

Recent Development of Personal Income Distribution Models : Application on the Case of Rural China

He, Yan
Graduate School of Economics, Kyushu University

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Recent Development of Personal Income Distribution Models: Application on the Case of Rural China

Yan He[†]

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1 . Introduction

Inequality has been an issue of concern of social scientists and policy makers in China. China has achieved high economic growth for over 20 years with rising income inequality (Rozelles, 1994; Yao *et al.*, 2004, Wan and Zhang, 2006). To describe the income inequality in China, different inequality index have been employed such as Theil index (Akita, 2003; Cheng and Li, 2006) and Gini coefficient (Yao, 1997, 1999; Wan, 2001). More popular, measure of inequality is the Gini coefficient, which is more intuitive to many people since it is based on the Lorenz curve.

For the case of China, the calculation of Gini coefficient is mostly concentrated on decomposition. Attempt on geographic targeting (Wagstaff, 2005) spatial decomposition has firstly been drawn attention since the coastal areas experienced phenomenal growth due to the open policy in 1978 (Yao, 1997; Yang, 1999; Shorrocks and Wan, 2005). Moreover, regression-based decomposition has also been applied to estimate Gini ratio by Zhang and Zhang (2003), Wan (2004), Wan and Zhou (2005) and Deng and Li (2009). The most popular studies devote to inequality decompo-

[†] Ph.D. Program 2nd Year, Graduate School of Economics, Kyushu University, Japan; Lecturer of Business School in Xiangtan University, Hunan, China. I wish to thank Prof. Saeki for helpful comments. The views and any remaining errors in this paper are solely author's responsibility. I would like to thank financial support from "Humanities and Social Sciences Planning Fund Supported by Chinese Ministry of Education: The study on the Linkage mechanism between coordination and regional urbanization under the goals of common rich development (No.12YJC790053, 2012~2013 year)".

sition by the means of subgroups. However, there are some drawbacks on subgroups to calculate Gini coefficient, which is hard to define and interpret surplus term R (Mookherjee and Shorrocks, 1982), although many scholars tried to offer reasonable interpretation for R (Siber, 1989; Yitzhaki and Lerman, 1991; Lambert and Aronson, 1993). Another debate of is that the subgroups decompositions have variable types on weight, such as Rao (1969) demonstrates weights should be population share of subgroup, while Mangahas (1975) debates it is proper to use income share of subgroup for weights. Targeting above issues of estimating Gini coefficient, parameter estimation can solve well. The key point is to choose an effective model to fit the family income data.

There are a great mount of literatures concentrating on model specification from two parameters to five parameters, even more. Two-parameter function is too simple to reflect the impact of economic fluctuation on the size distribution, whereas, five-parameter model is easy to loss simplicity. Thus, the purpose of this paper is to estimate Gini ratio by applying Dagum (1997) distribution (four-parameter model) on Chinese family income data for its various merits comparing with other models. Relative studies such as, Bandourian *et al.* (2003) find that, in a study utilizing 82 data sets, the log-logistic model (Dagum, 1975) is the best 3-parameter model in no less than 84% of the cases. The Dagum often provides a better fit to income data than the closely related Singh-Maddala distribution, which Kleiber (1996) provides a heuristic explanation.

On the whole, the purpose of this paper is to apply Chinese family income data by employing a 4-parameter model proposed by Dagum (1977a). In China, the statistical data is separated to rural part and urban parts. In this paper, I employ family income data in rural part only. We also apply two methods proposed by Dagum (1977a) for comparison. The rest of the paper will be organized as follows. Section 2 represents the development of parameter estimation, and points out the merits of Dagum distribution. Section 3 discusses the income distribution model and methodology used in this study. Section 4 illustrates applications of the results to rural household survey data in rural China. Section 5 concludes.

2 . A Survey of Income Distribution Function

Pareto (1895) first proposed a model of income distribution in the form of a probability density function, which started the exploration of the field of personal income distribution. The apparent attractions of the Pareto distribution evaporate somewhat when one considers its implications for the distribution of income amongst the population as a whole (Clementi *et al.*, 2008). However, empirical studies showed that the Pareto distribution accurately models only high levels of income, but poor in describing the lower end of the distribution since the income distribution is right-skewed and has a fat right-hand tail. As research continued, a variety of probability functions were proposed as suitable in describing the distribution of income by size

using both a combination of known statistical distributions (Nirei and Souma, 2004; Clementi *et al.*, 2005) and parametric functional forms for the distribution of income as a whole.

Two-parameter model is first proposed by Pareto (1895) known as the first Pareto law proved by Champernowne (1953) which demonstrated that under certain assumptions the stationary income distribution will approximate the Pareto distribution irrespective of the initial distribution, other contributions of mentioned area see Mandelbrot (1961) and Wold and Whittle (1957). Aitchison and Brown (1954) made efforts to complicate the matter with the applicability of the Champernowne's model. Sargan (1957) built the most complicated model but made it unwieldy and unintelligible (Kleiber and Kotz, 2003). For better describing the observed data, Gibrat (1931) suggested lognormal distribution, which was modified by Kalecki (1945) and examined by Aitchinson and Brown (1969) found that the lognormal distribution fit the whole range of income distribution but is quite poor in describing both the upper and lower tails of the actual distribution. To present the satisfactory goodness of fit, Ammon (1895) proposed the gamma distribution, which was applied to fit income data by March (1898) and further promoted by the work from Salem and Mount (1974) and Bordley *et al.* (1996) for USA and Bartels and van Metelen (1975) for the Netherlands showing that empirical evidence favors the Gamma over the lognormal distribution. Moreover, Rutherford (1955) incorporated birth-death considerations into a Markov model which provides a better fit than the lognormal. In the study of Branchman *et al.* (1996) shows that the gamma distribution emerges as the best two-parameter model by utilizing German household incomes data. The log-Gompertz distribution appears to be used mainly in income and size distributions and was noticed by Dagum (1980) in this connection, which has been proved to be an excellent two-parameter model by Cummins *et al.* (1990). Other contributions for two-parameter distribution are made by Bartels and van Metelen (1975) for the Weibull distribution and Kloeck and van Dijk (1976) the log t (where t is the Student distribution).

