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Application of the Weibull Distribution to Thinning*

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In order to reduce the coefficient of variation of diameters within plantations, two thinning methods were devised to increase the Weibull parameter c after thinning. One is the TDC thinning method. Another is the TLD thinning method. It was recognized that both methods are valid for practical use. These results show that the Weibull distribution is applicable to studies on thinning of plantations.

INTRODUCTION

Thinning is an important silvicultural tool for the plantation manager. Many studies on thinning have been undertaken. However, no analysis has been made based on the diameter distribution, which would serve to express the stand composition more distinctly. One of the reasons for the lack of research is that a probability density function suitable for the diameter distribution has not been found. Thirteen years ago (in 1973), the Weibull distribution was found to suitable for the assessment of the diameter distributions of plantations (Clutter and Allison, 1973 ; Robert, 1973). This made it possible to explain quantitatively the characteristics of the diameter distributions (Kakihara, 1982 ; Kakihara and Kinashi, 1984). It further suggests the possibility of devising a thinning method based on the diameter distributions, using the Weibull distribution. In this research, a new thinning method is to be examined based on Sugi (*Cryptomeria japonica* D. Don), Hinoki (*Chamaecyparis obtusa* S. et Z.) and Kuromatsu (*Pinus thumbergii* Parl.) in the attempt to increase the Weibull parameter c.

Samples were taken from the experimental forests set up by Forest and Forest Products Research Institute throughout Japan.

EFFECT OF THINNING ON THE DIAMETER DISTRIBUTION USING THE WEIBULL PARAMETERS

The Weibull distribution used to express diameter distributions is shown by equation (1).

$$n_i = Nw(c/b)\{(d_i-a)/b\}^{c-1} \exp[-\{(d_i-a)/b\}^c] \tag{1}$$

where n_i is the number of trees belonging to each diameter class, d_i stands for each

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diameter class, N for the total number of trees, w for the width of diameter class, and a, *b* and c are parameters.

Parameter a is the minimum stand diameter and *b* is the value for a cumulative distribution of diameters at the 63 % point. c expresses the shape of the distribution. Therefore, any of the experimental forests where thinning has taken place can be assessed by analyzing *a, b* and c respectively. The most common characteristics of normal thinning (so far known) are as follows (Kakihara and Kinashi, 1984 ; Kakihara, 1985) :

1. A thinning method by which parameter a changes and c decreases is generally used.
2. This results in plantations with the parameter c having a low value.

If the purpose of forest management is to create a plantation with the parameter c (variable diameters) low in value, the present method of thinning is valid. However, where the management is aimed at the production of plantations with a smaller variation coefficient of diameters (homogeneous diameters), a different method of thinning needs to be devised, in which the parameter c is increased in its value.

THINNING METHODS TO INCREASE THE WEIBULL PARAMETER C

Thinning based on the thinning percentage for each diameter class

According to the results of thinning of many experimental forests, a few examples are observed, in which parameter a changes and c increases in its value (Kakihara and Kinashi, 1984). Examinations were made as to the type of tree selection in these instances. Two examples of this type of thinning are shown in Table 1 (Sugi and Hinoki).

Table 1. Examples of thinning by which a changes and *c* increases.

Exp. For. : Shirami Species : Sugi Age: 15 years Area: 0.02 ha

d (cm)	4	6	8	10	12	14	16	18	20	22	24	Total	a	*c*
N(1)	28	71	74	109	120	118	73	45	20	5	1	664	3	2.3
N(2)	28	60	51	64	34	6						243		
N(3)		11	23	45	86	112	73	45	20	5	1	421	5	2.9
P (%)	100	85	69	59	28	5								
R (%)	0	10	20	30	40	50	60	70	80	90	100			

Exp. For. : Nasumichi Species : Hinoki Age: 46 years Area: 0.02 ha

d (cm)	12	14	16	18	20	22	24	26	28	30……36	Total	*a*	*c*
N(1)	1	6	16	29	33	46	53	38	30	14	267	11	3.1
N(2)	1		13	16	15	13	7	4	2		77		
N(3)		1	3	13	18	33	46	34	28	14	190	13	3.4
P (%)	100	86	81	55	46	28	13	11	7				
R (%)	0	8	17	25	33	42	50	58	67	……100			

d, Diameter; N(1), The number of trees before thinning; *N*(2), The number of thinned trees; N(3), The number of trees after thinning.

