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Shimojo, Masataka
Laboratory of Animal Feed Science, Faculty of Agriculture, Kyushu University

Bungo, Takashi
Laboratory of Animal Feed Science, Faculty of Agriculture, Kyushu University

Imura, Yoshimi
Laboratory of Animal Feed Science, Faculty of Agriculture, Kyushu University

Tobisa, Manabu
Laboratory of Animal Feed Science, Faculty of Agriculture, Kyushu University

他

https://doi.org/10.5109/24260

出版情報：九州大学大学院農学研究院紀要. 43 (1/2), pp.137-142, 1998-11. Kyushu University

J. Fac. Agr., Kyushu Univ., **43** (1·2), 137–142 (1998)

Use of Complex Number in the Analysis of Increase in Dry Matter Indigestibility with Growth of Forages

Masataka Shimojo, Takashi Bungo, Yoshimi Imura, Manabu Tobisa, Naoki Koga, Shao Tao, Muhammad Yunus, Yutaka Nakano*, Ichiro Goto, Mitsuhiro Furuse and Yasuhisa Masuda

Laboratory of Animal Feed Science, Faculty of Agriculture,
Kyushu University, Fukuoka 812–8581, Japan
(Received July 31, 1998 and accepted August 7, 1998)

This study was conducted to give a trigonometric representation, using complex numbers and plane, of the increase in dry matter indigestibility [IDMI] with growth of forages. The following results were obtained. Divisional calculations between two coordinates of dry matter indigestibility [DMI] expressed using complex numbers were related to the subtraction theorem of tangent. This relation might give some information on DMI changes, namely IDMI, the geometric mean of DMI [GMDMI] and relative IDMI (=IDMI/GMDMI). Changes in dry weight and indigestible dry weight with forage growth were visualized by the geometric representation. Therefore, there were four information on IDMI given by the trigonometric representation of DMI on the complex plane. It was suggested that the complex representation of DMI of forages derived, from the subtraction theorem of tangent, some information on IDMI with forage growth.

INTRODUCTION

The data on the nutritive value of forages are normally described using real numbers. There are probably no or few cases where imaginary numbers have been used in this field. It seems to be strange, therefore, that the present study will deal with the complex representation of indigestibility increases with growth of forages. The present motive of the use of complex numbers comes from an attempt at the geometric representation of forage indigestibility using trigonometric functions. It is suggested that trigonometric functions, when combined with complex numbers, give some useful information, for example, the description of electric power on the electric circuit.

In our previous two brief reports, some aspects on the digestibility decrease with forage growth were shown by its trigonometric representation on the complex plane (Shimojo *et al.*, 1998c) and on the orthogonal coordinates (Shimojo *et al.*, 1998e). The analysis of indigestibility increases seems to be more attractive than that of digestibility decreases (Masuda, 1985), because the former describes the forage maturation process using the increase in anti–digestion components (Shimojo *et al.*, 1995, 1997a, b, 1998a, b, d, f).

The present study was designed to suggest a trigonometric representation, using complex numbers and plane, of the increase in dry matter indigestibility with growth of forages.

* Kyushu University Farm, Fukuoka 811–2307.

DRY MATTER INDIGESTIBILITY INCREASES EXPRESSED USING COMPLEX NUMBERS

The increase in dry matter indigestibility with growth of forages [IDMI] expressed using real numbers is as follows (Masuda, 1985; Shimojo *et al.*, 1995):

$$\mathrm{IDMI} = \frac{I_2}{W_2} - \frac{I_1}{W_1}, \qquad (1)$$

where W=forage dry weight ($W_1 < W_2$), I=dry weight of indigestible materials in forage ($I_1 < I_2$), I_k/W_k=dry matter indigestibility [DMI] at time t_k.

Then, the following two coordinates are taken up and plotted on the complex plane (Fig. 1),

$$Z_1 = W_1 + jI_1, \qquad Z_2 = W_2 + jI_2, \qquad (2)$$

where j=the imaginary unit, $I_1/W_1=\tan\theta_1$ [DMI at t_1], $I_2/W_2=\tan\theta_2$ [DMI at t_2], another angle ($\theta_2-\theta_1$) is also shown, $0< \theta_1< \theta_2<\pi/4$, $0<\tan\theta_1<\tan\theta_2<1$.