Metcalf (1972) argued that a two-parameter function is too simple to reflect the impact of economic fluctuation on the size distribution. Thus, three-parameter functions are studied by scholars. These include the generalized gamma distribution proposed by Taille (1981) and Amoroso (1924-1925) as well as Beta distribution studied by Thurow (1970) and Kakwani and Podder (1976). Singh and Maddala (1976) and Dagum (1977a) proposed two closely related model which are the members of the Burr family of distributions known as Burr VII and Burr III in statistics, respectively. The Singh-Maddala distribution is also known as, the beta-P distribution (Mielke and Johnson, 1974), Burr distribution (Hogg and Klugman, 1983, 1984), the Pareto IV distribution (Arnold, 1983), or a generalized log-logistic distribution (El-Saidi *et al.*, 1990). Two-parameter models, including Lomax (1954) distribution, Log-logistic distribution suggested by Fisk (1961) and Para-logistic distribution proposed by Klugman *et al.* (1998), are the special case of Singh-Maddala distribution. Pareto (1896) also specified the Type II (three-parameter) to be

considered as alternative to the Type I. Furthermore, Vinci (1921) presented Pearson Type V distribution as a model of income distribution. These distributions allowed for intersecting Lorenz curves, a phenomenon observed in the data that could not be modeled by any of the two-parameter distributions considered (Bandourian *et al.*, 2002).

For an accurate description of empirical distributions with an associated measure of income inequality, four-parameter models were also taken into consideration. Pareto (1897) developed the Type III (four-parameter) models on the basis of Type I (Pareto, 1895) and Type II (Pareto, 1896). Champernowne (1952) studied the hyperbolic distribution. However, the four-parameter distributions that were not only very successful in fitting the data, such as the means and Gini ratio appear to be slightly underestimated (Branchmann *et al.*, 1996), but also included all of the previously mentioned distributions as special or limiting cases by studying the generalized beta of the first (GB1) and the second (GB2) kinds (McDonald, 1984). To solve the problems mentioned, Dagum (1977a) proposed a new model of person income distribution, empirical work for USA data shows that the results are the best comparing with the Lognormal, the Gamma, and the Singh-Maddala models.

Furthermore, Five-parameter distributions were presented by Champernowne (1953) and McDonald and Xu(1995). As is mentioned above, two-parameter models usually failed to be empirically corroborated, while five-parameter models are prone to loss simplicity. For the merit of the model proposed by Dagum (1977a), I will apply to Chinese data to estimate the inequality of China for the time period of 1985 to 2010.

3 . Basic properties and Methodology

3.1 Properties of Reasonable Mathematical Model

The specified model may be governed by its capacity to account fairly well to a set of economic, econometric, stochastic and mathematical properties. Dagum (1977a) proposed a personal income distribution model which supports intersecting Lorenz Curves and convergence to the weak form of Pareto distribution¹⁾ and satisfies all the properties as follows.

Firstly, requirement for the parameter set. The mathematical form of the distribution function, which must be simple from a technical point of view and fundamental from the point of view of model-building, can be derived from an elementary set of logic-empirical postulates or assumption. For the setting of unknown parameters, the principle is to make use of the smallest possible number of parameters for adequate and meaningful representation. The number should not be either too simple to empirically corroborated with data or too complicated to loss

1) For details about the interpretation of intersecting Lorenz Curves and convergence to the Pareto distribution, please refer to Dagum (1977a)

simplicity. Thus, three- or possible four- parameter function is taking into consideration. Furthermore, all the parameters should have well-defined economic meaning and simple and efficient of parameter estimation, which is always an advantage from the point of view of computer cost and the acceptance of model in applied economics.

Secondly, concerns on model flexibility. Specified model solves the problem of the model being identified and the actual observation, and provides a good fit of whole range of the distribution. An ideal model is able to deal with a positive, and not predetermined, minimum income without truncating the distribution. Especially, the existence of negative and nil income, which strongly restricts the descriptive power, is considered. Furthermore, the properties considered, such as the shape of distribution though changes in parameter values, unimodal and strictly decreasing (non-modal) income distribution, are also important.

Thirdly, conditions on the parameter estimation. Cumulative distribution function approach was chosen by Dagum to overcome the assumption of equi-distribution within each interval of income required when dealing with the method of parameter estimation. Dagum distribution gives the explicit solution for Lorenz curve and Gini concentration ratio, which allows computing Gini coefficient directly and verifying whether it is proposed as a goodness of fit test.

