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The Analysis and Verification of Tree Volume Equations of *Pinus elliottii* in the State Forest of Águas de Santa Bárbara, São Paulo†

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The state forests in São Paulo are mostly covered by eucalyptus and pine stands. Pine was introduced into Brazil and planted on a large scale, during the period from 1958 until the beginning of 1965. The cutting cycle for these pine stands was planned to be 25 or 30 years, so that the cutting time will come soon. However, the volume and yield tables which are the necessary tools for the cutting planning, has not been prepared yet. The local volume table and the volume table made on the basis of variety has never been prepared at all. In relation to this, the analysis *on* tree volume equation, which is an aspect of the study on *volume* table compilation method, has been conducted for the pine stands of state forest in Águas de Santa Barbara. São Paulo state is located in subtropical region, where the season is distinguished into rainy and dry seasons. In connection to this condition, the bark of pine trees is extremely thick. Therefore, the volume equations are classified into the equations for volume with bark and for volume without bark. Further, these are classified into the equations for total volume, for commercial volume and for real commercial volume. The commercial volume consists of the timber until the smallest diameter of 5 cm, while the real commercial volume is timber volume where the bed of truck of 2.4 m is taken into consideration (the timber until the length as smallest as 1.2 m or half of the truck bed is counted). The analysis on these mentioned 6 different counted-based volumes was conducted and the corresponding 6 different volume equations were obtained.

INTRODUCTION

São Paulo state is covering area of 247,898 km² in which 5,035,070 ha are the forest area (in 1973). This forest area corresponds to 20.3 % of total area. The artificial forests cover 641,420 ha, which correspond to 12.7 % of forest area or 2.6 % of total area. Firther, the artificial forests consist of eucalyptus stands 490,560 ha, pine plantations 142,070 ha and the residual of 8,790 ha is covered by other species. These are corresponding to 76.5 %, 22.1 % and 1.4 % of total artificial forests, respectively. The eucalyptus stand is overwhelming majority in area. However, in the recent period,

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pine plantation is being gradually increased.

In São Paulo state, from 1958 to 1965, tropical or subtropical pine trees of North or Middle America were introduced to encourage artificial plantation. Since the history is so short as about 20 years or so, tree volume tables have not been made for each district. Hence, the study of the preparation of tree volume table, taking up *Pinus elliottii* in state forest of Águas de Santa Barbara which is one of state forests, as the subject. At the same time some examination was attempted on this topic.

MATERIALS AND METHODS

1. Collection of materials

The whole area of the state forest of Águas de Santa Barbara were regarded as the subject for the area of sample trees, which would be basic material for tree volume table. Their diameter (*DAP*) and height (H) were intentionally selected so that they could be almost equally taken out from maximum to minimum. As the result, 100 sample trees of *Pinus elliottii* in total were collected as in Table 1.

2. Calculation of volume of sample trees

Six cases of the volumes are used in São Paulo state, i. e. A : the whole tree volume, B : the commercial volume of end diameter without bark of up 5 cm, C : the real commercial volume of end diameter without bark of up to 5 cm which was lumbered into the length of 2.4 m, equal to the truck bed length (there are also some cases marking use of as short as the half length, 1.2 m), and with bark and without bark for each of above 3 cases, all of which make 6 cases. These 6 volumes as below are being used for each purpose.

① Total tree volume with bark
② Total tree volume without bark
③ Commercial volume with bark
④ Commercial volume without bark
⑤ Real commercial volume with bark
⑥ Real commercial volume without bark

Therefore, cutting of sample trees, measurement in case of clean cutting, and volume calculation thereafter, were planned to furnish materials for each case. The measurement at cutting down and clean cut, volume calculation, etc. was carried out by simplified analysis method as shown in Table 2, based on the method of stem analysis. Table 3 is a list of values on sample trees, as above obtained. Various calculations were carried out by computer, as in the flow in Fig. 1. Table 4 indicated one example of sample tree No. 12, and Table 5 show a list of the results of various volume calculations of all sample trees (100 trees).

3. Examination on the volume equation

In the past, various volume equations have been proposed. Properly speaking, it is appropriate to examine each formula separately and select the most adaptable formula. However, in this study, the examination was made on the following five formulae which had been relatively often used.

Table 1. Distribution of sample trees by the grades of diameter at breast height with bark (*DAP*, Com casca) and tree height.

H	*DAP* (Com casca / cm)																							Total
(m)	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	
7	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
8	1	—	—	1	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	3
9	1	1	1	—	1	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	6
10	1	—	1	—	1	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	5
11	1	—	2	—	—	—	1	—	—	—	2	—	—	—	1	—	—	—	—	—	—	—	—	7
12	—	—	1	—	2	—	—	—	1	—	1	—	1	—	—	—	—	—	—	—	—	—	—	6
13	—	—	1	—	1	—	3	1	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	8
14	—	—	—	—	—	—	2	—	1	1	2	—	2	—	2	—	1	—	—	—	—	—	—	11
15	—	—	1	—	1	—	1	—	1	—	1	—	1	—	—	1	1	—	1	—	—	—	—	9
16	—	—	—	—	2	—	—	—	1	1	—	—	1	—	2	—	—	—	—	—	—	—	—	7
17	—	—	—	—	—	—	1	—	—	1	1	—	—	—	1	—	—	—	—	—	1	—	1	6
18	—	—	—	—	—	—	—	—	2	—	1	—	2	1	—	—	2	—	1	—	—	—	—	9
19	—	—	—	—	—	—	1	—	—	1	1	—	2	1	—	—	—	—	—	—	—	—	—	6
20	—	—	—	—	—	—	—	—	—	1	—	—	—	—	2	—	—	1	—	—	1	—	1	6
21	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1	—	1	—	—	—	—	3
22	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1	—	—	—	—	2
23	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	1	—	1	4
Total	5	1	8	1	8	—	11	1	9	6	11	—	10	2	9	1	6	1	4	—	3	—	3	100