P stands for the thinning percentage for each diameter class and *R* for the relative scale for the diameter. *R* is expressed by equation (2).

$$R=\{(d_i-d_{min})/(d_{max}-d_{min})\}\times 100\ (\%) \tag{2}$$

where d_i stands for each diameter class, d_{min} is the minimum diameter class and d_{max} represents the maximum diameter class.

Form the date in Table 1 the regression equation $P = A + BR$ is induced, where A and B are parameters. The parameters and correlation coefficient (r) between P and

Table 2. Experimental forests in which a changes and c increases by thinning.

Experimental forests	Age (years)	No. of trees per 0.1 ha	Mean diameter (cm)	a B. T.	a A. T.	c B. T.	c A. T.
			Species : Sugi				
Shirami	15	332	11.8	3	5	2.3	2.8
Mizunashihira	30	503	21.2	9	11	2.6	3.5
Kikuchisuigen	21	271	13.6	3	5	2.8	4.4
Teradoko	25	172	17.5	3	7	4.5	5.2
			Species : Hinoki				
Hontano	65	100	30.0	17	19	2.5	2.6
Nasumichi	46	131	22.7	11	13	3.1	3.4
Maruyama	65	205	19.3	11	13	2.0	2.5

B.T., Before thinning; A.T., After thinning.

Table 3. Thinning percentage for each diameter class of experimental forests.

R	Shirami*	Mizunashihira	Kikuchisuigen	Teradoko
		Species : Sugi		
0 (0–5)	100	100	100	100
10 (6–15)	85	89, 83	92, 96	100
20 (16–25)	69	79	96, 77	89
30 (26–35)	59	43	75, 64	80
40 (36–45)	28	12, 1	35, 20	59
50 (46–55)	5	85	11	
60 (56–65)		103, 13	7, 8, 3	20
70 (66–75)				8
80 (76–85)				4

R	Hontano	Nasumichi**	Maruyama
		Species: Hinoki	
0 (0–5)	100	100	100
10 (S-15)		86	88
20 (16–25)	86, 55	81, 55	66
30 (26–35)	5	46	41
40 (36–45)	11	28	26
50 (46–55)		13	
60 (56–65)		11	
70 (66–75)		7	

*,**, See Table 1; Unit, %; R, The relative scale for the diameter.

R were calculated using 4 experimental forests of Sugi and 3 experimental forests of Hinoki shown in Table 2. The value of P for each R are shown in Table 3.

Equation (3), (4) and Table 4 indicate the results of this analysis.

Table 4. Thinning percentage for each diameter class calculated by equation (3) and (4).

R	0	10	20	30	40	50	60
Sugi	98	84	69	55	41	26	12
Hinoki	96	80	64	48	32	16	

R, Relative scale for the diameter; Unit, %.

Sugi: $P = 98.15 - 1.44\ R$ (3) $r = -0.89$

Hinoki: $P = 96.05 - 1.69\ R$ (4) $r = -0.95$

These results show that the following selections should be made, if parameter c is to be increased after thinning.

1) Fell all the trees in the minimum diameter class.
2) Reduce P gradually according to the increase in diameter.
3) Set P about 50 % where R is approximately 30 %.
4) Reduce P to 0 where R is 60-70 %.

With this method of thinning, a change in parameter a and an increase in c can be attained. In its application, the shape and arrangement of trees must be taken into consideration. This new method will be called the TDC Thinning Method ; namely, based on the Thinning percentage for each Diameter Class.

