jump-diffusion processes which are triggered by a Poisson event [9]-[14]. Because of the tractability provided by the affine structure of the model, we can reduce the Hamilton-Jacobi-Bellman (HJB) partial differential equations (PDEs) which are allowing us to obtain the optimal solution for investment [9]-[14]. In the model, it is assumed that VaR is bounded at time t by an exogenous limit proportional to the current wealth directly for a given time horizon, then the problem becomes to be tractable enough. By using the first-order approximation of the wealth process, we find the optimal dynamic portfolio in which we switch the weight for the risky asset depending on the boundaries of weight[5][7][8]. As a result, the suppression of loss in investments and increase of profit are realized by VaR regulation compared to cases without regulation. Since the estimation of parameters defining diffusion processes may affect the VaR regulation results, we also show the fuzzy based (multi-stage fuzzy) inference for estimating the jump diffusion processes [14]-[17].

In the followings, in Section 2, we treat the basic model of asset price dynamics and the optimal investment. Section 3 gives the first order approximation of evaluation function after a elapse of time and the impact of VaR regulation. In Section 4, we describe applications for the dynamic asset allocation having risky assess with jump diffusion processes under VaR regulation.

2 Basic Model of Asset Price Dynamics and Optimal Investment

2.1 Asset price dynamics and budget equation

Asset prices usually follow ordinary Brownian motion, however, sometime prices exhibit enormous spikes in which prices may jump several order of magnitude in a short period of time, and then return to normal levels just as quickly [9]-[14]. Then, the period of price jumps is assumed to be small enough. The price spike is comprised of sudden rise (denoted as "go" in equations), but closely followed by a backward fall (denoted as "back" in equations) by which price will return quickly to the level close to the previous value of $P_i(t)$. For simplicity, we restrict ourselves to the cases where we have only two kinds of jumps, namely, the upward jump and the downward jump. In the upward (downward) jump, sudden rise (fall) in price $P_i(t)$ is followed by quick fall (rise) forcing the prices move to previous levels. The most general form of continuous time, Markov process for the price $P_i(t), i = 1, 2, ..., n$ for ith asset in one dimension can be written as.

$$dP_i(t) = \alpha_i P_i(t)dt + \sigma_i P_i(t)dz_i + H_i(P_i)dt. \tag{1}$$

$$H_i(P_i) = (\gamma_{go,i} - P_i)\lambda_{go,i}(P_i) + (\gamma_{back,i} - P_i)\lambda_{back,i}(P_i). \tag{2}$$

where α_i, σ_i are constant values which are different from the definition in reference [9][13][14]. In the equation, $\lambda_{go,i}(.), \lambda_{back,i}(.)$ are the probabilities of occurrence of jumps per unit of time, and $\gamma_{go,i}, \gamma_{back,i}$ are the amounts of jumps characterized by random variables drawn from some probability distribution functions $J_{go,i}, J_{back,i}$ (for example, price moves from $P_i(t)$ to $P_i(t)+\gamma_{go,i}$. The variable $dz = (dz_1, dz_2, ..., dz_n)$ are the standard increments of Brownian motions.

It is assumed that $\lambda_{go}(.), \lambda_{back}(.)$ have simple forms with piecewise linear characteristics. Fig.1 shows the schematic diagram of occurrence probability $\lambda_{go,i}(.), \lambda_{back,i}(.)$ of upward/downward jumps. As is seen from figures the probability $\lambda_{go}(P), \lambda_{back}(P)$ are changed depending on the threshold values such as PT_{11}, and except for these transient region they have constant values. The probability distribution of jumps J_{go}, J_{back} are defined by normal distribution $N(a, s)$ having mean a and standard deviation s.

- upward jumps : $J_{go} \sim N(a_{11}, s_{11}), J_{back} \sim N(a_{12}, s_{12})$
- downward jumps : $J_{go} \sim N(a_{21}, s_{21}), J_{back} \sim N(a_{22}, s_{22})$

On the other hand, the model of price for the risk-free asset such as savings is represented by $dP/P = rdt$, where r is the ordinary interest rate.

Fig.2 shows examples of upward jump and downward jump.