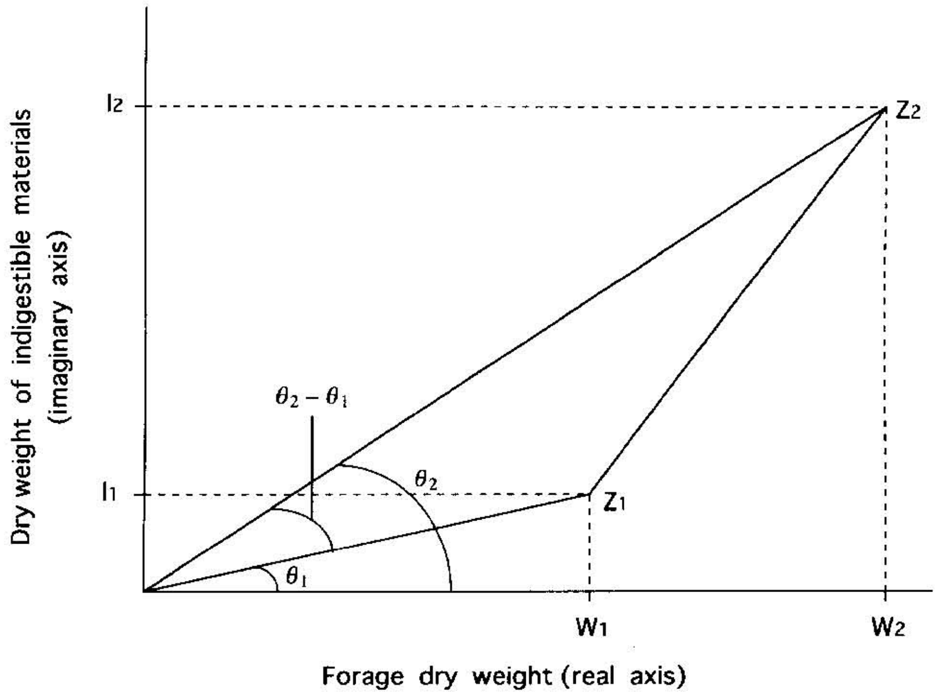

Fig. 1. Plotting of forage dry weight [W] and dry weight of indigestible materials [I] on the complex plane (θ shows the angle).

Thus, equation (1) is rewritten as follows:

$$\mathrm{IDMI} = \tan\theta_2 - \tan\theta_1 \,. \qquad (3)$$

The two coordinates (equation (2)) rewritten using trigonometric functions are as follows:

$$Z_1 = \sqrt{W_1^{\,2} + I_1^{\,2}} \cdot (\cos\theta_1 + j\sin\theta_1), \qquad Z_2 = \sqrt{W_2^{\,2} + I_2^{\,2}} \cdot (\cos\theta_2 + j\sin\theta_2). \qquad (4)$$

How equation (3) will be related to equation (4) is the present subject. For this purpose we take up the following equation that is known as the subtraction theorem of tangent,

$$\tan(\theta_2 - \theta_1) = \frac{\tan\theta_2 - \tan\theta_1}{1 + \tan\theta_2 \cdot \tan\theta_1}$$

$$= \frac{\tan\theta_2 - \tan\theta_1}{1 + \left(\sqrt{\tan\theta_2 \cdot \tan\theta_1}\right)^2} \,. \qquad (5)$$

Equation (5) shows that the numerator of the right–hand side gives IDMI. It is also shown that the denominator includes the geometric mean of two DMIs at t_1 and t_2 [GMDMI], namely $(\tan\theta_2 \cdot \tan\theta_1)^{0.5}$. GMDMI is also described using $\tan\theta_2$, $\tan\theta_1$, and $\tan(\theta_2 - \theta_1)$ as follows:

$$\mathrm{GMDMI} = \sqrt{\tan\theta_2 \cdot \tan\theta_1} = \sqrt{\frac{\tan\theta_2 - \tan\theta_1 - \tan(\theta_2 - \theta_1)}{\tan(\theta_2 - \theta_1)}} \,. \qquad (6)$$

GMDMI seems to show, as it were, the indigestion level associated with the two DMIs. Then, the IDMI related to its indigestion level [relative IDMI] might be estimated as follows:

$$\text{Relative IDMI} = \frac{\tan\theta_2 - \tan\theta_1}{\sqrt{\tan\theta_2 \cdot \tan\theta_1}} \,. \qquad (7)$$

Thus, some information on DMI changes might be derived from the right–hand side of the subtraction theorem of tangent (equation (5)), namely IDMI, GMDMI and relative IDMI.