Table 2. Field book for simplified stem analysis for the purpose of measurement of sample trees.
—— Ficha para o Cálculo do Volume --

No. Árvores : 12				Espécie : *P. elliottii*						Idade : 18		
Data da Medição : 3 de junho de 1982							Local : Águas de Santa Bárbara					
No.	*Alt.*	Diâmetro à Altura do Peito (*d*)						Área Basal (*g*)		Volume (*v*)		
		Com casca (C/c)			Sem casca (S/c)			C/c	S/c	C/c	S/c	Porc.
		Máx.	Min.	Med.	Máx.	Min.	Med.	(G_b)	(G_i)	(V_b)	(V_i)	
	m	cm	cm	cm	cm	cm	cm	m²	m²	m³	m³	%
0	1.3			28.5			24.1					
1	1.1			29.3			24.1	0.0674	0.0456	0.1348	0.0912	67.66
2	3.1			25.7			22.5	.0519	.0398	.1038	.0796	77.69
3	5.1			25.0			22.2	.0491	.0387	.0982	.0774	78.82
4	7.1			24.5			21.7	.0471	.0370	.0942	.0740	78.56
5	9.1			24.0			21.2	.0452	.0353	.0904	.0706	78.10
6	11.1			21.7			19.8	.0370	.0308	.0740	.0616	83.24
7	13.1			21.7			19.3	.0370	.0293	.0740	.0586	79.19
8	15.1			16.4			15.0	.0211	.0177	.0422	.0354	83.89
9	17.1			13.0			12.0	.0133	.0113	.0266	.0226	84.96
10	19.1			7.2			6.8	.0041	.0036	.0082	.0072	87.80
11	21.1			4.5			3.8	.0016	.0011	.0032	.0022	68.75
12	22.1			2.1			1.9	.0003	.0003	.0001	.0001	100.00
13												
14												
15												
16												
17												
18												
(19)	19.3			7.5			6.5	.0044	.0033	.0067	.0051	76.12
(20)	20.5			5.9			5.0	.0027	.0020	.0025	.0019	76.00
D A P (*D*)				cm 28.5			cm 24.1					
Altura (*H*)				m 22.8			m 22.8					
Comp. do Ponteiro (l_T)				m 0.7			m 0.7					
Comp. de Acima de 5.0 cm (l_{05})				m 2.3			m 2.3					
Área Basal Total (ΣG_i)								m² 0.3748	m² 0.2902			
①Vol. das Seccoes (ΣV)										m³ 0.7496	m³ 0.5804	
②Vol. do Cone (V_T)										.0001	.0001	
①+②Vol. Total (V_A)										.7497	.5805	% 77.43
③Vol. Acima de 5.0 cm (V_{05})										.0025	.0019	
V_A-③Vol. Comercial (V_B)										.7472	.5786	77.44
Porcentagem (%)										% 99.67	% 99.67	

(Responsável : Chyo, Haga, Aoki, Adauto, Ataide)

Table 3. List of sample trees (A part of 100 trees data).