2.2 Optimal portfolio selection

We consider a continuous-time stochastic economy on the finite horizon $t \sim T$ with investment opportunities represented by n long-lived securities. By assuming the price process described in previous section, the increment of wealth $dW(t)$ for the wealth process $W(t)$ of an investor at time t is given by the following equation (details are shown in Appendix A).

$$dW(t) = [\sum_{i=1}^{m} w_i\eta_i + r]W(t)dt + \sum_{i=1}^{m} \sigma_i w_i W(t)dz_i + \sum_{i=1}^{m} w_i P_i^{-1}(t)H_i(t)W(t)dt. \tag{3}$$

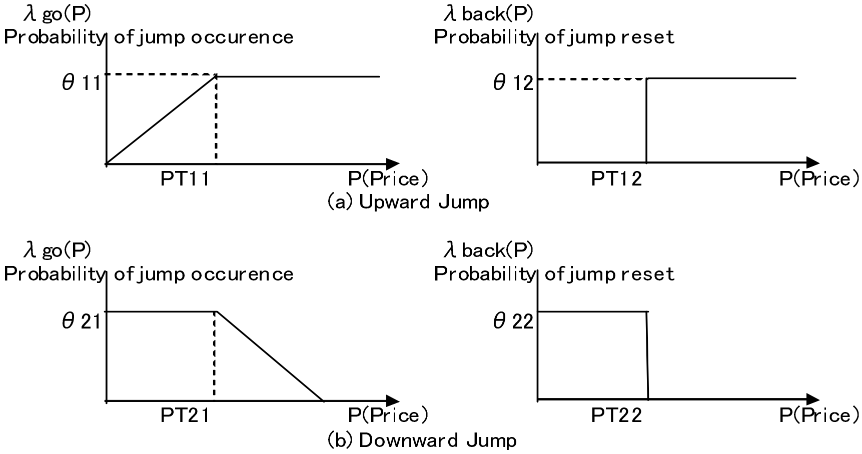

Figure 1: Probabilities $\lambda_{go}, \lambda_{back}$ for upward and dowmward jump (upper:upward, lower:downward)

where $w = (w_1, w_2, ..., w_n)$ is the proportion of wealth invested in i-th asset at time t, and $\eta_i = \alpha_i - r$. For simplicity, the nth asset is assumed to be risk-free asset and remaining $m = n-1$ assets are risky assets.

There are many reductions for the problem of choosing optimal investment based on the stochastic dynamic programming, which leads us to the following HJB (Hamilton Jacobi Bellman) equation (reduction of formula is found in [9]-[11], and the overview of the reduction is shown in Appendix B. The underlying problem is to maximize following evaluation function.

$$V(W,t) = \max_w E[\int_t^T (W(t)dt]. \tag{4}$$

The solution of the problem is given by the following partial differential equation (PDE) as shown in Appendix B.

$$\begin{aligned} 0 \ &= \ \max_w [V_t + \sum_{i=1}^m w_i \eta_i + rWV_W + \frac{1}{2} \sum_{i=1}^m \sum_{j=1}^m \sigma_{ij} w_i w_j W^2 V_{WW} \\ &+ \sum_{i=1}^m w_i [V_{go,i}(W,t)^+ - V)\lambda_{go,i} + (V_{back,i}(W,t)^+ - V)]\lambda_{back,i}]. \end{aligned} \tag{5}$$

where we use the notations such as: $V_t = \partial V/\partial t, V_W = \partial V/\partial W, V_{WW} = \partial V^2/\partial W^2$. The values such as $V_{go,i}(W,t)^+$ are the value of $V(.)$ where in price P_i jump process has occurred.

In the usual fashion of maximization of equation (4) under constraint, we definer the Lagrangian $F = \phi + \zeta(1 - w_1 - w_2 - ... - w_n)$ where ϕ is the function in the bracket on the right hand side of equation (5), and ζ is the multiplier and find the extreme points from the first-order conditions. We obtain the optimal values for $w^* = (w_1^*, w_2^*, ..., w_n^*)$ based on following equations.

$$0 = -\zeta + \eta_i WV_W + \sum_{j=1}^n \sigma_{ij} w_j^* W^2 V_{WW} + I_{go,i}\lambda_{go,i} + I_{back,i}\lambda_{back,i}, i = 1, 2, ..., n. \tag{6}$$

