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Hydrograph Recession Analysis using Wavelet Transforms

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A technique for determining hydrograph recession characteristics is presented based on the wavelet transforms analysis. Like a signal, a flood hydrograph can be decomposed into a number of wavelet components. By using the wavelet component values in the form of wavelet maps, the recession characteristics such as a recession constant and a cut-off frequency are easily determined. The technique is demonstrated on three catchments ranging from small to large catchment areas at eastern New South Wales, Australia. The proposed method gives promising results and minimizes a number of problems associated with hydrograph recession analysis.

INTRODUCTION

Hydrograph recession analysis has long been a topic of research in hydrology. The recession curve contains valuable information about natural storage properties and aquifer characteristics. Recession analysis has been applied in many areas including low flow forecasting to benefit the management of irrigation, water supply allocation and waste dilution scheme (Tallaksen, 1995). Hydrograph recession characteristics are required for hydrological modeling such as a unit hydrograph method.

A number of techniques has been developed and used for analyzing the hydrograph recession curve. Semi–logarithmic plot of the recession curve (discharge on the logarithmic scale) is useful for investigating the components of hydrograph recession and for determining the end of surface runoff (Pilgrim and Cordery, 1993). On the semi–logarithmic plots, the recession curve can be approximated by three straight lines (Linsley et al., 1958). The end of direct flow (surface flow and interflow) is generally assumed to be the point of intersection of the two lowest lines, since the different slopes represent different flow components. Often, however, the transition from one line to the next line is so gradual, hence it is difficult to select the points of change in slope as well as the slope itself as shown in Fig. 1a.

Another method for analyzing the recession curve can also be carried out by plotting the recession parts on semi–logarithmic plots. Instead of plotting directly discharge data on the logarithmic scale, discharge ratio values (Q_0/Q) are used as given in Fig. 1b. Q_0 is

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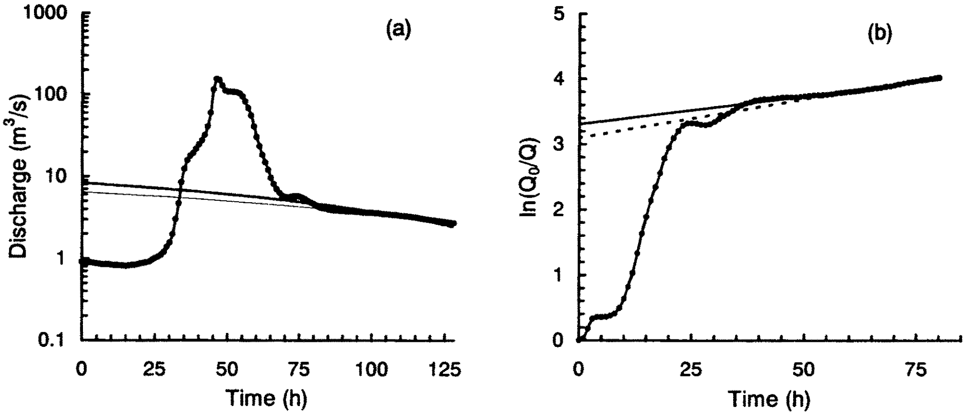

Fig. 1. Graphical methods for estimating (a) recession constant K and (b) cut-off frequency f_c

initial flow and Q is flow at t time after Q_0. Hino and Hasebe (1984) used this approach to determine the cut-off frequency of the component separation of hourly hydrologic data by finding the slope of the recession curve. As a graphical method, the method has also high level of subjectivity, as the slope is not easily defined.

In this paper, a new technique for determining hydrograph recession characteristics is presented based on the wavelet transform analysis. By analyzing the wavelet maps, the recession characteristics, hopefully, can be determined more accurate and less subjective.

METHODS AND DATA USED

Recession Curve

The hydrograph recession curve represents withdrawal of water from storage within the catchment. The shape of the recession curve can be expressed by (e.g. Hino and Hasebe, 1984; Nathan and McMahon, 1990):

$$Q_t = Q_0 \exp(-\alpha t) \quad \text{or} \quad Q_t = Q_0 \exp(-\frac{t}{T_c}), \tag{1}$$

where Q_t is the discharge at time t, Q_0 is the initial discharge, α is a constant or it is also called a cut-off frequency (f_c) and T_c is a recession period; the term $\exp(-\alpha)$ may be replaced by K which is called a recession constant. The above equation is then written in the following form:

$$Q_t = Q_0 K^t. \tag{2}$$

The relationship between the cut-off frequency (f_c) and the recession constant (K) can be written in the following form:

$$K = e^{-f_c} \quad \text{or} \quad f_c = -\ln(K). \tag{3}$$

Typical value of hourly recession constant of groundwater flow component for catchments ranging from 300 to 16,000 km^2 in eastern Australia and United States is 0.998 (f_c=0.002) (Pilgrim and Cordery, 1993).