No.	Talhão *No.*	Idade	*DAP*		*H*	Altura da *COPA*	Comp. do Ponteiro	Comp. Acima de 5.0 cm	Pont. Final (l_F)
			C/c	S/c					
			cm	cm	m	m	m		m
1	4	20	21.0	17.8	19.1	11.5	1.00	2.11	2.20
2	16	〃	13.8	12.0	17.1	10.9	1.04	2.69	2.64
3	〃	〃	25.0	21.0	19.8	10.8	1.74	2.56	2.94
4	10	〃	12.5	10.2	15.1	8.5	0.95	2.95	2.95
5	〃	〃	8.2	6.7	10.5	5.6	0.40	6.64	5.60
6	〃	〃	7.7	6.2	8.5	1.8	0.35	4.75	3.55
7	15	〃	13.6	12.0	13.3	1.9	1.20	3.07	3.60
8	〃	〃	15.7	14.3	15.7	7.8	1.60	3.04	3.60
9	〃	〃	17.5	15.3	16.2	7.8	0.10	1.70	1.70
10	35	18	21.2	19.0	17.6	8.7	1.50	2.60	3.10
	〃	〃	16.8	14.8	20.0	10.0	1.90	2.90	3.10
12	31	〃	28.5	24.1	22.8	14.5	0.70	2.30	3.50
13	12	20	21.8	18.8	19.7	12.2	1.60	2.60	2.80
14	〃	〃	17.3	15.2	19.5	11.9	1.40	2.65	3.80
15	〃	〃	15.8	12.8	18.0	11.7	1.90	2.74	3.50
16	〃	〃	12.0	10.1	16.4	11.3	0.30	3.95	4.30
17	123	13	8.1	7.2	11.5	8.1	1.40	4.40	5.40
18	〃	〃	10.1	8.4	10.8	6.3	0.70	4.70	4.70
19	〃	〃	12.0	10.7	12.5	8.3	0.40	2.80	2.80
20	〃	〃	10.0	8.0	11.2	7.8	1.10	5.95	6.30
21	85	17	14.0	11.8	14.0	6.8	1.90	2.70	3.10
22	〃	〃	14.0	11.2	12.6	7.4	0.50	3.23	4.10
23	〃	〃	16.0	13.4	14.0	8.2	1.90	2.65	3.10
24	〃	〃	18.0	15.2	14.5	7.8	0.40	2.40	2.40
25	〃	〃	20.0	17.0	15.6	7.8	1.50	2.30	2.30
26	〃	〃	14.3	12.0	14.8	8.1	0.70	2.90	3.90
27	118	14	23.4	20.0	15.4	8.0	1.30	2.00	2.10
28	〃	〃	21.8	18.5	14.4	6.0	0.30	2.30	2.30
29	〃	〃	17.5	14.8	14.1	7.2	2.00	2.50	3.30
30	〃	〃	15.2	13.1	13.2	7.0	1.10	3.50	3.50
31	12	20	26.0	22.5	22.4	11.7	0.30	2.40	3.20
32	〃	〃	20.0	17.0	21.0	14.6	0.90	2.90	3.00
33	〃	〃	22.0	19.0	22.7	13.7	0.60	3.30	3.50
34	〃	〃	30.0	26.8	22.7	14.0	0.60	2.00	2.30
35	〃	〃	24.0	20.2	23.0	13.1	0.90	3.30	3.80
36	〃	〃	18.0	16.5	22.0	15.4	1.90	2.70	2.80
37	〃	〃	14.2	12.0	18.6	11.6	0.50	2.20	3.00
38	103	17	18.0	15.5	17.6	9.5	1.50	2.80	3.20
39	〃	〃	18.0	15.2	17.2	12.6	1.10	2.70	2.80
91	12	20	16.0	13.2	17.8	9.2	1.70	2.80	3.40
92	〃	〃	18.0	14.8	19.2	13.5	1.10	3.60	3.60
93	〃	〃	22.0	18.0	19.7	12.0	1.60	3.20	4.10
94	〃	〃	28.0	24.7	20.5	10.2	0.40	1.90	2.50
95	〃	〃	30.0	25.0	20.1	9.7	2.00	2.10	2.10
96	〃	〃	26.0	23.5	21.5	12.8	1.40	2.00	2.30
97	117	13	16.0	14.0	9.1	4.0	1.00	1.90	1.90
98	〃	〃	22.0	19.0	15.7	7.8	1.60	2.50	2.50
99	〃	〃	14.0	11.0	9.1	5.3	1.00	3.90	4.30
100	〃	〃	22.0	18.5	14.1	7.1	1.90	2.80	3.20

$$g_m = \pi \cdot (d_m/2)^2$$

$$v_m = g_m \cdot l$$

$$l_t = h_n - h_{n-1}$$

$$v_t = (g_{n-1} \cdot l_t)/3$$

$$V_A = \sum_{m=1}^{n-2} v_m + v_t$$

$$l_{.05} = h_{n-1} - h_{.05}$$

$$v_{.05} = \{(g_{.05} + g_{n-1})/2\} \cdot l_{.05} + v_t$$

$$V_B = V_A - v_{.05}$$

$$l' = h_{n-1} - h'$$

$$v' = \{(g' + g_{n-1})/2\} \cdot l' + l_t$$

$$V_C = V_A - v'$$

g_m : cross section area (m^2)
π : the ratio of the circumference of a circle to its diameter (3.1415926)
d_m : diameter (cm)
v_m : separated trunk volume (m^3)
l : separated length (2.0 m) (m)
l_t : length to tree neck (m)
h_n : total tree height (m)
h_{n-1} : tree height for the part for measurement after cutting (m) (corresponding to the tree height of the subject of separated trunk volume)
v_t : volume to tree neck (m^3)
g_{n-1} : cross section area at tree neck (m^2)
V_A : total trunk volume (m^3)
$l_{.05}$: length from the part of 5 cm without bark diameter to tree neck (m)
$h_{.05}$: tree height to the part of 5 cm without bark diameter (m)
$v_{.05}$: volume of less than 5 cm without bark diameter (m^3)
V_B : commercial volume (m^3)
l' : length from the last lumbered part to tree neck (m)
h' : total lumber length separated by every 2.4 m (or 1.2 m) (m)
v' : volume not to be lumbered (m^3)
g' : cross section area of last lumbered part (m^2)
V_c : real commercial volume (m^3)

Fig. 1. Flow of volume calculation by computer.

Table 4. Example of result of volume calculation by computer (sample tree No. 12)

Ficha para o Cálculo do Volume –

N.ARVORES : 12-(1)
ESPECIE : P.ELLIOTTII
IDADE : 18
DATA : 03/06/1982
LOCAL : AGUAS DE SANTA BARBARA (31)