3.2 The Four-parameter Dagum Distribution

He motivates his model from the empirical observation that the income elasticity of the cumulative distribution function (CDF) F of income is a decreasing and bounded function of F . Starting from the differential equation:

$$\frac{d \log[F(x) - \alpha]}{d \log x} = \beta \delta \left[1 - \left(\frac{F - \alpha}{1 - \alpha} \right)^{1/\beta} \right] \quad (1)$$

where $x > 0$ if $0 \leq \alpha < 1$ and $x > x_0 > 0$; $F(x_0) = 0$, if $\alpha < 0$, subject to $\beta > 0$, $\alpha < 1$, and $\beta \delta > 1$.

The solution of function (1) can get cumulative distribution function (c.d.f):

$$F(x) = \alpha + \frac{1 - \alpha}{(1 + \lambda x^{-\delta})^\beta}, \lambda > 0 \quad (2)$$

where λ is a scale parameter and strictly positive because it is the antilog of the constant of integration. α , β and δ are dimensionless parameters. α is an inequality parameter and β and δ can be called equality parameters because the Gini ratio is an increasing function of the former and a decreasing function of the latter.

3.3 Gini Concentration Ratio

Cumulative distribution function (2) associated with the Lorenz curve argued by Dagum (1977b) can be noted as:

$$L(y) = \frac{B(y^{1/\beta}; \beta + \frac{1}{\delta}, 1 - \frac{1}{\delta})}{B(\beta + \frac{1}{\delta}, 1 - \frac{1}{\delta})}, \quad \beta\delta > 1, \quad 0 \leq \alpha < 1 \quad (3)$$

where

$$y = \frac{F(x) - \alpha}{1 - \alpha}, \quad y \in [0, 1]$$

and its corresponding Gini concentration ratio is:

$$G = (2\alpha - 1) + (1 - \alpha) \frac{\Gamma(\beta)\Gamma(2\beta + \frac{1}{\delta})}{\Gamma(2\beta)\Gamma(\beta + \frac{1}{\delta})} \quad (4)$$

where $\Gamma(\cdot)$ denotes the complete Gamma function.

3.4 Methods of Parameter Estimation

In order to estimate the parameters of the above model, there are five methods proposed, including iterative method I, II and III; unconstrained function minimization and the method of maximum likelihood. For comparison, this paper will choose iterative method I and maximum likelihood to apply on Chinese data.

Iterative method I: According to the transformation of equation (2), we can get:

$$\log F(x) = -\beta \log(1 + \lambda x^{-\delta}) + \sum_{k=1}^{\infty} a^k \frac{1 - F^k}{k F^k} \quad (5)$$

Equation (10) can be approximated by the following linear form:

$$\log F(x) \approx \alpha \frac{1 - F(x)}{F(x)} - \beta \log(1 + \lambda x^{-\delta}) \quad (6)$$

To estimate the parameter vector $(\alpha, \beta, \lambda, \delta)$ in equation (2), start with an initial value of α, β, λ and δ by nonlinear least square using equation (6) to search for convergence by the iterative procedure.

The method of maximum likelihood (ML): According to Wold (1961, 1963), assume $F(x)$ in equation (2) be an unbiased predictor of the sample realization $\eta(x)$. Let ε is the purely random variable and normally distributed, Hence,

$$\eta(x) = F(x) + \varepsilon, \quad \varepsilon \stackrel{d}{=} N(0, \sigma^2) \quad (7)$$

The log of the likelihood equation is

$$\begin{aligned} \log L = & -\frac{n}{2} \log 2\pi - \frac{n}{2} \log \sigma^2 - \frac{1}{2\sigma^2} \\ & \sum (\eta(x) - \alpha - (1 - \alpha)(1 + \lambda x^{-\delta})^{-\beta})^2 \end{aligned} \quad (8)$$

The ML estimator $\hat{\mu}$ of μ is obtained by solving the likelihood equation $\frac{d \log L}{d\mu} = 0$, where $\mu = (\alpha, \beta, \lambda, \delta)$.

4 . Application of Dagum Distribution: Household Income Data in Rural China

4.1 Data Explanation

I will apply Dagum distribution on family income data of rural China for the time period from 1985 to 2010. Income data is gathered from “China Statistical Yearbook” and “China Rural Household Survey Yearbook”, which grouped by the percentage of households. Hence, it is need to transfer household proportion to population proportion. To simplify the mathematical expression, the variables defined are as follows.

PP: Population proportion

SHE: The surveyed households of each group

TSH: Total surveyed households

APE: Average population in each group

TAP: Average population in whole surveyed households

Household population is transferred to individual population proportion by the function:

$$PP_i = SHE_i * APE_i / \sum_{n=1}^{20} (SHE_i * APE_i) = SHE_i * APE_i / TSH * TAP \quad (9)$$

where i denotes income class. The data structure in Yearbook is arranged by “Percentage of Households Grouped by per Capita Annual Net Income (%)”, which we denote as PH. Thus, equation (9) can be expressed as:

$$PP_i = SHE_i * APE_i / TAP \quad (10)$$

Furthermore, we take mean income of net income as income variable. For comparison to native studies, the mean value of each income range will also be used.

4.2 Estimation of Gini Coefficient

The methods of Iterative I and Maximum Likelihood will be used to estimate the parameters and calculate Gini ratio. RATS software of version 7 is employed to compute the parameters. The sum of squared residuals of the observed cumulative distribution function ($F(x)$) from their corresponding predicted probabilities are denoted by:

$$\sum \epsilon^2 = \sum \left[F(x) - \hat{F}(x) \right]^2 \quad (11)$$

The estimated parameter vector ($\alpha, \beta, \lambda, \delta$) by the method of Iterative I lists in table 1. The values of all parameters are positive and significant. β and δ are significant at the level of 1%. λ and α are at least in the significant level of 10%. It also can be seen that the sums of squared residuals show a satisfied goodness fit.