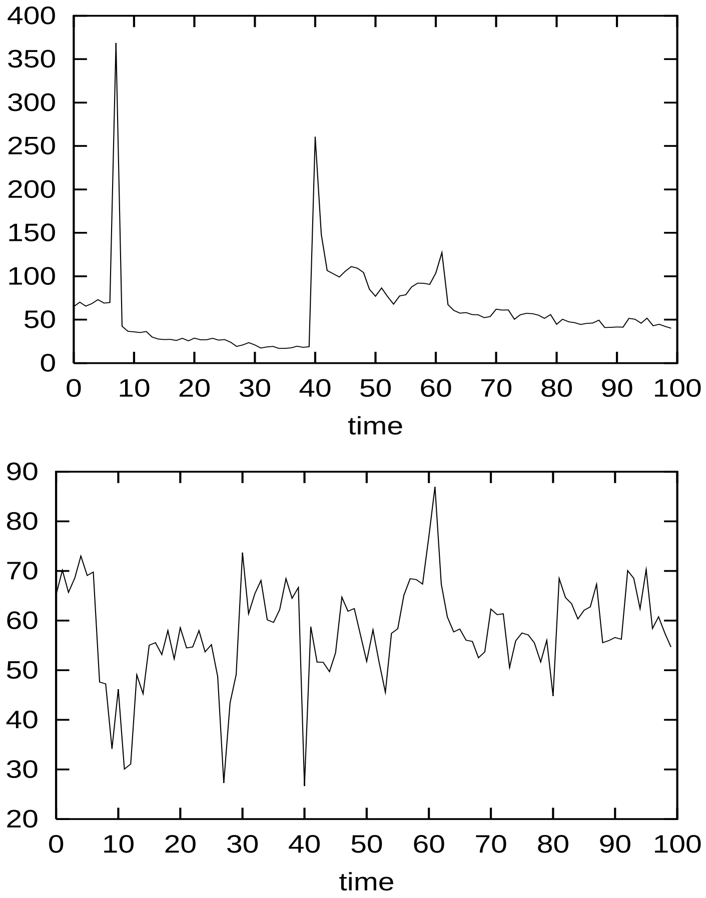

Figure 2: Examples of upward jump and downward jump (upper:upward, lower:downward)

$I_{k,i}, k = go, back$ included in equation (6) is obtained by evaluating $E[V_{go,i}(W,t)^{+} - V(W,t)), V_{back,i}(W,t)^{+} - V(W,t))]$ as.

$$I_{k,i} = \int_{-\infty}^{\infty} V(\gamma_{k,i}) d\gamma_{k,i}, k = go, back. \tag{7}$$

where we utilize the relaltions $V_k^{+}(P_i + dP_i) = V(P_i + \gamma_{k,i} - P_i) = V(\gamma_{k,i})$ in the calculation of expectations $V_{k,i}^{+}$. Moreover, since the nth asset is the risk-free asset, we have $0 = -\lambda + rV_W$. By substituting the relation into equation (6), we have further the representation for the weight $w_i, i = 1, 2, ..., m$ as

$$[w^*]^T = -\Omega[A^T W V_W + B^T]\frac{1}{W^2 V_{WW}}, \Omega = [\Sigma]^{-1}, \Sigma = [\sigma_{ij}]. \tag{8}$$

$$A = [\alpha_1 - r, \alpha_2 - r, ..., \alpha_m - r], B = [I_{go,1} + I_{back,1}, I_{go,2} + I_{back,2}, ..., I_{go,m} + I_{back,m}]. \tag{9}$$

where Ω is the inverse matrix for the matrix Σ having the elements σ_{ij} (diffusion matrix).

As is seen in equations (6)-(9), PDEs themselves are represented by the optimal values of allocation w_i, it is hard to solve direct higher order nonlinear equations. Then, we apply successive approximation to solve PDEs as an alternative. The successive approximation procedure is summarized as follows.
(1) Give initial values for w_i at random.
(2) Solve PDEs and obtain current solutions w_i.
(3) Replace these variables by new solutions, and iterate solution procedures of PDEs.
(4) Terminate if the solutions w_i are not changed, otherwise iterate solution process.

3 Modeling of impact of VaR regulation

3.1 Simplified VaR regulation

Now consider the problem of a trader who starts with an endowment W_0 and must select a portfolio w_i so as to maximize the expected utility of the trading portfolio, subject to that at any time the VaR of its portfolio in no larger than some prespecified level.