The TDC thinning method was actually applied to experimental plots in plantations of Sugi and Hinoki in Kyushu district. Table 5 shows the stand composition of both experimental plots.

Table 5. Stand composition of experimental plots.

Species	Age (years)	Area (ha)	Mean height (m)	Mean diameter (cm)	No. of trees per ha	Volume per ha (m^3)
Sugi	25	0.06	13	20	1,800	350
Hinoki	19	0.05	8	11	2,900	140

The method of selecting the Sugi trees for thinning is as follows. As shown in Table 6, the minimum diameter class is 12 cm. Thus, all trees 12 cm in diameter are felled. The diameter class $R = 70\%$ is 20 cm. All trees 20 cm or more in diameter, except for one inferior sample 22 cm, are left unfelled. As for the trees with diameter in the range of 14-20 cm, the value of P for each R is calculated from equation (3), and the number of thinned trees for each diameter class is obtained by multiplying P by the number of trees before thinning for each diameter class. The selection of trees to be felled was made considering their shape and arrangement. The same method of selection was applied for Hinoki trees.

Table 6 and Fig. 1 indicate the results. The parameter c are increased in both cases.

Table 6. Results of thinning by the TDC thinning method at experimental plots.

Species: Sugi											
d (cm)	12	14	16	18	20	22	24	Total	*a*	*b*	*c*
N (1)	6	11	23	18	24	24	13	119	11	8.8	2.5
N (2)	6	7	12	5		1		31			
N (3)		4	11	13	24	23	13	88	13	7.9	2.8
R (%)	0	17	33	56	67	83	100				

Species: Hinoki												
d (cm)	6	8	10	12	14	16	18	20	Total	a	*b*	*c*
N (1)	6	18	42	42	26	10	1	1	146	5	7.3	2.7
N (2)	6	3	20	4			1		45			
N (3)		14	22	38	26	10	85	1	101	7	6.2	2.9
R (%)	0		29	43	57	71		100				

See below Table 1.

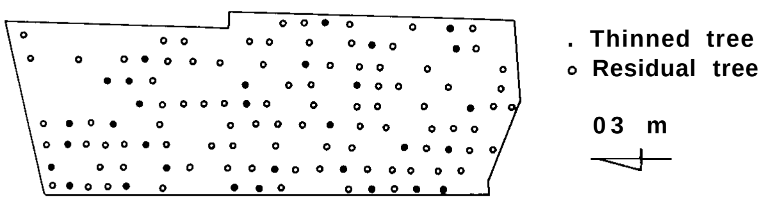

Fig. 1 Arrangement of stumpage at experimental plot (Sugi).

Thinning method for large diameter trees

Assuming **that** the forests with a smaller variation cocefficient have diameter values lying within the normal distribution, thinning should be undertaken in such a manner in which the Weibull parameter c is **3.6** after the operation. Since the value of c for plantations is generally below 3.6 in many cases, the above thinning method will generally serve to increase parameter c. In this experiment Sugi and Kuromatsu plantations in Kyushu district with parameter c low in value were selected to examine the possibility of this method. Table 7 shows the stand composition of both experimental plots.

Table 7. Stand composition of experimental plots.

Species	Age (years)	Area (ha)	Mean height (m)	Mean diameter (cm)	No. of trees per ha	Volume per ha (m^3)
Sugi	17	0.06	13.2	22.5	2,017	495
Kuromatsu	13	0.04	4.1	5.5	4,400	31

The mathod of tree selections for thinning sugi is as follows.

1) Firstly, parameter c is determined as 3.6.
2) The number of trees (N) after thinning is determined to be 80 with reference to

normal methods.

3) It is assumed that the mean diameter (d) after thinning is apporximately the same as before thinning, that is, $\bar{d}=23$ cm.

4) Where $a=13$, Γ_1 corresponding to $c=3.6$ is 0.9 (Nishizawa, 1978).
Then, $b=(\bar{d}-a)/\Gamma_1=(23-13)/0.9=11.1$.