N.	ALT.	DIAMETRO A ALTURA DO PEITO – COM CASCA (C/C) MAX.	MIN.	MED.	SEM CASCA (S/C) MAX.	MIN.	MED.	AREA BASAL C/C	S/C	VOLUME C/C	SIC	PORC.
0	1.30	0.0	0.0	28.5	0.0	0.0	24.1					
1	1.10	0.0	0.0	29.3	0.0	0.0	24.1	0.0674	0.0456	0.1348	0.0912	67.66
2	3.10	0.0	0.0	25.7	0.0	0.0	22.5	0.0519	0.0398	0.1038	0.0796	76.69
3	5.10	[illegible]	[illegible]	[illegible]	0.0	0.0	[illegible].22	[illegible]	0387[illegible]	09 82	0.0774	78.82
4	7.10	0.0	0.0	24.5	0.0	0.0	21.7	0.0471	0.0370	0.0942	0.0740	78.56
5	9.10	0.0	0.0	24.0	0.0	0.0	21.2	0.0451	0.0353	-0.0904	0.0706	78.10
6	11.10	0.0	0.0	21.7	0.0	0.0						
7	13.10	0.0	0.0	21.7	0.0	0.0	19.8 19.3	0.0370 0.0370	0.0308 0.0293	0.0740 0.0740	0.0616 0.0566	83.24 79.19
8	15.10	0.0	0.0	16.4	0.0	0.0	15.0	0.0211	0.0177	0.0422	0.0354	83.89
9	17.10	[illegible]	[illegible]	1.	[illegible]	00	0 .0	12.0	0 0.0113	[illegible]	0.0226	84.96
10	19.10	0.0	0.0	7.2	0.0	0.00.0	60.0	0.0041	0.0036	0.0082	0.0072	87.80
11	21.10	0.0	0.0	-4[illegible]	0.0	0.0	3.8	0.0016	0.0011	0.0032	0.0022	68.75
12	22.10	0.0	0.0.	2.1			1.9	0.0003	0.0003	0.0001	0.0001	100.00
13	19.30	0.0	0.0	7.5	0.0	0.0	6.5	0.0044	0.0033	0.0067	0.0051	76.12
14	20.50	0.0	0.0	5.9	0.0	0.0	5.0	0.0027	0.0020	0.0025	0.0019	7b.00

		C/C	S/C	AREA BASAL C/C	S/C	VOLUME C/C	SIC	PORC.
D. A. P.	(CM)	28.50	24.10					
ALTURA	(M)	22.80	22.80					
COMP. DO PONTEIRO	(M)	0.70	0.70					
COMP. ACIMA DE 5CM	(M)	.	2.30					
AREA BASAL TOTAL	(M**2)			0.3748	0.2902			
VOL. DAS SECCOES	(M**3)					0.7496	0.5804	
VOL. DO CONE	(M**3)					0.0001	0.0001	
YOL. TOTAL	(M**3)					0.7497	0.5805	77.43
VOL. ACIMA DE 5CM	(M**3)					0.0025	0.0019	
VOL. COMERCIAL	(M**3)					0.7472	0.5786	77.44
PORCENTACEN	(%)					99.67	99.67	

$$V = a_0 D^{a_1} \quad (1)$$

$$V = a_0 + a_1 (D^2 H) \quad (2)$$

$$V = a_0 + (D^2 H)^{a_1} \quad (3)$$

$$V = a_0 + D^{a_1} H^{a_2} \quad (4)$$

$$V = a_0 + a_1 D^2 + a_2 H + a_3 (D^2 H) \quad (5)$$

In order to solve these 5 equations by linear least square method, we apply logarithmic conversion to equation (1), (3) and (4), and the following are obtained.

$$\log V = \log a_0 + a_1 \log D \quad (1')$$

$$\log V = \log a_0 + a_1 \log (D^2 H) \quad (3')$$

$$\log V = \log a_0 + a_1 \log D + a_2 \log H \quad (4')$$

When y is substituted for dependent variant, x_i for an independent variable, a_0 for a regression constant, a_i for a regression coefficient, these formulae can be expressed in a general formula as below.

Table 5. List of the calculation results of various volume and ratios on sample trees (A part of 100 trees data).