Usually, VaR is defined as the probability level of loss after a elapse of time τ about the current wealth at time t. If $W(t)$ is defined as a function, then the VaR is described by the following form with allowable level ν.

$$Prob[(W(t) - W(t+\tau)) \geq L_{loss}] < \nu. \tag{10}$$

for a given loss probability L_{loss} and the time horizon τ. For example, if the random variable to describe the change of wealth obeys to the normal distribution, then $v = F^{1-}(\nu)$ where $F(x)$ is the integration of probability function of normal distribution from $-\infty$ to x ,

There exist several models for evaluating VaR regulation such as the Variance-Covariance method (called Delta method), but these rigid definitions are not relevant to estimate the impact of VaR regulations. Following the research by Leippold et al., we use the definition of VaR as follows [5][7][8].

$$L_{loss} = \beta W(t). \tag{11}$$

We work with a VaR regulation proportional to current wealth $W(t)$. Even though the VaR regulation in equation (11) is a legitimate but certainly not unique choice. In practice, different risk regulation specifications are used. However, the definition in equation (11) has some nice tractability properties when we perform the optimization. Then, we restrict our analysis to a proportional VaR regulation to mimic the regulation framework.

We must note that the wealth dynamics depends on the stochastic state variable $P_i(t)$, then we cannot expect to obtain closed form solutions for the bank's intertemporal decision problem in the presence of VaR regulations. To retain analytical tractability, we approximate the VaR constraint shown in equation (10)(11).

So as to approximate the VaR constraints implied by equation (10)(11), we apply the Ito Tayler expansion formula to define the first-order approximation.

$$\log W(t+\tau)^{(1)} \approx \log W(t) + [\sum_{i=1}^{m} w_i \eta_i + r - \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m} w_i w_j \sigma_{ij} + E(\sum_{i=1}^{m} w_i P_i^{-1} H_i)]\tau + \sum_{i=1}^{m} \sigma_i w_i dz_i. \tag{12}$$

In this relation, the term $E(\sum_{i=1}^{m} P_i^{-1} H_i)$ is evaluated by taking the expectation within the range of $P_i(t)$ and the random variable $\gamma_{k,i}$. As Leippold et al. discussed, the approximation error using the first-order approximation is relatively good [5]. They define the approximation error as the probability of the first-order approximation $W(t+\tau)^{(1)}$ for the value $W(t)$ of a fixed weight portfolio with initial weight $w(t)$ which is bounded by

$$Prob[\log W(t+\tau)^{(1)} - \log W(t+\tau)] \geq M]. \tag{13}$$

As they suggested the conditional probability that the logarithmic difference between the approximated wealth and the true wealth exceeds the amount M at time $t+\tau$ can be bounded by a certain measure. If we assume the mean-reverting geometric Brownian motion for the volatility process, the experimental results show the approximation error M is usually bounded below 1%.

The quality of the approximation ensures us to use the VaR approximation to investigate the constrained dynamic portfolio. Moreover, market practice usually confines itself to regulatory VaR figures reported based on a conditional normal distribution. The approximation implies us the possibility of the direct portfolio bounds on the optimal policy of VaR-constrained bank. More over we checked the availability of first order approximation under jump diffusions provide us good approximation (details are omitted here) [8].

3.2 VaR regulation by changing asset allocations

To reduce the bank optimal behavior under the VaR constraints to adjust the weight w_i, we start with assuming the first order approximation of change of wealth $W(t)$. Assuming that the maximum loss of current wealth $W(t)$ to be βW at time $t+\tau$, then we have the relation $G(w) \leq 0$ based on the equation (10) $\sim$ (12).

$$G(w) = \log(1-\beta) - [\sum_{i=1}^{m} w_i \eta_i + r - \sum_{i=1}^{m}\sum_{j=1}^{m} \frac{1}{2} w_i w_j \sigma_{ij} + E(\sum_{i=1}^{m} w_i P_i^{-1} H_i)]\tau - v \sum_{i=1}^{m} w_i \sigma_i \sqrt{\tau}. \tag{14}$$

where by definition we have $v = F(\nu)^{-1}$. Here we denote w_i^v as the solutions of $G(w) = 0$, and w_i^f as the optimal value given by equations (6)(7). Then, it is expected that variables (allocations) must be selected as

$$w_i^* = \begin{cases} w_i^v, & w_i^f \geq w_i^v, \\ w_i^f, & \text{otherwise.} \end{cases} \tag{15}$$

4 Applications

4.1 Examples of VaR regulation

At first, we show examples of VaR regulation based on simulation studies. We assume following parameters for numerical examples. $T = 8(year), \tau = 25/256, n = 101, \nu = 0.01, \beta = 0.05, r = 0.05, \alpha_i = 0.08, \sigma_{ii} = 0.5, \sigma_{ij} = \delta\sigma_{ii}, i \neq j$.

where δ are selected from uniformly distributed random numbers ranging between 0.01 and 0.08. For the jump diffusions we assume following characteristics.