Wavelet Transforms

Any arbitrary signal $f(x)$ can be decomposed into an infinite summation of wavelets at different levels according to the wavelet expansion as the following (Newland, 1993):

$$f(x)=\sum_{j=-\infty}^{\infty}\sum_{k=-\infty}^{\infty}c_{j,k}\,W(2^jx-k), \tag{4}$$

where $c_{j,k}$ are the wavelet coefficients, $W(2^jx-k)$ is wavelet function and k is integer. The integer j describes different level of wavelets. Those coefficients of a number wavelet functions such as Daubechies, Symlet can be found in some textbooks (e.g. Sakakibara, 1995; Vidakovic, 1999).

By assuming $f(x)$ is zero outside the interval $0\leq x<1$, scaling function and wavelet function are wrapped around the interval, the wavelet expansion of $f(x)$ in $0\leq x<1$ can be written as:

$$f(x)=a_0+\sum_j\sum_k a_{2^j+k}\,W(2^jx-k), \tag{5}$$

where a_0 is the wrapped scaling function (the lowest wavelet) that is equal to the average value of signal $f(x)$, $W(2^jx-k)$ are wrapped wavelet functions. The integer j describes different levels of wavelets, starting with j=0; the integer k covers the number of wavelets in each level, so that it covers the range k=0 to 2^j-1.

The mean–square value of $f(x)$ can be calculated by squaring both sides of the Eq. (5) and integrating over the interval $0\leq x<1$. The final result can be expressed in the following form (Newland, 1993):

$$\int_0^1 f^2(x)dx=a_0^2+\sum_j\sum_k a_{2^j+k}^2\left(\frac{1}{2^j}\right). \tag{6}$$

Writing out the first few terms of the double summation, the mean square of $f(x)$ can be written as:

$$\int_0^1 f^2(x)dx=a_0^2+a_1^2+\frac{1}{2}(a_2^2+a_3^2)+\frac{1}{4}(a_4^2+a_5^2+a_6^2+a_7^2)+\frac{1}{8}(a_8^2+a_9^2+\cdots+a_{15}^2)+\cdots. \tag{7}$$

Eq.7 shows that the mean square of $f(x)$ is distributed between different wavelet levels and between different wavelets within each level. Also, Eq. 7 can be illustrated graphically by a three–dimensional plot called mean–square wavelet map. Instead of using wavelet levels in drawing the map, the use of their frequency may be more appropriate for analyzing the recession curve behavior. The centered frequency in each level may be computed by using the following expression:

$$f=\frac{2^jf_s}{N}, \tag{8}$$

where f is centered frequency, integer j is wavelet level, $f_s=1/\Delta t$, Δt is unit time interval, and N is length of the data.

Data Used

The proposed technique for determining hydrograph recession characteristics is applied to three catchments at eastern New South Wales (NSW), Australia. The catchment areas range from 39 km^2 to 2,670 km^2. Details of the catchment characteristics such

Table 1. Catchment characteristics used in the study

Gauging Station		Area	MAR	MAF
Number	Name	(km^2)	(mm)	(m^3/s)
203012	Byron Creek at Binna Burra	39	1830	165
204015	Boyd River at Broadmeadows	2670	860	555
416008	Beardy River at Haystack	866	840	236

as catchment area, mean annual rainfall (MAR), mean annual flood (MAF) are given in Table 1. For each catchment, three floods event of hourly interval are selected for the analysis. The data were extracted from Pinneena version 6.1 published by the Department of Land and Water Conservation, NSW (1999).

RESULTS AND DISCUSSION

By using the wavelet component values in the form of wavelet maps, the recession characteristics were analyzed. Fig. 2 to Fig. 4 show the mean–square value maps together with their original signals (flood event hydrographs) for three catchments studied. The figures provide valuable information of the recession curve behavior. Change in the recession line affect to the mean square values of the signal characterized by change of the wavelet maps. As the baseflow component has a recession time much longer than other components (surface flow and interflow), thus the baseflow component has the lowest frequency compared with the other components.

All the wavelet maps show that the lowest frequencies can easily be distinguished from the others. As a result, the cut–off frequency or the recession constant can be determined more objective than the existing methods (semi–logarithmic plot methods). Moreover, the end of the direct flow T_b' may be determined by looking at the time scale where the mean–square values reach maximum.