This leads us to the investigation into how equation (4) will be related to equation (5). The two coordinates, Z_1 and Z_2, will be related with the subtraction theorem of tangent according to the following calculation:

$$\frac{Z_2}{Z_1}=\frac{\sqrt{W_2^{\,2}+I_2^{\,2}}\cdot\left(\cos\theta_2+j\sin\theta_2\right)}{\sqrt{W_1^{\,2}+I_1^{\,2}}\cdot\left(\cos\theta_1+j\sin\theta_1\right)}$$

$$=\frac{\sqrt{W_2^{\,2}+I_2^{\,2}}}{\sqrt{W_1^{\,2}+I_1^{\,2}}}\cdot\left\{\cos\left(\theta_2-\theta_1\right)+j\sin\left(\theta_2-\theta_1\right)\right\}$$

$$=\frac{\left(\sqrt{W_2^{\,2}+I_2^{\,2}}\cdot\cos\theta_2\right)\cdot\left(\sqrt{W_1^{2}+I_1^{2}}\cdot\cos\theta_1\right)}{\sqrt{W_1^{\,2}+I_1^{\,2}}\cdot\sqrt{W_1^{2}+I_1^{2}}}\cdot\left\{\left(1+\tan\theta_2\cdot\tan\theta_1\right)+j\left(\tan\theta_2-\tan\theta_1\right)\right\}$$

$$=\frac{W_2\cdot W_1}{W_1^{\,2}+I_1^{\,2}}\cdot\left\{\left(1+\tan\theta_2\cdot\tan\theta_1\right)+j\left(\tan\theta_2-\tan\theta_1\right)\right\},$$

therefore,

$$\frac{W_1^{\,2}+I_1^{\,2}}{W_2\cdot W_1}\cdot\frac{Z_2}{Z_1}=\left(1+\tan\theta_2\cdot\tan\theta_1\right)+j\left(\tan\theta_2-\tan\theta_1\right). \qquad (8)$$

In the right–hand side of equation (8), the real part [Re(Z_2/Z_1)], 1 + $\tan\theta_2\cdot\tan\theta_1$, gives the denominator of the right–hand side of equation (5), and the imaginary part [Im (Z_2/Z_1)], $\tan\theta_2-\tan\theta_1$, gives the numerator of the right–hand side of equation (5).

Z_2/Z_1 is also related with the left–hand side of equation (5) as follows:

$$\frac{Z_2}{Z_1}=\frac{\sqrt{W_1^{\,2}+I_1^{\,2}}}{\sqrt{W_2^{\,2}+I_2^{\,2}}}\cdot\left\{\cos\left(\theta_2-\theta_1\right)+j\sin\left(\theta_2-\theta_1\right)\right\}$$

$$=\frac{\sqrt{W_2^{\,2}+I_2^{\,2}}\cdot\sqrt{W_1^{\,2}+I_1^{\,2}}\cdot\cos\left(\theta_2-\theta_1\right)}{\sqrt{W_1^{\,2}+I_1^{\,2}}\cdot\sqrt{W_1^{\,2}+I_1^{\,2}}}\cdot\left\{1+j\tan\left(\theta_2-\theta_1\right)\right\}$$

$$=\frac{W_2\cdot W_1+I_2\cdot I_1}{W_1^{\,2}+I_1^{\,2}}\cdot\left\{1+j\tan\left(\theta_2-\theta_1\right)\right\},$$

therefore,

$$\frac{W_1^{\,2}+I_1^{\,2}}{W_2\cdot W_1+I_2\cdot I_1}\cdot\frac{Z_2}{Z_1}=1+j\tan\left(\theta_2-\theta_1\right). \qquad (9)$$

In the right–hand side of equation (9), Re(Z_2/Z_1), 1, gives the denominator of the left–hand side of equation (5), and Im(Z_2/Z_1), $\tan(\theta_2-\theta_1)$, gives the numerator of the left–hand side of equation (5).

Therefore, equations (8) and (9) suggest a sort of relationship between Z_2/Z_1 and the subtraction theorem of tangent (equation (5)), when some modifications are made on the calculation of Z_2/Z_1. This relationship might give some information on dry matter indigestibility changes with growth of forages, namely IDMI, GMDMI and relative IDMI. This seems to be due probably to the characteristics of complex representation combining things which are different and do not seem to be related. Another benefit obtained from the geometric representation of DMI seems to be the visualization of changes in dry weight and those in indigestible dry weight with growth of forages (Fig. 1). Thus, there are four information on IDMI derived from the trigonometric representation of DMI on the complex plane. The present results are comparable to those for the digestibility decrease with forage growth (Shimojo *et al.*, 1998c).