(upward jump)

$PT_{11} = 100, PT_{12} = 100, a_{11} = 300, s_{11} = 100, a_{12} = 100, s_{12} = 20, \theta_{11} = 0.05, \theta_{12} = 0.85$

(downward jump)

$PT_{21} = 50, PT_{22} = 50, a_{21} = 30, s_{21} = 10, a_{22} = 50, s_{22} = 10, \theta_{21} = 0.05, \theta_{22} = 0.85$

Note that these parameters were not fit to actual data, but merely serve as a backdrop to illustrate the method developed in the paper. In other words, when prices are low they follow a time-dependent, mean reverting, stochastic Brownian motion. As price rise (fall), so does the probability of an upward (or downward) price spike. For example, in the upward jump, when a spike occurs, the price instantly rise into the high price regime normally distributed with mean 300 and standard deviation 100; while in this regime the probability of backward fall which bring the price back into the low price regime. In the following, we denote Case U (Case D) corresponding to the cases where price including upward jump (prices including downward jumps).

Most important feature of VaR regulation is the limitation of loss, and also increase of gain by changing the allocation among assets. Fig.3 shows the probability density of the value of wealth process $W(t)$ for the cases including upward and downward jumps in asset prices , respectively (the horizontal axis is $\log W(t)$). In these figures, the solid lines correspond to the wealth under the VaR regulation (denoted as Case U(Y), Case D(Y)), and dashed lines mean the wealth obtained by applying no regulation (denoted as Case U(N), Case D(N)). As is seen from the results, the variance of the solid line is lower than that of dashed lines, then the VaR regulation provide us lower probability of extreme losses in the investment strategy than under unconstrained strategy. For simplicity, we omit the figures depicting the behavior of w_i, however, w_i are switched from w_i^v to w_i^f, or vise versus depending on the risk exposure.

We define the number of cases N_B of violation of VaR regulation where $W(t) - W(t+\tau) > \beta W(t)$, and obtain the ratio of violation of regulation as $R = N_B/N_T$ where N_T is the total number of observation. Then, the ratio of these examples are summarized as follows.

Case U(Y):$R = 0.011$, Case U(N):$R = 0.031$

Case D(Y):$R = 0.011$, Case D(N):$R = 0.032$

By varying the parameter σ_{ii}, it is expected that the difference of R among cases will be changed. Table 1 summarizes the comparison of R depending on several values of σ_{ii}. As is seen from the result, the difference included in R becomes larger along the increase of σ_{ii}, and we see that the increase of wealth in the region of relatively small σ_{ii}, and also the suppress of the risk in the region of higher value of σ_{ii} are realized.

Table 1: Comparison of ratio R (violation of VaR regulation)

σ_{ii}	Case U(Y)	Case U(N)	Case D(Y)	case D(N)
0.1	0.012	0.021	0.012	0.020
0.5	0.011	0.031	0.011	0.034
1.0	0.013	0.071	0.012	0.051
2.0	0.015	0.122	0.014	0.120

4.2 Effects of jump diffusion on VaR regulation

Now we examine the effects of the jump diffusion processes in the VaR regulation by changing the parameters of simulation studies. Parameters are assumed to be taken from following ranges of values.

$a_{11} = 50 \sim 700, s_{11} = 20 \sim 100, a_{12} = 20 \sim 100, s_{12} = 10 \sim 30$

$\theta_{11} = 0.005 \sim 0.1, \theta_{12} = 0.3 \sim 0.85$

$a_{21} = 10 \sim 50, s_{21} = 10 \sim 20, a_{22} = 30 \sim 60, s_{22} = 10 \sim 20$

$\theta_{21} = 0.005 \sim 0.1, \theta_{22} = 0.3 \sim 0.85$

Other parameters such as r, α, PT_{11} are assumed to be fixed to the initial values. However, the number of changeable parameters are relatively large, then we restrict simulation studies by imposing the assumption that the occurrence of jump (probability) is rare (frequent) if the amplitude of the jump is large (small). Then, for example the parameter a_{11} is inversely proportional