The last column reports the Gini concentration ratio applying the formula deduced from model (2) and presented in (4). Gini ratio in rural China shows an increasing trend from 0.25151 in 1985

to 0.40117 in 2009 which was over the international warning level of 0.4. In 2010, Gini coefficient is slow down. Whatever, high Gini coefficient shows high income inequality and serious social problem in rural China.

Of special interest are methods fitting to the same data, with an eye on relative performance. To avoid the approximation in equation (6), Table 2 represents the estimation results of maximum likelihood. Comparing with Iterative I, the estimated values of all parameters in Table 2 are significant at the level of 1% except parameter α . Variance is approximate to zero, which shows an excellent goodness of fit.

Furthermore, Gini ratio in Table 2 is higher than that of in Table 1 correspondingly, which is can be seen in Figure 1. The line of maximum likelihood is above iterative I. The possible reason accounts for approximate (6). Gini ratio estimated by maximum likelihood is more reliable, although the results of two methods show the increasing trend.

Native studies also estimated Gini coefficient by different methods. For example, Hong (2008) developed a new decomposition method to estimate Gini coefficient of China. With increasing trend, Gini coefficient estimated by Hong (2008) is lower than either of the methods proposed by Dagum (1977a) except the year of 1990, which demonstrate that the Gini coefficient is probably underestimated by traditional calculation method.

Table 1 Method: Iterative I

Year	<i>Estimated parameters</i>				$\sum \epsilon^2$	No. of intervals	<i>Gini Ratio</i>
	α	β	λ	δ			
2010	0.045565***	1.446203***	508440452.6205***	2.604936***	0.049	20	0.38891
2009	0.011531*	0.933395***	589092908.5638**	2.57269***	0.296	20	0.40117
2005	0.019391**	1.108717***	507402433.8652**	2.562319***	0.652	20	0.39461
2004	0.011069***	1.194235***	332116907.2456**	2.568267***	0.431	20	0.38357
2002	0.01384***	1.21635***	241181256.4741*	2.595083***	0.230	20	0.38019
2000	0.007069***	1.082874***	243540044.8901**	2.603951***	0.064	20	0.38251
1999	0.004360***	1.222829***	278013552.4498**	2.643996***	0.053	20	0.36688
1998	0.005991***	1.273749***	331088981.7561***	2.683727***	0.041	20	0.35990
1995	0.007274***	1.072681***	562480884.9372***	2.859753***	0.024	20	0.34938
1990	0.005364***	1.228693***	4223896822.4205***	3.631742***	0.129	12	0.26690
1985	0.021245***	1.338987***	3526068765.6278***	3.980777***	0.026	12	0.25151

Note:***, ** and * represent at the significant level of 1%, 5% and 10%, respectively.

Table 2 Method: Maximum likelihood

Year	<i>Estimated parameters</i>				VAR	No. of intervals	<i>Gini Ratio</i>
	α	β	λ	δ			
2010	0.05418***	42.82797***	613891.1279***	2.2247***	0.00075***	20	0.40182
2009	-0.110558***	0.357861***	3214082008.5156***	2.5848***	0.003127***	20	0.45142
2005	0.005896**	1.105807***	101718795.0429***	2.3644***	0.000069***	20	0.41879
2004	-0.022947***	0.741795***	1077813740.4201***	2.6314***	0.000122***	20	0.39248
2002	-0.015134**	0.612957***	7588416842.9366***	2.9048***	0.000230***	20	0.38008
2000	0.002252*	1.107039***	88424121.0815***	2.4690***	0.000081***	20	0.39879
1999	0.014402***	2.177649***	11318825.7531***	2.3067***	0.000157***	20	0.39796
1998	0.005275*	1.265124***	214724501.5825***	2.6185***	0.000062***	20	0.36895
1995	0.004793**	1.127685***	145360478.6224***	2.6746***	0.000121***	20	0.36824
1990	-0.0128***	0.564200***	1.2334e+012***	4.3199***	0.000030***	12	0.26784
1985	0.009374*	1.277819***	1545345040.0909***	3.8216***	0.001768***	12	0.25467

Note:***, ** and * represent at the significant level of 1%, 5% and 10%, respectively.

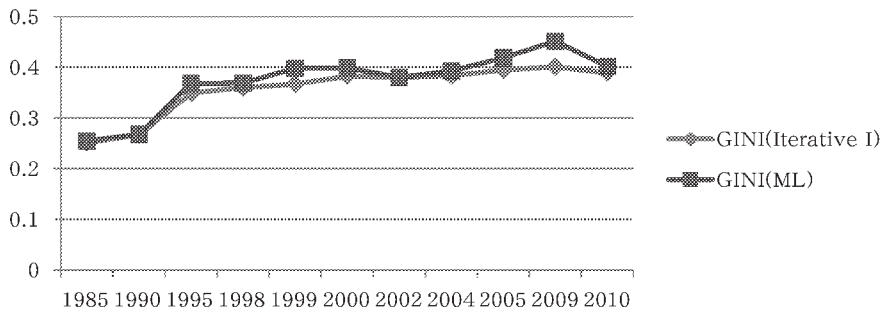


Figure 1 The comparison of Gini Coefficient between Iterative I and Maximum Likelihood (ML)