Table 2. shows the cut–off frequency values for all data estimated both using wavelet transforms and semi–logarithmic plot method. The table gives clear indication that the semi–logarithmic plot or the graphical method has high level of subjectivity in determining the recession characteristics, whereas the wavelet transforms produces exact values. Since the accuracy of the cut–off frequency computed using the wavelet transforms depends on the length N of the data used (see Eq. 8), more accurate results may be obtained by using the longer data. In case of flood events analysis, smaller time interval may be required.

CONCLUSION

A new technique for determining the hydrograph recession characteristics was successfully applied to three catchments at eastern New South Wales, Australia. The recession constant, the cut off frequency and the end of surface runoff of a flood hydrograph are easily determined using the proposed technique. Further application of the technique to other catchments and study of their wavelet maps behavior are needed to give more definite results of the hydrograph recession characteristics.

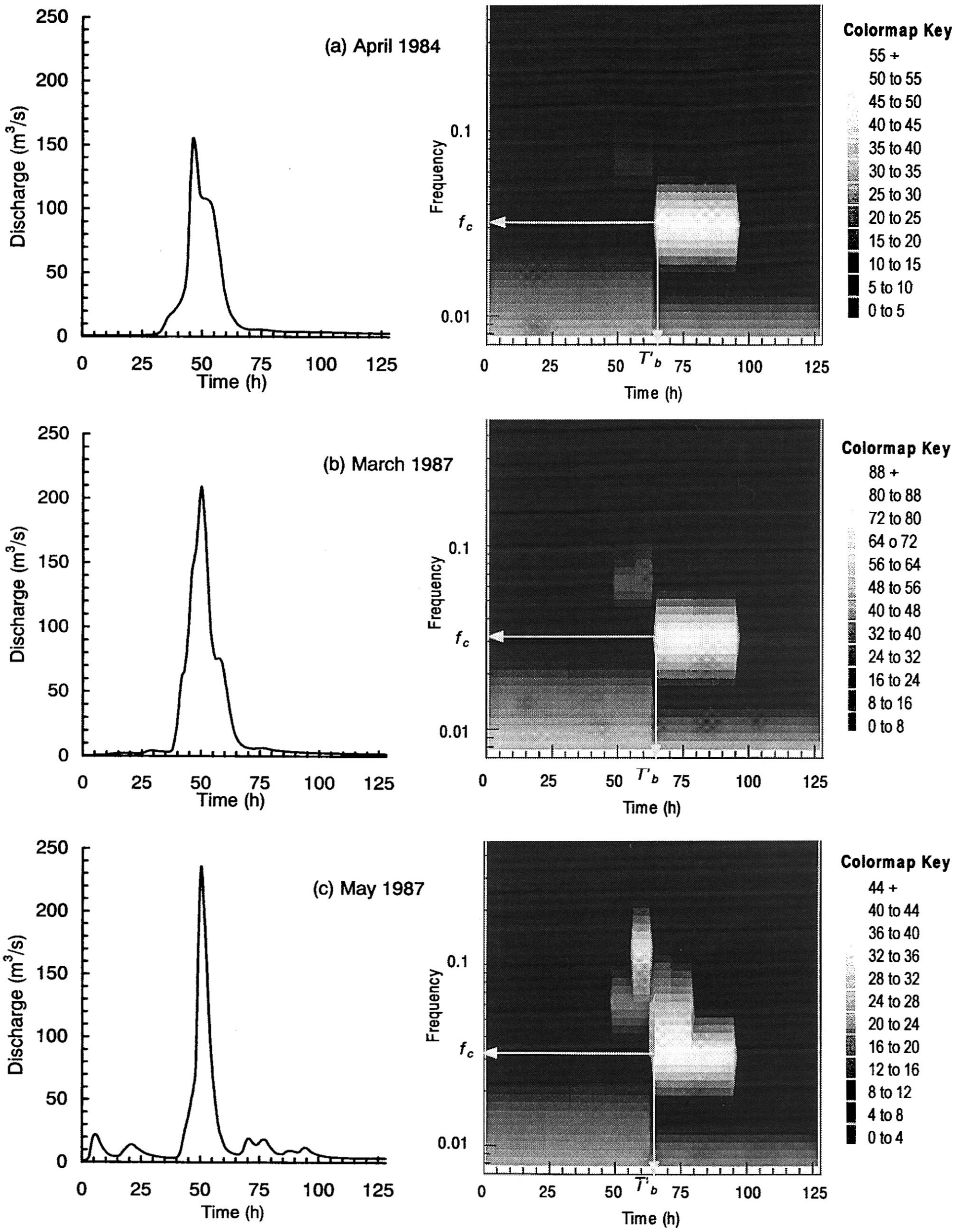

Fig. 2. Flood events and their wavelet maps of station 203012 (A=39 km^2) (a) event April 1984, (b) event March 1987 and (c) event May 1987