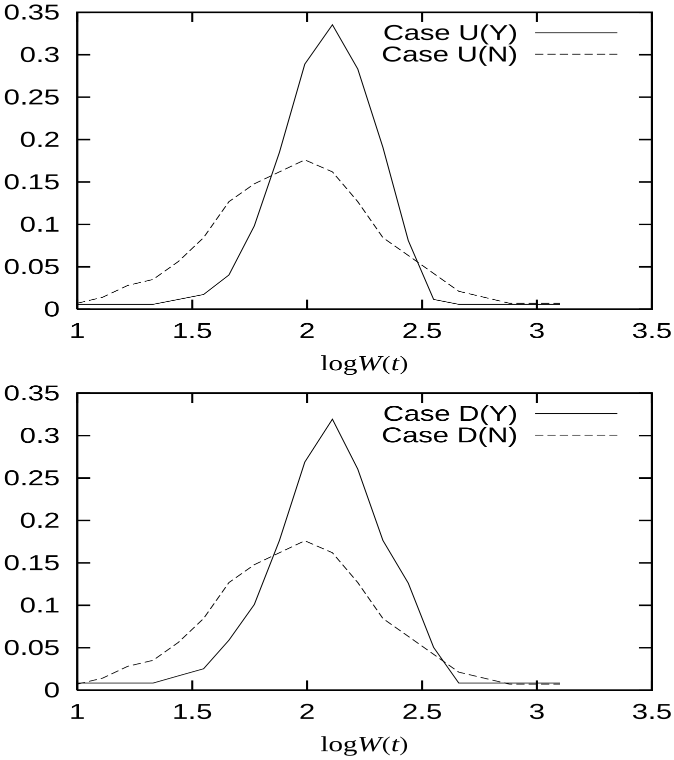

Figure 3: Porbabiliy density of $\log W(t)$ (upper:Case U, lower:Case D)

to θ_{11}, and a_{12} is inversely proportional to θ_{12}. In a similar manner, other sets of parameters are combined through this kind of relations. We also use the ratio R (rate of violation of VaR regulation) to examine the effect of VaR regulation on asset allocation. In Table 2 we summarize the difference of ratio R between the cases with VaR regulation and without VaR regulation. As is seen from the result, according to the increase of amplitude of jumps, the value of R grows rapidly for cases without VaR regulation, however, in cases with VaR regulation the ratio R is stable and almost lower than 0.05.

Table 2: Comparions of R depending on parameters of jumps

θ_{11}	Case U(Y)	Case U(N)	θ_{21}	Case D(Y)	case D(N)
0.01	0.011	0.037	0.01	0.011	0.031
0.05	0.012	0.037	0.05	0.012	0.032
0.10	0.017	0.061	0.10	0.015	0.070

4.3 Application to real data

Then, we discuss the applicability of the method proposed in the paper to real data of asset allocations. Different form artificially generated data, we have no models about the generation processes of prices, and they are necessary to be estimated from observed data. We apply the estimation scheme about the generation models of prices including jump diffusions based on the Genetic Programming (GP) and the multi-stage fuzzy inferences[14]-[18]. However, the comprehensive description of the estimation scheme is not appropriate for the studies of applications, then we only summarize the overview of estimation scheme (details are shown in references [14]).

The estimation process is composed of two subsystems: Subsystem F corresponds to the fuzzy inference, and Subsystem S corresponds to the estimation of Brownian motion.

Subsystem F

Since the occurrence of jump diffusion is usually rare, and available observation is restricted. Then, we use a learning scheme based on artificially generated data. We prepare a certain learning time series for price $P(t)$ including jump diffusion processes. We assume that the center of the jump diffusion occurs at time t_c, and the time series data for price $P(t)$ around t_c (denoted as vector $x(t) = (P(t_c - K), P(t_c - K + 1), ..., P(t_c + K - 1), P(t_c + K))$ are used as the input to Subsystem F. Since we know the time t_c as the occurrence of jump, we can organize the Subsystem so that the output $y(t)$ is to be 1 (0) if the input vector $x(t)$ to the system includes (does not include) the jump diffusion process. Briefly speaking, in the multi-stage fuzzy inference system (Subsystem F), the weights of the membership functions are adjusted in the way that the output $y(t)$ becomes to be either 1 or 0 depending on the input segment $x(t)$ of the time series. In general, the number of input variables for the fuzzy inference system is limited to suppress the explosion of the number of rules, however, in the multi-stage fuzzy inference system the input vector $x(t)$ is provided to the several different stages of inference system in a distributed manner, then the number of rules is remarkably small compared to ordinary single stage fuzzy inference systems [14]-[16].