No.	DAP c/c	DAP s/c	H	Volume total C/c	Volume total s/c	Volume total P	Vol. commercial C/c	Vol. commercial s/c	Vol. commercial P	Vol. comme. real C/c	Vol. comme. real s/c	Vol. comme. real P
	cm	cm	m	m³	m³	%	m³	m³	%	m³	m³	%
1	21.0	17.8	19.1	0.3542	0.2817	79.5	0.3523	0.2803	79.6	0.3517	0.2801	79.6
2	13.8	12.0	17.1	0.1411	0.1096	77.7	0.1380	0.1073	77.8	0.1382	0.1076	77.9
3	25.0	21.0	19.8	0.4758	0.3776	70.4	0.4734	0.3757	79.4	0.4719	0.3745	79.4
4	12.5	10.2	15.1	0.0928	0.0729	78.6	0.0896	0.0705	78.7	0.0896	0.0705	78.7
5	8.2	6.7	10.5	0.0248	0.0178	71.8	0.0158	0.0116	73.4	0.0191	0.0134	70.2
6	7.7	6.2	8.5	0.0200	0.0140	70.0	0.0136	0.0096	70.6	0.0171	0.0119	69.6
7	13.6	12.0	13.3	0.1090	0.0897	82.3	0.1060	0.0874	82.5	0.1039	0.0856	82.4
8	15.7	14.3	15.7	0.1602	0.1357	84.7	0.1569	0.1330	84.8	0.1546	0.1311	84.8
9	17.5	15.3	16.2	0.2450	0.1998	81.6	0.2429	0.1981	81.6	0.2429	0.1981	81.6
10	21.2	19.0	17.6	0.2985	0.2439	81.7	0.2955	0.2418	81.8	0.2941	0.2406	81.8
11	16.8	14.8	20.0	0.2307	0.1886	81.8	0.2274	0.1858	81.7	0.2268	0.1854	81.8
12	28.5	24.1	22.8	0.7497	0.5805	77.4	0.7472	0.5786	77.4	0.7430	0.5754	77.4
13	21.8	18.8	19.7	0.4025	0.3291	81.8	0.3999	0.3271	81.8	0.3990	0.3264	81.8
14	17.3	15.2	19.5	0.2548	0.2035	79.9	0.2523	0.2016	79.9	0.2500	0.2000	80.0
15	15.8	12.8	18.0	0.1859	0.1425	76.7	0.1829	0.1405	76.8	0.1804	0.1387	76.9
16	12.0	10.1	16.4	0.0982	0.0740	75.4	0.0935	0.0703	75.2	0.0906	0.0684	75.5
17	8.1	7.2	11.5	0.0340	0.0283	83.2	0.0294	0.0247	84.0	0.0276	0.0234	84.8
18	10.1	8.4	10.8	0.0389	0.0268	68.9	0.0324	0.0222	68.5	0.0324	0.0222	68.5
19	12.0	10.7	12.5	0.0732	0.0560	76.5	0.0694	0.0531	76.5	0.0694	0.0531	76.5
20	10.0	8.0	11.2	0.0389	0.0255	65.6	0.0320	0.0198	61.9	0.0310	0.0189	61.0
21	14.0	11.8	14.0	0.1050	0.0782	74.5	0.1028	0.0764	74.3	0.1014	0.0756	74.6
22	14.0	11.2	12.6	0.0911	0.0664	72.9	0.0872	0.0634	72.7	0.0847	0.0617	72.9
23	16.0	13.0	14.0	0.1514	0.1077	71.1	0.1489	0.1058	71.1	0.1475	0.1049	71.1
24	18.0	15.2	14.5	0.2014	0.1536	76.3	0.1987	0.1516	76.3	0.1987	0.1516	76.3
25	20.0	17.0	15.6	0.2476	0.1973	79.7	0.2448	0.1949	79.6	0.2448	0.1949	79.6
26	14.3	12.0	14.8	0.1195	0.0924	77.3	0.1161	0.0900	77.5	0.1127	0.0876	77.7
27	23.4	20.0	15.4	0.3485	0.2797	80.3	0.3466	0.2784	80.3	0.3462	0.2781	80.3
28	21.8	18.5	14.4	0.2674	0.2048	76.6	0.2646	0.2027	76.6	0.2646	0.2027	76.6
29	17.5	14.8	14.1	0.1656	0.1315	79.4	0.1632	0.1297	79.5	0.1608	0.1279	79.5
30	15.2	13.1	13.2	0.1090	0.0837	76.8	0.1047	0.0804	76.8	0.1047	0.0804	76.8
31	26.0	22.5	22.4	0.6696	0.5372	80.2	0.6666	0.5350	80.3	0.6641	0.5330	80.3
32	20.0	17.0	21.0	0.3188	0.2627	82.4	0.3155	0.2603	82.5	0.3153	0.2602	82.5
33	22.0	19.0	22.7	0.4219	0.3484	82.6	0.4182	0.3454	82.6	0.4166	0.3449	82.8
34	30.0	26.8	22.7	0.8803	0.7575	86.1	0.8780	0.7557	86.1	0.8773	0.7550	86.1
35	24.0	20.2	23.0	0.5134	0.4324	84.2	0.5096	0.4292	84.2	0.5075	0.4274	84.2
36	18.0	16.5	22.0	0.3095	0.2620	84.7	0.3064	0.2596	84.7	0.3062	0.2594	84.7
37	14.2	12.0	18.6	0.1613	0.1384	85.8	0.1359	0.1185	87.2	0.1331	0.1158	87.0
38	18.0	15.5	17.6	0.2281	0.1906	83.6	0.2254	0.1884	83.6	0.2239	0.1873	83.7
39	18.0	15.2	17.2	0.2200	0.1755	79.8	0.2172	0.1735	79.9	0.2171	0.1734	79.9
91	16.0	13.2	17.8	0.1723	0.1350	78.4	0.1695	0.1327	78.3	0.1681	0.1316	78.3
92	18.0	14.8	19.2	0.2325	0.1812	77.9	0.2283	0.1779	77.9	0.2283	0.1779	77.9
93	22.0	18.0	19.7	0.3546	0.2809	79.2	0.3512	0.2781	79.2	0.3479	0.2755	79.2
94	28.0	24.7	20.5	0.6355	0.5308	83.5	0.6333	0.5291	83.6	0.6310	0.5272	83.6
95	30.0	25.6	20.1	0.6868	0.5590	81.4	0.6849	0.5576	81.4	0.6849	0.5576	81.4
96	26.0	23.5	21.5	0.6971	0.5919	84.9	0.6956	0.5908	84.9	0.6948	0.5902	85.0
97	16.0	14.0	9.1	0.0898	0.0708	78.8	0.0878	0.0694	79.0	0.0878	0.0694	79.0
98	22.0	19.0	15.7	0.2708	0.2189	80.8	0.2686	0.2171	80.8	0.2686	0.2171	80.8
99	14.0	11.0	9.1	0.0593	0.0407	68.6	0.0550	0.0374	68.0	0.0529	0.0360	68.1
100	22.0	18.5	14.0	0.2468	0.1906	77.2	0.2440	0.1884	77.2	0.2414	0.1862	77.1

$$y = a_0 + \sum_{i=1}^{P} a_i x_i \qquad (6)$$

where, P = 1, 2, 3

Here formula (6) is called a linear multiple regression formula. Therefore, in order to determine a volume formula to be adopted, apply each material obtained to five formulae, evaluate their adaptability, and determine the formula furnishing the least value of standard errors for each of residual. Table 6 indicates only the standard errors of residual, out of series of calculation results due to the multiple regression formula programme. In addition, standard error (SE) are acquired by formula (7) (In this formula, n: number of samples, k: number of independent variables, v: real volume, $\hat{v}$: estimated volume).