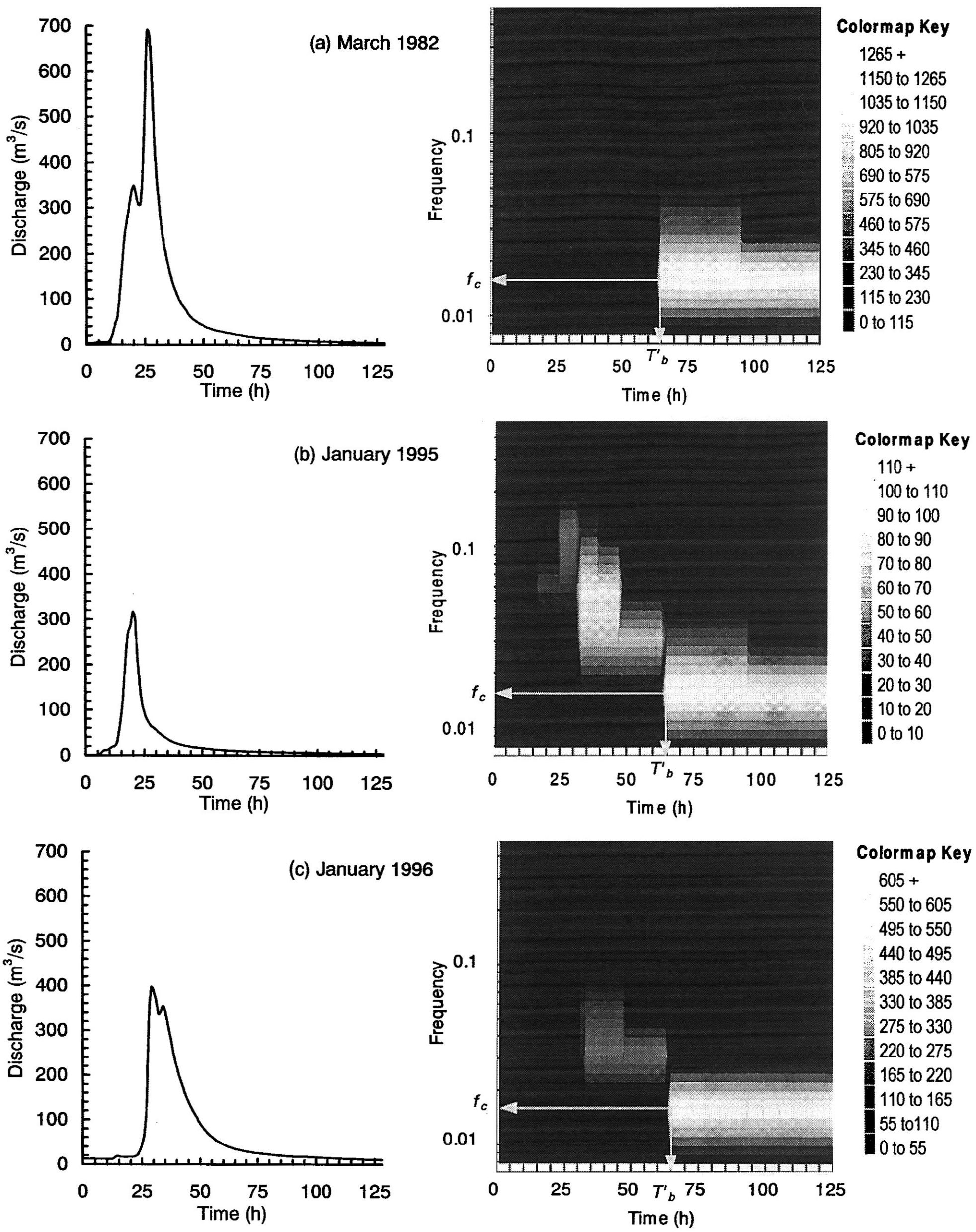

Fig. 3. Flood events and their wavelet maps of station 416008 (A=866 km²) (a) event March 1982, (b) event January 1995 and (c) event January 1996