After learning process, we then apply Subsystem F to the observation where in similar manner we divide the time series of price $P(t)$ into segments $x(t)$. If the output $y(t)$ of Subsystem F is greater than 0.5 (lower than 0.5), then we conclude that the jump in included (not included) in the underlying segment $x(t)$.

Subsystem S

We assume that the time period of jump diffusion is quite short and remaining observations (jumps are removed from the time series) are available to estimate σ_{ij} for Brownian motions.

Recent observation of financial time series such as stock prices and exchange rates are used for real application. We have following financial time series at hand.

(1) Two exchange rates:dollar-yen and euro-yen rate

(2) nine stock prices:average stock prices in nine stock markets such as New York stock exchange market, however average stock prices.

The time periods of observation are taken as following two period:

Period I: 50 trading days before October 20, 2011.

Period II: 50 trading days after October 20, 2011.

Fig.4 shows an example of time series in Period I and Period II. We know that the stock prices in Period I are relatively stable, and in Period II after a large event stock prices bear fluctuation (sudden rise followed by sudden fall). For simplify the analysis, we normalize these time series so that the average value of each time series is to be around 35. Details of estimation of σ_{ij} and the features of jump diffusion are omitted here, and we only use the result of estimation in the following.

Parameters are taken as $\beta = 0.05$, $\tau = 1/250$. Similar to artificial data, we evaluate the VaR regulation scheme by using the indicator R which mean the ratio of the number of violation of VaR regulation to the total observation. In two periods, we have R as follows.

with VaR regulation:$R = 0.011$ in Period I, $R = 0.012$ in Period II

without VaR regulation:$R = 0.041$ in Period I, $R = 0.067$ in Period II

Then, it is clear that the investment accompanied by VaR regulation provide us better result than ordinary investment scheme.

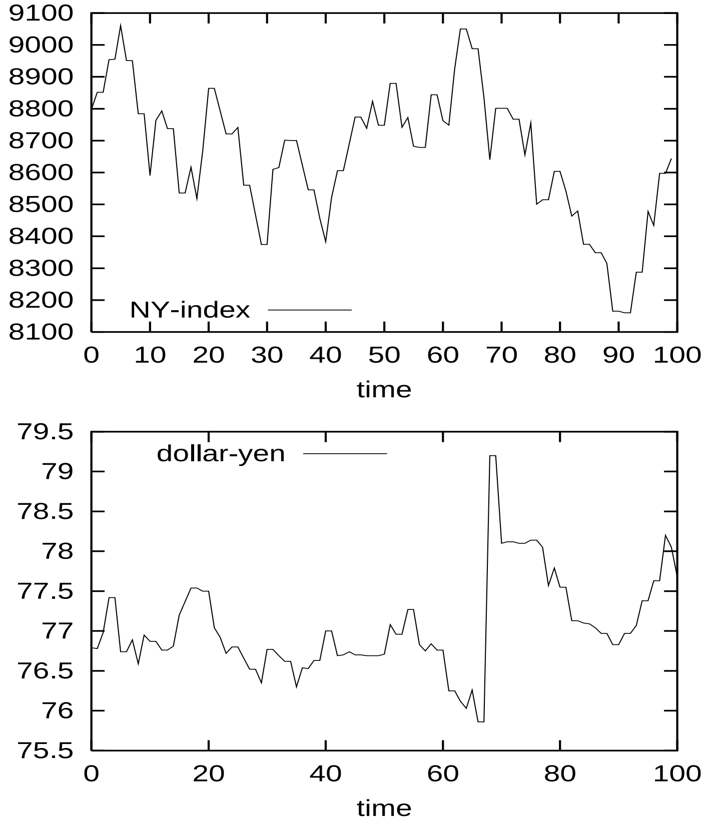

Figure 4: Examples of real price time series (upper:NY-index, lower:dollar-yen)

5 Conclusion

This paper showed the implications of event-related jumps in security prices and the dynamic portfolio strategies under VaR regulation. With the jump-diffusion processes triggered by a Poisson event, we reduced the HJB PDEs. By assuming that VaR is proportional to current wealth directly, we found the optimal dynamic portfolio by switch the weight for the risky asset depending on the boundaries of weight. We described examples application for the proposed methods.