$$SE = \left\{ \frac{1}{n-(k+1)} \sum_{i=1}^{n} (v-\hat{v})^2 \right\}^{\frac{1}{2}} \qquad (7)$$

Table 6. Standard errors of residual

Kind		(1′)	(2)	(3′)	(4′)	(5)
Total tree volume formula	①with bark	0.05828	0.02134	0.02107	0.01990	0.01089
	@without bark	0.05105	0.02129	0.02109	0.02000	0.01986
Commercial volume formula	@with bark	0.05998	0.02135	0.02345	0.02264	0.02020
	@without bark	0.05388	0.02120	0.02566	0.02485	0.01999
Real commercial volume formula	@with bark	0.06011	0.02126	0.02322	0.02219	0.02028
	@without bark	0.05417	0.02143	0.02580	0.02488	0.02008

However, standard error determined by the logarithmic formula cannot be simply compared with that by the non-logarithmic formula. For such purpose, when standard errors in (1′), (3’) and (4’) formula in Table 6 are demanded, calculate by substituting $\hat{v}$ multiplied by correction factor (c. *f.*), which is given in formula (8), for $\hat{v}$ in formula (7). (Where, $S_{yx_1x_2^2}$: standard error of residual).

$$c.f. = 10^{\frac{n-1}{n} S_{yx_1x_2^2} \cdot 1.151293} \qquad (8)$$

As the result of the above calculation, the standard error of residual became minimum in formula (5) for any of volume formula (See Table 6). Hence, we decided to adopt formula (5), which is called Australian formula, for our volume calculation.

Further, in order to examine the effect of independent variable in formula (5) to dependent variable, a significance test in partial regression coefficient in each volume formula was conducted by the formula as below, under a null hypothesis $(H: a_i = 0)$, where partial regression coefficient, a_i is assumed 0,

$$t = \frac{a_i}{\sqrt{S^{ii} V_e}} \geqq t\,(n-P-1;\ \alpha)$$

where, S^{ii}: reverse matrix in matrix of deviation square sum and product sum, V_e: error variance

The result was obtained as in Table 7, showing the second variable ($x_2 = H$) was eliminated in all formulae of volume. Therefore, the formula of volume was determined as formula (9).

$$V = a_0 + a_1 D^2 + a_2 (D^2 H) \qquad (9)$$

Next, in order to prevent the effect of abnormal material to the estimated volume, a rejecting zone, when formula (9) was applied, was calculated by formula (10) (where, t: value of t at 99 % level when freedom is n-3, $s_{yx_1x_2}$: error variance, $V(\hat{v})$: variance of estimated value $\hat{v}$).

$$E_{yx_1x_2} = t\,\{s_{yx_1x_2} - V(\hat{v})\}^{\frac{1}{2}} \qquad (10)$$

Compare $E_{yx_1x_2}$ obtained by formula (10) with variance $v - \hat{v}$ from regression, and reject i-th sample when $E_{yx_1x_2} < |v_i - \hat{v}_i|$ exists. As the result, material No. 94 was rejected in total volume formulae, and material No. 98 was rejected in formulae with bark.

RESULTS

In linear regression model, there is a provision that the relation among variable is linear over the whole range of materials. But in this case, since the number of samples are only 100, and the range is narrow, as 8 cm-30 cm in diameter, and 7 m- 23 m in height, one volume formula is taken for the whole diameter grade, as follows. (Application of volume formula is omitted.)

① Total tree volume with bark formula :

$$\hat{V} = 0.00674609 - 0.00012281D^2 + 0.00004552D^2H \qquad (E = 14.96)$$

② Total tree volume without bark formula :

$$\hat{V} = 0.00226291 - 0.00009136D^2 + 0.00003656D^2H \qquad (E = 16.66)$$

③ Commercial volume with bark formula :

$$\hat{V} = 0.00113217 - 0.00010899D^2 + 0.00004506D^2H \qquad (E = 15.46)$$

④ Commercial volume without bark formula :

$$\hat{V} = -0.00204230 - 0.00008071D^2 + 0.00003621D^2H \qquad (E = 17.05)$$

⑤ Real commercial volume with bark formula :

$$\hat{V} = 0.00116770 - 0.00011814D^2 + 0.00004539D^2H \qquad (E = 15.61)$$

⑥ Real commercial volume without bark formula :

$$\hat{V} = -0.00200872 - 0.00008761D^2 + 0.00003644D^2H \qquad (E = 17.20)$$

The analysis resulted in 6 different volume equations, and based on these equations, 6 different stumpage volume tables for *Pinus elliottii* has been contracted (The

Table 7. Test of partial regression coefficient.