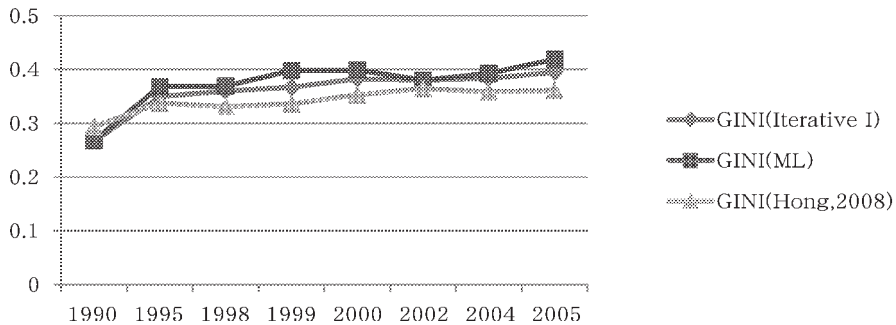


Figure 2 The comparison of Gini Coefficient between Dagum (1977a) and Hong (2008)

4.3 Lorenz Curve

Lorenz curve offer an intuitive path to identify inequality. We take Excel to draw Lorenz curve by original data. To investigate the trend of change, take the years of 1990, 2000, 1995, 2000, 2005 and 2010 for example.

At the beginning of drawing the graphs, we calculate cumulative proportion of income. It can be seen from Figure 3 to Figure 8 that the shape of the Lorenz Curves is changing through the time period. The situation of income inequality is worsening. In 1990, approximate 50% of the population own 20% of the income. High income class, about 1% of the population takes up 10% of the income which caused by the reform of open policy in 1978. Income inequality began to increase. In 2000, the condition of low income class improved, almost 34% of the population has 20% of the income. For high income class, about 10% of the population has 30% of the income.

The following year, income inequality increase gradually for the labor flows from rural areas into urban areas. In 2005, about 18% of the population has 21% of the income for the lower class. A series of policies prone to rural China and surplus labor force forcing farmers flowing back to rural areas, the situation of income inequality become well in 2009. In 2010, the situation is better for a series policies bias to develop rural areas. Overall, the income inequality in China is an increasing trend. For all time range, high income class owns the majority of the income. Income inequality with increasing trend in rural China is an issue of policy maker.