There will be a strong objection to the complication, using complex representation, of simple subjects such as changes in dry matter digestibility and indigestibility with growth of forages. It seems to be, however, of interest to make analytic approaches to this sort of subjects using various methods we can try. This will lead to an understanding of interrelationships between some aspects of forage utilization.

Conclusions

It is suggested from this study that the complex representation of DMI of forages derives, from the subtraction theorem of tangent, some information on IDMI occurring with forage growth.

ACKNOWLEDGEMENTS

The present work is based on our previous works on forage indigestibility which were supported by skilled technical assistance by Mr. Yasukatsu Yano.

REFERENCES

Masuda, Y. 1985 Analysis of dry matter digestibility of green panic as affected by the change in temperature. *Proc. XV Int. Grassl. Congr.*, 1009–1011

Shimojo, M., Y. Masuda, T. Bungo, T. Kawamura and I. Goto 1995 Analytical expression for formation of indigestible materials and increase in dry matter indigestibility with growth of some tropical grasses. *J. Fac. Agr., Kyushu Univ.*, **40**: 179–188

Shimojo, M., T. Bungo, Y. Imura, M. Tobisa, N. Koga, Y. Nakano, I. Goto and Y. Masuda 1997a An analysis of digestibility change with growth of forage. *In* "Present and future of rumen research", *Proc. First Joint Symp. Japan and Korea on Rumen Metab. Physiol.*, P–11

Shimojo, M., Y. Imura, M. Tobisa, N. Koga, T. Bungo, Y. Nakano, T. Nishihira, I. Goto and Y. Masuda 1997b Formation of indigestible materials and increase in dry matter indigestibility in the growth of two tropical forages. *J. Fac. Agr., Kyushu Univ.*, **42**: 95–99

Shimojo, M., Y. Imura, T. Bungo, M. Tobisa, N. Koga, S. Tao, M. Yunus, Z. Yin, Y. Nakano, I. Goto and Y. Masuda 1998a Formation of indigestible materials from digestible materials and photosynthates in the growth of Rhodes grass (*Chloris gayana* Kunth). *J. Fac. Agr., Kyushu Univ.*, **42**: 355–360

Shimojo, M., T. Bungo, M. Tobisa, Y. Imura, N. Koga, S. Tao, M. Yunus, Y. Nakano, I. Goto, M. Furuse and Y. Masuda. 1998b Analyses of formation of indigestible materials and increase in dry matter indigestibility with growth of two tropical forages. *Proc. 8th World Conf. Anim. Prod.*, Seoul, Korea, **Vol. II**: 508–509

Shimojo, M., T. Bungo, Y. Imura, M. Tobisa, N. Koga, S. Tao, M. Yunus, Y. Nakano, I. Goto, M. Furuse and Y. Masuda. 1998c Digestibility decrease with forage growth as interpreted using complex plane. *Proc. 8th World Conf. Anim. Prod.*, Seoul, Korea, **Vol. II**: 514–515

Shimojo, M., T. Bungo, Y. Imura, M. Tobisa, N. Koga, S. Tao, M. Yunus, Y. Nakano, I. Goto, M. Furuse and Y. Masuda. 1998d Some aspects on the analysis of the formation of indigestible materials with growth of forages. *Proc. 8th World Conf. Anim. Prod.*, Seoul, Korea, **Vol. II**: 518–519

Shimojo, M., T. Bungo, Y. Imura, M. Tobisa, N. Koga, S. Tao, M. Yunus, Y. Nakano, I. Goto, M. Furuse and Y. Masuda. 1998e Digestibility decrease with forage growth as interpreted using orthogonal coordinates. *Proc. 8th World Conf. Anim. Prod.*, Seoul, Korea, **Vol. II**: 520–521

Shimojo, M., T. Bungo, Y. Imura, M. Tobisa, N. Koga, S. Tao, M. Yunus, Y. Nakano, I. Goto, M. Furuse and Y. Masuda 1998f Accumulation rate of digestible materials and formation rate of indigestible materials in the description of relative growth rate of forages. *J. Fac. Agr., Kyushu Univ.*, in press