Kind		Variable	Partial regression coefficient	95 % confidence limit of a_i		Standard error	Standard partial regression coefficients	t-test
				Upper limit	Lower limit			
Total tree volume formula	with bark ①	x_1	−0.00015592	−0.000242	−0.000069	0.0000	−0.1768836	− 3.581**
		x_2	−0.00044804	−0.002418	0.001522	0.0010	−0.0099104	− 0.451
		x_3	0.00004781	0.000042	0.000052	0.0000	1.1713886	19.412**
	without bark ②	x_1	−0.00015530	−0.000241	−0.000069	0.0000	−0.2113804	− 3.573**
		x_2	−0.00046669	−0.002433	0.001500	0.0010	−0.0123852	− 0.471
		x_3	0.00004095	0.000036	0.000045	0.0000	1.2036333	16.654**
Commercial volume formula	with bark ③	x_1	−0.00014332	−0.000231	−0.000055	0.0000	−0.1619246	− 3.241**
		x_2	−0.00048609	−0.002486	0.001514	0.0010	−0.0107079	− 0.482
		x_3	0.00004744	0.000042	0.000052	0.0000	1.1576738	18.971**
	without bark ④	x_1	−0.00014617	−0.000233	−0.000059	0.0000	−0.1982326	− 3.340**
		x_2	−0.00051635	−0.002469	0.001463	0.0010	−0.0136536	− 0.518
		x_3	0.00004070	0.000035	0.000045	0.0000	1.1921310	16.442**
Real commercial volume formula	with bark ⑤	x_1	−0.00015342	−0.000241	−0.000065	0.0000	−0.1736585	− 3.455**
		x_2	−0.00051834	−0.002527	0.001490	0.0010	−0.0114392	− 0.512
		x_3	0.00004874	0.000042	0.000052	0.0000	1.1693535	19.047**
	without bark ⑥	x_1	−0.00015377	−0.000241	−0.000066	0.0000	−0.2088885	− 3.498**
		x_2	−0.00053076	−0.002519	0.001458	0.0010	−0.0140581	− 0.530
		x_3	0.00004099	0.000036	0.000045	0.0000	1.2024670	16.484**

volume tables are not presented in this paper). Volume tables were made, corresponding to the forest stands, for the range of diameter of 6 cm-34 cm graded by 1 cm, and for the tree height of 5 m-25 m graded by 1 m. The volume tables were indicated in any case in values of diameter with bark, because it was impossible to measure diameter without bark. Therefore, by measuring a diameter with bark, volumes without bark are estimated.

By the way, the relation between diameter with bark (x) and diameter without bark (Y) is as shown in Fig. 2 and the regression formula and the coefficient of correlation are as follows.

$$Y = 0.962 + 0.909x \qquad (r = 0.989)$$

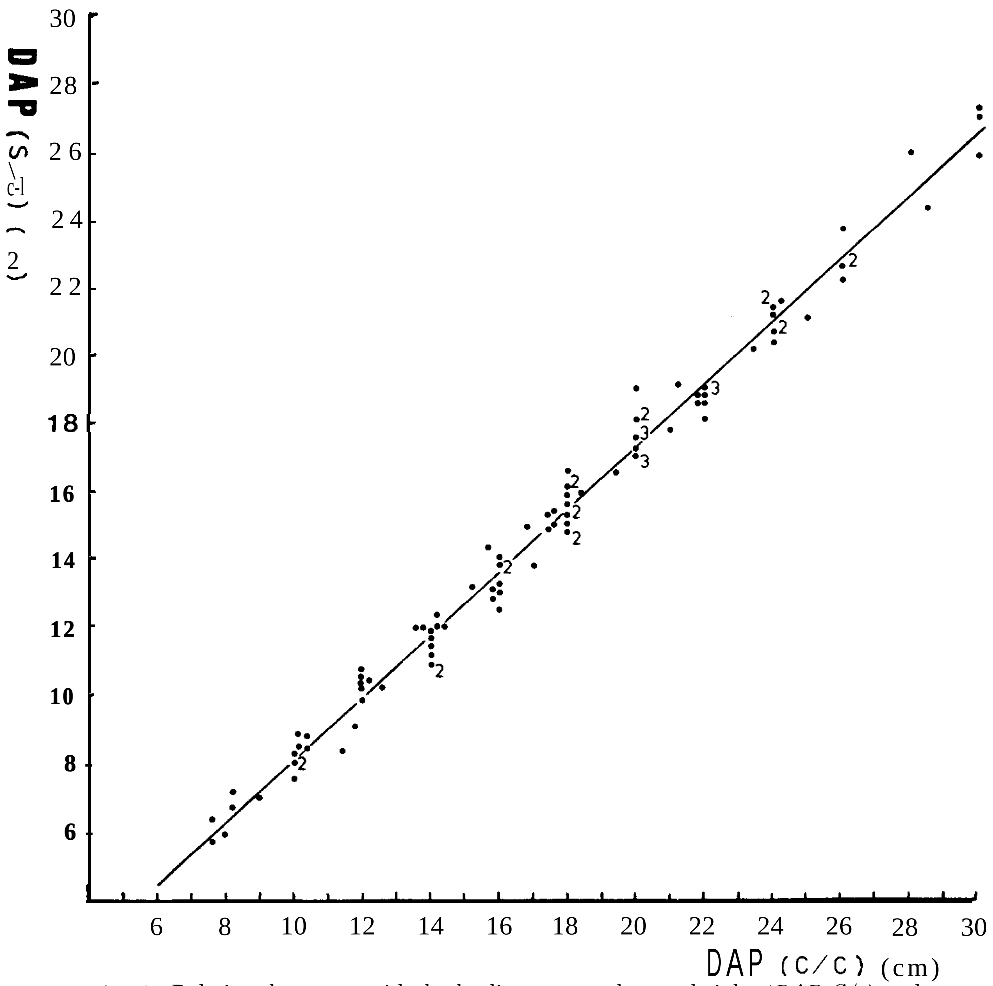

Fig. 2. Relation between with bark diameter at breast height (*DAP*, C/c) and without bark diameter at breast height (*DAP*, S/c).
$(Y = -0.962 + 0.909x,\ r = 0.989)$

DISCUSSION

The six kinds of tree volume equations were made for study of adjustment methods, but hereby some consideration was attempted on the adjusted volume equations.