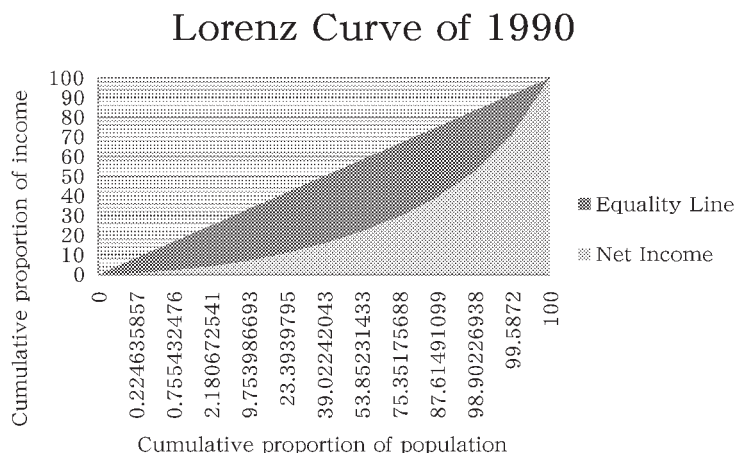


Figure 3 Lorenz curves in 1990

Lorenz Curve of 1995

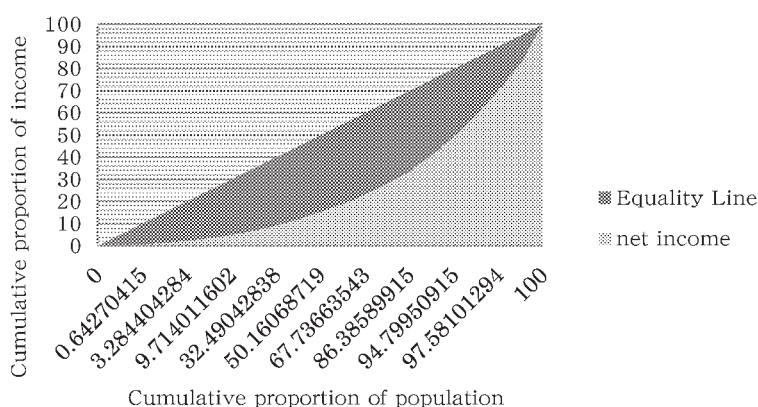


Figure 4 Lorenz curves in 1995

Lorenz Curve of 1995

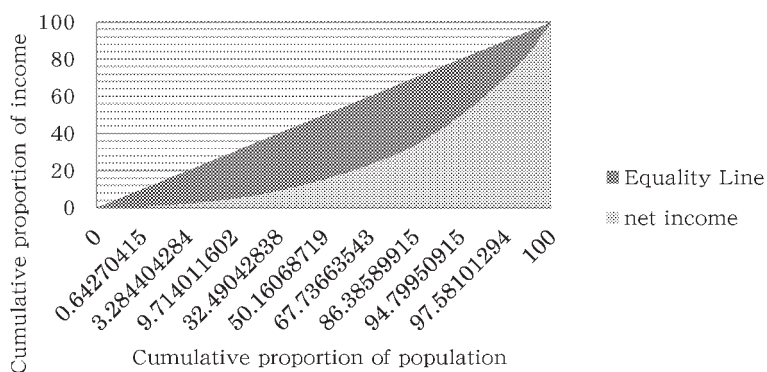


Figure 5 Lorenz curves in 1995

Lorenz Curve of 2000

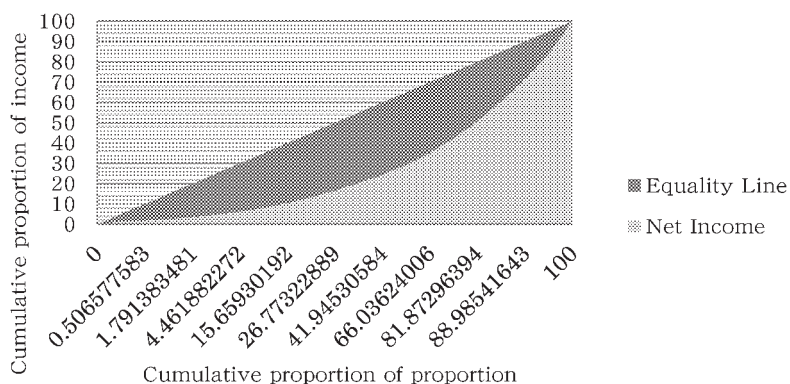


Figure 6 Lorenz curves in 2000

Lorenz Curve of 2005

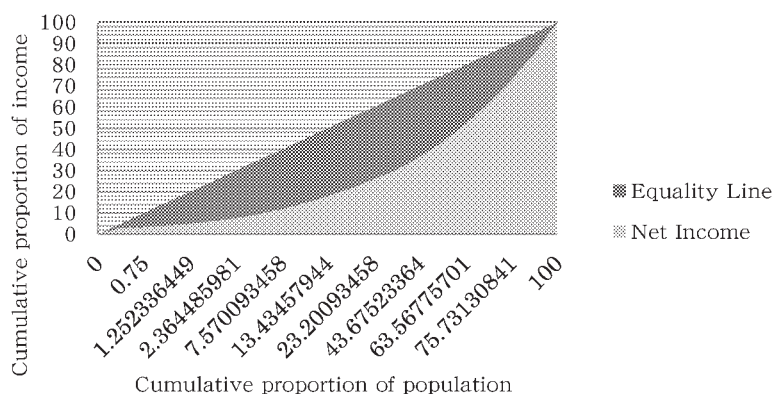


Figure 7 Lorenz curves in 2005

Lorenz Curve in 2010

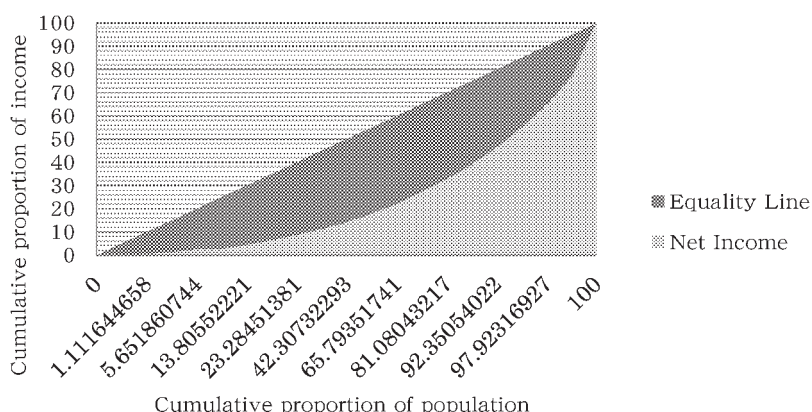


Figure 8 Lorenz curves in 2010

5 . Conclusion

This Chapter has provided a brief introduction to the development of models for income distribution and applies Dagum (1977a) distributions to household data in rural China for the time period of 1985 to 2010. Two methods including Iterative I and maximum likelihood are employed to estimate the parameters of Dagum model. I conclude that Dagum distribution fits household data well in the case of rural China. Maximum likelihood prefers to offer more reliable results for our study since it avoids the approximate problem in Iterative I method which will underestimate the inequality level. Moreover, comparing Dagum distribution with native study, we found that Gini coefficient calculated by Dagum distribution is higher than native estimation by traditional paths. That is, Gini coefficient might be underestimated by the native estimation. This study shows that Gini coefficient is mainly decided by the method applied. Various models offer many kinds of results. It is necessary to choose the suitable model to fit different types of data.

References

- Aichinson, J., and J.A.C. Brown (1969) "The Lognormal Distribution with Special Reference to Its Uses in Economics", Cambridge University Press.
- Aitchison, J. and C. Brown (1954) "On Criteria for Descriptions of Income Distribution", *Metroeconomica*, 6(3), pp. 88-107.
- Akita, T. (2003) "Decomposing Regional Income Inequality in China and Indonesia using two-stage nested Theil Decomposition Method", *AnnRegSci*, 37, pp.55-77.