5) Substituting $a=13, b=11.1, c=3.6, N=80$ and $w=2$ in equation(l), equation (5) is deduced as

$$n_i=80\times2\times(3.6/11.1)\{(d_i-13)/11.1\}^{2.6}\exp[-\{(d_i-13)/11.1\}^{3.6}] \tag{5}$$

6) The number of trees for each diameter class after thinning is obtained by using equation (5).

7) The number of thinned trees for each diameter class is obtained by subtracting the number of trees after thinning form the original number before thinning.

8) The number of thinned trees corresponding to each diameter class are selected by considering their shape and arrangement.

Selections for kuromatsu thinning were also made by this method. The results are shown in Table 8 and Fig. 2. As expected, parameter c was increased in both **cases** of Sugi and Kuromatsu. It is clear from Table 8, the characteristics of this metod is that trees of not only small diameters but also of large diameters are felled. This new method will be called the TLD Thinning Method ; namely, Thinning for Large Diameter trees.

Table 8. Results of thinning by the TLD thinning method at experimental plots.

Species : Sugi

d (cm)	12	14	16	18	20	22	24	26	28	30	32	Total	a	b	c
N(1)	4	3	6	10	19	21	24	18	6	4	3	118	11	12.7	2.9
N(2)	4	3	4	4	6	2	5	5		2	3	38			
N(3)			2	6	13	19	19	13	6	2		80	13	11.1	3.6

Species : Kuromatsu

d (cm)	3	4	5	6	7	8	9	10	11	12	Total	a	b	c
N(1)	16	38	41		38	25	11	4	2	1	176	2.5	3.4	1.9
N(2)	15	25	5		1	13	10	4	2	1	76			
N(3)	1	13		36		37		12	1		100	2.5	3.3	3.6

See below Table 1.

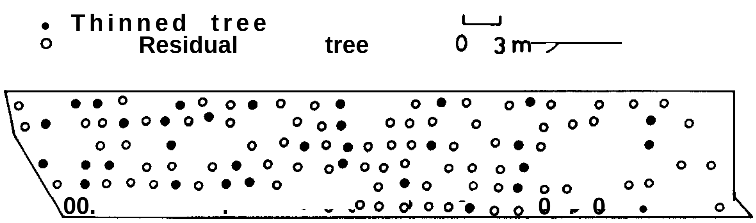

Fig. 2 Arrangement of stumpage at experimental plot (Sugi).

DISCUSSION

Where forest management is aimed at the production of a forest with smaller variation coefficient in diameter, it is necessary to devise methods of thinning to increase the Weibull parameter c. In searching for ways in which parameter c can be increased, two new methods of thinning were devised. It was also ascertained that these new methods are applicable to actual forests.

There is, however, the following problem in the new methods of thinning. As shown in Fig. 1 and 2, the trees after thinning are not evenly located. It is too early at this stage to dismiss this method as useless, as it is still unknown how these trees will perform. To answer this question, continuous surveys of the experimenal thinning plots set up during this experiment are essential.

Despite the abve-mentioned question, it is possible to devise a new method of thinning by using the Weibull distribution from the results of this study.

From the forest management point of view, the target diameter distribution should be first determined, and a forest with this diameter distribution should be created. For this purpose, the change in the diameter distribution with age or by thinning must be examined. The thinning method to attain the target forest must be determined based on these results. This problem cannot be solved by merely analyzing the diameter distribution change in figures and tables. It is only possible through quantitative analysis of the change in the diameter distribution by means of a probability density function well-suited for the diameter distribution. In this study, the Weibull distribution was found to be effective for solving this problem. This indicates that the Weibull distribution was valuable for studies on thinning methods, and that it is applicable for developing a new thinning method, which satisfy the demand of forest management. Thinning is the most crucial management technology. Furthermore, with the economic development, forest management is becoming more diversified. Therefore, it is probable that the Weibull distribution will be of great value for future studies, particularly on the systematization of the growing range of thinning plans.

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