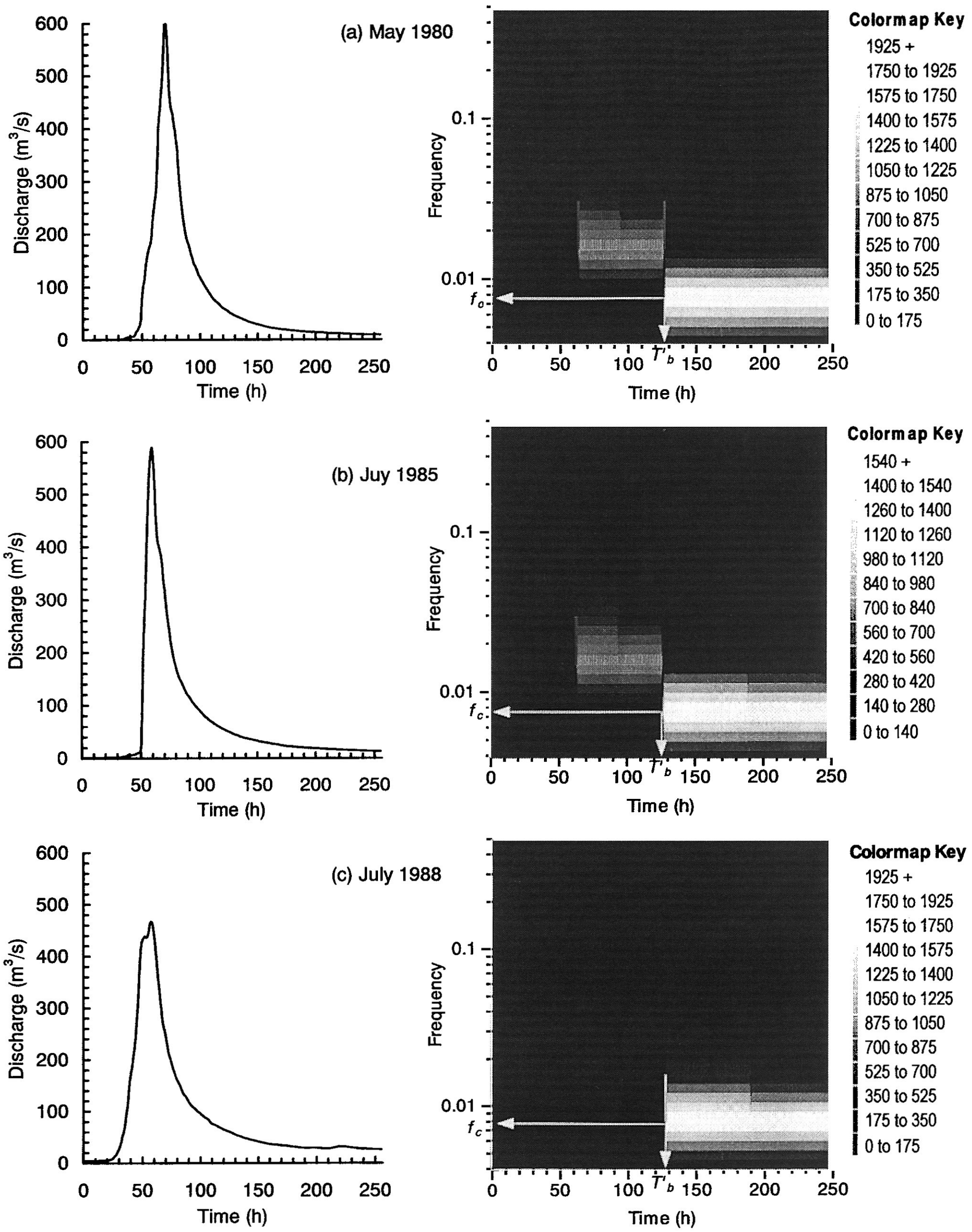

Fig. 4. Flood events and their wavelet maps of station 204015 (A=2670 km²) (a) event May 1980 (b) event July 1985, and (c) event July 1988

Table 2. Cut–off frequency f_c using different methods

Station	Flood event	Graphical method	Proposed method
203012	March 1984	0.0086 – 0.0120	0.0313
	March 1987	0.0112 – 0.0311	0.0313
	May 1987	0.0315 – 0.1042	0.0313
204015	May 1980	0.0094 – 0.0100	0.0078
	July 1985	0.0072 – 0.0089	0.0078
	July 1988	0.0023 – 0.0146	0.0078
416008	March 1982	0.0262 – 0.0310	0.0156
	January 1995	0.0161 – 0.0293	0.0156
	January 1996	0.0150 – 0.0184	0.0156

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