- Ammon, O. (1895) “Die Gesellschaftsordnung und Ihre Naturlichen Grundlagen”, Jena.
- Amoroso, L. (1924-1925) “Ricerche Intorno alla Curva dei Redditi”, *Annali di Matematica, Pura ed Applicata*, Series 4-21, vol.II, 123-157.
- Arnold, B.C. (1983) “Pareto Distributions”, International Co-operative Publishing House. ISBN 0-899974-012-1.
- Bandourian, R., Madonald, J.B. and R.S. Turley (2002) “A Comparison of Parametric Models of Income Distribution across Countries and Over Time”, URL: <http://www.lisproject.org/publications/liswps/305.pdf>.
- Bartels, C.P.A. and H. van Metelen (1975) “Alternative Probability Density Functions of Income”, Vrije University Amsterdam: Research Memorandum, 29, pp.30 pages.
- Bordley, R.F., J.B. McDonald and A. Mantrala (1996) “Something New, Something Old: Parametric Models for the Size Distribution of Income”, *Journal of Income Distribution*, 6, pp.91-103.
- Brachmann, K., A. Stich and M. Trede (1996) “Evaluating Parametric Income Distributions Models”, *Allgemeines Statistisches Archiv*, 80, pp.285-298.
- Champernowne, D.G. (1952) “The Graduation of Income Distribution”, *Econometrica*, 20(4), pp.591-615.
- Cheng, Y.S. and S.K. Li. (2006) “Income Inequality and Efficiency: A Decomposition Approach and Applications to China”, *Economics Letters*, 91(1), pp.8-14.
- Clementi, F. and M. Gallegati, in: A. Chatterjee, S. Yarlagadda, B.K. Chakrabarti (Eds.) (2005) “Econophysics of Wealth Distributions”, Springer-Verlag, Italia, Milan, pp.3-14.
- Clementi, F., T. Di Matteo, M. Gallegati and G. Kaniadakis (2008) “The Kappa-generalized Distribution: A New Descriptive Model for the Size Distribution of Incomes”, *Physica-Statistical Mechanics and its Application*, 387, pp.3201-3208.
- Cummins, J.D., G. Diome, J.B. McDonald and B. Michael Pritchett (1990) “Applications of GB2 Family of Distribution in Modeling Insurance Loss Processes”, *Insurance: Mathematics and Economics*, 9, pp.257-272.
- Dagum, C. (1975) “A Model of Income Distribution and the Conditions of Existence of Moments of Finite Order”, *Bulletin of the International Statistical Institute*, 46(Proceedings of the 40th Session of the ISI, Warsaw, Contributed Papers), 1997205.
- Dagum, C. (1977a) “A New Model of Personal Income Distribution: Specification and Estimation”, *Economie Appliquée*, 30, pp.413-437.
- Dagum, C. (1977b) “A New Model of Income Distribution, the Lorenz Curve and the Gini Concentration Ratio”, forthcoming paper.
- Dagum, C. (1980) “Sistemas Generadores de Distribution del Ingresos y la Ley de Pareto”, *El Trimestre Economico*, 188, pp.877-917.
- Deng Q.H. and S. Li (2009) “What Lies behind Rising Earnings Inequality in Urban China? Regression-based Decompositions”, *Cesifo Economic Studies*, 55(3-4), pp.598-623.
- Fisk, P. R. (1961) “The Graduation of Income Distributions”, *Econometrica*, 29(2), pp.171-185.
- Gibrat, R. (1931) “Les Inegalites Economiques”, Sirely, Paris.
- Hogg, R.V. and S.A. Klugman (1983) “On the Estimation of Long Tailed Skewed Distributions with Actuarial Applications”, *Journal of Econometrics*, 23, pp.91-102.
- Hogg, R.V. and S.A. Klugman (1984) “Loss Distributions” (New York: Wiley).
- Hong X.J. (2008) “A New Subgroup Decomposition of the Gini Coefficient”, *The Quarterly Journal of Economics*, 8, pp.307-324.
- Kakwani, N.C. and N. Podder (1976) “Efficient Estimation of the Lorenz curve and Associated Inequality Measures from Grouped Observations”, *Econometrica*, 44(1), pp.137-149.
- Kalecki, M. (1945) “On the Gibrat distribution”, *Econometrica*, 13, pp.161-170.
- Kleiber, C. (1996) “Dagum vs. Singh-Maddala Income Distributions”, *Economics Letters*, 53, pp.265-268.
- Kleiber, C. and S. Kotz (2003) “Statistical Size Distributions in Economics and Actuarial Science”, John Wiley & Sons