For future works, it is necessary to extend the method to various fields of investment problems. Further researches will be done by the authors.

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Appendix A: Reduction of relation for dW

As first we assume that there is no term including jump diffusion processes. Define $N_i(t)$ as the number (amount) of shares of ith asset purchased during period between t and $t+h$. Then, the total wealth invested in assets is given by

$$W(t) = \sum_{i=1}^{n} N_i(t-h)P_i(t). \tag{16}$$

for simplicity, we assign the suffix n for the risk-free asset. If it is assumed that all trades are made at known current prices, then we have

$$0 = \sum_{i=1}^{n} [N_i(t) - N_i(t-h)]P_i(t). \tag{17}$$

Then, the investor comes into period $t+h$ with the wealth.

$$W(t+h) = \sum_{i=1}^{n} N_i(t)P_i(t+h). \tag{18}$$

By changing the equation (17) as

$$0 = \sum_{i=1}^{n} [N_i(t+h) - N_i(t)][P_i(t+h) - P_i(t)] + \sum_{i=1}^{n} [N_i(t+h) - N_i(t)]P_i(t). \tag{19}$$

By taking the limit as $h \to 0$, we arrive at the continuous version of equations (18) and (19) as

$$0 = \sum_{i=1}^{n} dN_i(t)dP_i(t) + \sum_{i=1}^{n} dN_i(t)P_i(t), W(t) = \sum_{i=1}^{n} N_i(t)P_i(t). \tag{20}$$

By using the Ito's lemma, we differentiate it to get

$$dW = \sum_{i=1}^{n} N_i dP_i + \sum_{i=1}^{n} dN_i P_i + \sum_{i=1}^{n} dN_i dP_i. \tag{21}$$

By arranging the equation, we have

$$dW(t) = \sum_{i=1}^{n} N_i(t)dP_i(t). \tag{22}$$

By substituting the relation for dP_i/P_i and arranging the equation, we have

$$dW(t) = [\sum_{i=1}^{n} w_i\eta_i + r]W(t)dt + \sum_{i=1}^{n} \sigma_i w_i W dz_i. \tag{23}$$

wherw $w_i(t) = N_i(t)P_i(t)/W(t)$ is the rate of allocation of total investment to the ith asset, and $\eta_i = \alpha_i - r$.

In case where we include the jump diffusion processes in prices, we merely need to add $\sum_{i=1}^{m}[(\gamma_{go,i} - P_i)/P_i + (\gamma_{back,i} - P_i)/P_i]W$ to the right hand size of the equation (23).

Appendix B: Optimization of evaluation function to derive PDE.

In equation (4), we divide the time period on integration into $t \sim t+dt$ and $t = t+dt \sim T$ (two parts).

$$V = \max_w E[\int_t^{t+dt} W(P,t)d\tau + \int_{t+dt}^{T} W(P,t)d\tau]. \tag{24}$$

By rewriting the second term in the right hand size of the equation, we have

$$V = \max_w E[\int_t^{t+dt} W(P,t)d\tau + V(W+\Delta W, t+dt)d\tau]. \tag{25}$$

By expanding the term $V(W+\Delta W, t+dt)$ based on the Ito's lemma and by rearranging them, we have

$$V = \max_w [\int_t^{t+dt} W dt + [V(W,t) + [V_W dW + \frac{1}{2}V_{WW}(dW)^2]dt]. \tag{26}$$

By removing common terms on both side of the equation and by substituting dW, dW^2 in Appendix A and taking $dt \to 0$, then we have

$$\begin{aligned} 0 \quad = \quad & \max_w [V_t + \sum_{i=1}^{m} w_i\eta_i + rWV_W + \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m} \sigma_{ij} w_i w_j W^2 V_{WW} \\ & + \sum_{i=1}^{m} w_i [V_{go,i}(W,t)^+ - V)\lambda_{go,i} + (V_{back,i}(W,t)^+ - V)]\lambda_{back,i}]. \end{aligned} \tag{27}$$

The values such as $V_{go,i}(W,t)^+$ are the value of $V(.)$ where in price P_i jump process has occurred.

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