First, on 98 samples left after 2 extraordinary ones were discarded out of 100 sample trees which were selected and cut in a forest stand, the relation between diameter at breast height ***(DAP*** with bark = D) and tree height ***(H)*** were as follows.

$$H = 3.759 + 0.790D - 0.008D^2 \qquad (r = 0.694)$$

When these data are shown on a graph, the distribution indicates scattering to some extent, so the coefficient of correlation cannot be regarded high. But this is the result of that sample trees were intentionally collected so that tree height might distribute in a certain range by each diameter grade. The scattering of this extent is often seen in any forest stand. Viewed from another angle, it means that such scattering as this extent always exists due to the different places or different land positions even in the same state forest of Âguas de Santa Barbara.

Next, the relation between bark and volume without bark are as in Table 8.

Table 8. List of parameter and correlation coefficients in the regression formula of diameter to volume.

	Kind of volume	Parameter			Correlation coefficient
		a_0	a_1	a_2	
With bark	①Tatol tree volume	0.08603	-0.01649	0.00127	0.895
	③Commercial volume	0.07084	-0.01591	0.00126	0.897
	⑤Real commercial volume	0.06875	-0.01567	0.00125	0.895
Without bark	②Total tree volume	0.03923	-0.01118	0.00122	0.890
	④Commercial volume	0.03400	-0.01106	0.00123	0.892
	⑥Real commercial volume	0.03480	-0.01117	0.00123	0.890

From Table 8, in case of with bark, the regression formula is formed with nearly similar tendency in ①, ③, and ⑤. The volume was as ①>③>⑤, so the values of parameter became a little smaller in order of ①, ③, and ⑤. In three of ②, ④, and ⑥, the values of parameter became smaller than those with bark. But the order is not definite as the former three. The coefficients of correlation are generally in a range between 0.890 and 0.897.

The relation between total volume with bark and other volume, between commercial volume with bark and those without bark, and between real volume with bark and that without bark are shown in Table 9.

From Table 9, in all formulae the coefficients of correlation are as high as in a range of 0.997-0.999, showing good correspondence. Regarding individual formula, (b) and (d) show hardly any difference on a graph. On the contrary, in (a), (c), and (e), volume values are much smaller than the total volume with bark. The cases of(f) and (g) show the same tendency. So it is inferable that the main cause for the difference

Table 9. List of parameters and correlation coefficients in regression formulae based on the corresponding volume.

Concerned formula	Parameter		Correlation coefficient
	au	a_1	
(a) Total volume with bark to total volume without bark	−0.006	0.832	0.998
(b) Total volume with bark to commercial volume with bark	−0.004	1.004	0.999
(c) Total volume with bark to commercial volume without bark	−0.006	0.829	0.999
(d) Total volume with bark to real commercial volume with bark	−0.005	1.002	0.999
(e) Total volume with bark to real commercial volume without bark	-0.010	0.834	0.997
(f) Commercial volume with bark to commercial volume without bark	0.006	0.832	0.998
(g) Real commercial volume with bark to real commercial volume without bark	0.004	0.829	0.998

exists in bark. It is guessed that is probably the reason why São Paulo state measures the volume with bark always together with the volume without bark. By the way, the result of stem analysis carried on *Pinus elliottii* of 21 years age *(DAP = 22.0* cm, $H =$ 22.1 m, Talhão No. 15) in the same state forest of Águas de Santa Barbara, indicated total volume without bark 0.3331 m^3 against total volume with bark 0.4109 m^3, showing bark ratio of 18.93 %.

Here, consideration is mainly limited to the relating formulae corresponding to breast height diameter and various volumes, on basis of 6 different volume formulase. However, the problem is whether the prepared volume value would be adaptable to the actual measurement or not. In this respect, we must wait for the field study result.

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REFERENCES

Aoki, H., M. Chyo and N. Haga 1986 Análise de tronco de *Pinus elliottii* VAR. *elliottii* através dos an&is de crescimento. *Public. I. F., SP, BR, n. 28* : 1-41 (in Portuguese with English abstract)

Chyo, M., T. Shiibayashi, N. Haga and H. Aoki 1984a A study on volume equations for *Pinus elliottii* growing in state forest of Águas de Santa Bárbara-São Paulo, Brazil. *Trans. 95th Mtg. Jap. For. Soc.*: 109-110 (in Japanese)

Chyo, M., T. Shiibayashi, N. Haga and H. Aoki 1984b Examination of the conformity between actual measurement of volume table, for *Pinus elliottii* in state forest of Águas de Santa Bárbara (Expert Report of International Cooperation *on* Study of Forestry in São Paulo, Brazil). *JICA, FDD • JR84-40*: 291-310 (in Japanese)

Chyo, M., M. Kobayashi, H. Suzuki, L. A. Bucci and J. M. Motta 1984c Examination of the conformity between actual measurement of volume and volume table used, for *Pinus elliottii* and *Pinus patula* in state forest of Campos do Jordão. *Ditto* : 311-335 (in Japanese)

Chyo, M., N. Haga and H. Aoki 1984d Stem analysis — Mannual by stem analysis of *Pinus elliottii* —. *Ditto* : 395-444 (in Japanese)

Chyo, M., T. Shiibayashi, N. Haga and H. Aoki 1986 Preparation of tree volume table (Synthetic Report of the Japanese Technical Cooperation Project for the Forestry Research in São Paulo, Brazil). *JZCA, FDD • JR86-30: 334-358*

Forest Management Division 1956 Explanatory book for the preparation of tree volume table. *For. and Forest Prod. Res. Inst.* (in Japanese with English summary)