- publication, Canada.
- Kloek, T. and H.K. van Dijk (1976) "Efficient Estimation of Income Distribution Parameters", Report 7616/E, Econometric Institute, Erasmus University, Rotterdam, 15 pages.
- Klugman, S. A., H.H. Panjer and G. Willmot (1998) "Loss Models: From Data to Decisions", Wiley, New York, ISBN 0-4712388-4-8.
- Lambert, P. and J. Aronson (1993) "Inequality Decomposition Analysis and the Gini Coefficient Revisited", *Economic Journal*, 103 (420), pp.1221-1227.
- Lomax, H. S. (1954) "Business Failures; Another Example of the Analysis of Failure Data", *JASA*, 49, pp.847-852.
- Mandelbrot, B.B. (1961) "Final Note on a Class of Skew Distribution Functions: Analysis and Critique of a Model Due to H.A. Simon", *Information and Control*, 4(2-3), pp.198-216.
- Mangahas, M. (1975) "Income Inequality in the Philippines: A Decomposition Analysis", World Employment Program Working Paper, No. 12, Geneva, ILO.
- McDonald, J.B. (1984) "Some Generalized Functions for the Size Distribution of Income", *Econometrica*, 52, pp.647-663.
- McDonald, J.B. and Xu, Y.J. (1995) "A Generalization of the Beta Distribution with Applications", *Journal of Econometrics*, 66, pp.133-152.
- Metcalfe, E.C (1972) "An Econometric Model of the Income Distribution", Markham Publishing Company, Chicago.
- Mielke, P.W., Jr., and E.S. Johnson, 1974: Some Generalized Beta Distributions of the Second Kind Having Desirable Application Features in Hydrology and Meteorology. *Water Resources Research*, 10, 223-226.
- El-Saidi, M.A., K.P. Singh and A. Bartolucci (1990) "On a Characterization of the Generalized Log-logistic Distribution", *Environmetrics*, 1, pp.337-342.
- Mookherjee, D. and A. Shorrocks, "A Decomposition Analysis of the Trend in U K Income Inequality", *Economic Journal*, 1982, 92(368), 886-902.
- Nirei, M. and W. Souma (2004) "Two Factor Model of Income Distribution Dynamics", Working Paper No. 04-10-029, Santa Fe Institute.
- Pareto, V. (1895) "La Legge della Domanda, Giornale degli Economisti", Genai: 59-68
- Pareto, V. (1897) "Cours d'Economie Politique", 2F, Rouge, Lausanne.
- Pareto, V. (1896) "Ecritsur la courbe de la répartition de la richesse", *Œuvres Complètes de Vilfredo Pareto* publiées sous la direction de Giovanni Busino, Genève, Librairie Droz, 1965.
- Rao, V. (1969) "Two Decompositions of Concentration Ratio", *Journal of the Royal Statistical Society, Series A*, 132, pp.418-425.
- Rozelle, S. (1994) "Rural Industrialization and Increasing Inequality: Emerging Patterns in China's Reforming Economy", *Journal of Comparative Economics*, 19, pp.362-91, 1994.
- Rutherford, R.S.G. (1955) "Income Distributions: A New Model", *Econometrica*, 23, pp.277-294.
- Salem, A.B. and T.D. Mount (1974) "A Convenient Descriptive. Model of Income Distribution: The Gamma Density", *Econometrica*, 42, pp.1115-1127.
- Sargan, J. D. (1957) "The distribution of wealth", *Econometrica*, 25, pp.568-590.
- Shorrocks, A. and G.H. Wan (2005) "Spatial Decomposition of Inequality" [J]. *Journal of Economic Geography*, Vol.5(1), pp.59-81.
- Silber, J. (1989) "Factor Components, Population Subgroups and the Computation of the Gini Index of Inequality", *Review of Economics and Statistics*, 71(1), 107-115.
- Singh, S.K. and G.S. Maddala (1976) "A Function for the Size Distribution of Incomes", *Econometrica*, 44, pp.963-970.
- Taille, C. (1981) "Lorenz Ordering within the Generalized Gamma Family of Income Distributions", in *Scientific Work*, Vol.6, ed. By C. Taille. G.P. Patil. and B. Baldessari. Boston: Reidel, 1981, pp.181-192

- Thurow, L.C. (1970) “Analyzing the American Income Distribution”, *The American Economic Review*, Papers and Proceedings, 60, pp.261-269.
- Vinci, F. (1921) “Nuovi contributi allo studio della distribuzione dei redditi”, *Giornale Econ. Rivista Stat.*, 61, pp. 365-369.
- Wagstaff, A. (2005) “An Inequality Decomposition and Geographic Targeting with Applications to China and Vietnam”, *Health Economics*, 14(6), pp.649-653.
- Wan, G.H. (2001) “Changes in Regional Inequality in Rural China: Decomposing the Gini Index by Income Sources”, *Australian Journal of Agricultural and Resource Economics* 43, pp.361-81.
- Wan, G.H. (2004) “Accounting for Income Inequality in Rural China: A Regression-based Approach”, *Journal of Comparative Economics*, 32, pp.48-63.
- Wan, G.H. and X.B. Zhang (2006) “Rising Inequality in China”, *Journal of Comparative Economics*, 34, pp.651-653.
- Wan, G.H. and Z.Y. Zhou (2005) “Income Inequality in Rural China: Regression-based Decomposition Using Household Data”, *Review of Development Economics*, 9(1), pp.107-120.
- Wold, H. and P. Whittle (1957) “A Model Explaining the Pareto Distribution of Wealth”, *Econometrica*, 25, pp.591-95.
- Yang, D.T. (1999) “Urban-biased Policies and Rising Income Inequality in China”, *American Economic Review*, 89(2), pp.306-310.
- Yao, S.J. (1997) “Decomposition of Gini Coefficients by Income Factors: A New Approach and Application”, *Applied Economics Letters*, 4(1), pp.27-31.
- Yao, S.J. (1997) “Industrialization and Spatial Income Inequality in Rural China, 1986-92”, *Economics of Transition*, 5(1), pp.97-112.
- Yao, S.J. (1999) “On the Decomposition of Gini Coefficients by Population Class and Income Source: a Spreadsheet Approach and Application”, *Applied Economics*, 31(10), pp.1249-1264.
- Yao, S.J., Z.Y. Zhang and L. Hanmer (2004) “Growing Inequality and Poverty in China”, *China Economic Review*, 15(2), pp.145-163.
- Yitzhaki, S. and R. Lerman (1991) “Income Stratification and Income Inequality”, *Review of Income and Wealth*, 37(3), 313-329.
- Zhang, X. and K. Zhang (2003) “How does Globalization Affect Regional Inequality within a Developing Country? Evidence from China”, *Journal of Development Studies*, 39, pp.47-67.