# Determination of Stand Basal Area and Volume Yield Functions for KARAMATSU (Larix leptolepis Gordon) Plantations

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The compatible equations for predicting stand basal area and stand volume yield were described as a system of first order, ordinary differential equations for an even-aged KARAMATSU (*Larix leptolepis* Gordon) plantations. Prediction equations were fitted with growth data from thinned KARAMATSU permanent sample plots established in Kyushu University Experimental Forest of Hokkaido. The proposed method adequately estimated per-unit values for stand basal area and stand volume yield under consideration.

## INTRODUCTION

About one thousand hectares of KARAMATSU (*Larix leptolepis* Gordon) plantations have been established in Kyushu University Experimental Forest of Hokkaido. Some of these plantations have been thinned, while some others are being considered for thinning. Stand basal area growth and volume yield functions are needed to predict growth and yield following thinning in these KARAMATSU plantations.

The purpose of this paper is: (1) to present on a method for simultaneously estimating stand basal area and stand volume yield, (2) to present the equations that resulted from applying this method to data from permanent sample plots which were thinned growing in Kyushu University Experimental Forest of Hokkaido, and (3) to evaluate the derived equations in terms of standard error of estimates and average bias in predicted values.

## MATERIALS AND METHODS

### KARAMATSU Data Base

Data used for equation development and testing consisted of 20x0. l-ha permanent sample plots from KARAMATSU plantations established in Kyushu University Experimental Forest of Hokkaido.

The initial spacing of most plantations was from 2.0 to 2.2 m. Measurements immediately before and after thinning, as well as remeasurement at some future time, were available for each of these 20 plots. Number of remeasurements varied from plot to plot, ranging from 1 to 3. The length of

time from the measurement following thinning to a remeasurement varied from 3 to 6 years. The stands were thinned 1 or 2 times and, for the most part, thinnings were from below.  However, some predominant trees were removed to improve the quality of the leave stand.

Measurements were taken in stands from 13 to 32 years of age, growing on sites with site indices ranging from 16 to 23 m. Site indices were determined as the average height of dominant and codominant trees at age 30 using a site index curve especially developed for KARAMATSU plantations in Kyushu University Experimental Forest of Hokkaido (Nogami, 1984).

In each plot, measurements of diameter at breast height (*dbh*) were taken but only some tree heights were measured. Height corresponding to each *dbh* class was estimated for each plot measurement using a regression equation (height-diameter curve) estimated for that plot.

Basal area growth for each growth period was the instantaneous annual growth. This growth was estimated using the periodic annual growth that was observed over the growth period.  Volumes per unit area were computed by applying individual tree volume equation developed by the Author (Nogami, 1985) to the measured *dbh* and predicted total height values on each plot.

Age of each plot was assessed from the forest record of Kyushu University Experimental Forest of Hokkaido.

### Mathematical Growth and Yield Model

To develop the stand basal area and volume yield models presented here, a basic relationship is needed.  This is an equation relating stand basal area to stand volume.

Preliminary investigations indicate that the function :

$$V = a_1 B^{a_2} \tag{1}$$

relates stand volume to stand basal area.

where $V$ is stand volume (m³/ha), $B$ is stand basal area (m²/ha), and $a_1$, $a_2$ are parameters to be estimated.

The result of fitting this model by non-linear least squares is:

$$V = 3.1358\, B^{1.2657} \tag{2}$$

The coefficient of determination for estimated vs actual volume is 0.830. And standard error of equation (2) is 14.785.

The instantaneous rate of change of stand volume can be derived from (1) or (2) by differentiating with respect to stand age (A) :

$$dV/dA = a_1 a_2 B^{a_2-1}(dB/dA) = a_2 V \cdot B^{-1}(dB/dA) = 1.2657\, V \cdot B^{-1}(dB/dA) \tag{3}$$

*To* develop the volume yield equation (3) through integration, the form of $(dB/dA)$ in (3) must be determined.

As used in the present study, this model is:

$$(dB/dA) + B(\ln B)A^{-1} = b_0 B \cdot A^{-1} + b_1 B \cdot S \cdot A^{-1} + b_2 B \cdot A^{-3} + b_3 B \cdot S \cdot A^{-3} \tag{4}$$

where B: stand basal area (m²/ha), $A$: stand age, S: site index, and ln denotes logarithm to the base e.

Equation (4) is Clutter's original full model (Clutter, 1963). By applying unweighted least squares stepwise regression procedure to equation (4), the following reduced model was determined as best for the data used:

$$(dB/dA) = -B (\ln B) A^{-1} + 4.5061 B \cdot A^{-1} - 532.6000 B \cdot A^{-3} + 27.3212 B \cdot S \cdot A^{-3} \quad (5)$$

The coefficient of determination for estimated vs actual growth is 0.924, standard error of equation (5) is 0.395.

The $-B (\ln B) A^{-1}$ term was transposed to the right hand side of the equation after the model had been fitted to the data. That is, in order to estimate the coefficients $b_i, i = 0, \ldots, 3$, the dependent variable used was $(dB/dA) + B (\ln B) A^{-1}$ rather than basal area growth by itself.

### System of Stand Basal Area Growth and Yield Models

By employing the equations (3) and (5), the stand basal area and stand volume yield may be modeled by solving the following system of differential equations:

$$(dB/dA) = -B(\ln B) A^{-1} + 4.5061 B \cdot A^{-1} - 532.6000 B \cdot A^{-3} + 27.3212 B \cdot S \cdot A^{-3} \quad (6)$$

$$(dV/dA) = 1.2657 V \cdot B^{-1} (dB/dA) \qquad (7)$$

## RESULTS

Solutions to first-order systems may be either analytic, graphic, or numeric. Since algorithms for numerical solutions are readily available, the Runge-Kutta method is used to obtain solutions for the above system.

The following criteria were used to evaluate the models in terms of stand basal area and stand volume yield.

(1) standard error of the estimate (m²/ha and m³/ha)

$$\sqrt{\sum_{i=1}^{n} (B_i - \hat{B}_i)^2 / (n-1)}$$

$$\sqrt{\sum_{i=1}^{n} (V_i - \hat{V}_i)^2 / (n-1)}$$

(2) average bias (m²/ha and m³/ha)

$$\sum_{i=1}^{n} (B_i - \hat{B}_i)/n$$

$$\sum_{i=1}^{n} (V_i - \hat{V}_i)/n$$

where $\hat{B}_i, \hat{V}_i$ are predicted stand basal area, and volume yield at remeasurement point $i$, respectively. $B_i, V_i$ are the actual measurement of stand

basal area and volume yield at remeasurement point *i,* respectively. *n* is the number of remeasurement points.

Result is presented in Table 1. For the components of both stand basal area and volume yield, the governing equations provide values that closely fit the actual values in general.

It is apparent that the trend of underprediction when using the system of equations (6) and (7) for stand volume yield is occurred.

On the other hand, stand basal area is slightly overestimated when using this system.

**Table 1.** Values of standard error and average bias for estimates computed by solving the system of equations (6) and (7).

|  | Stand basal area (m²/ha) | Stand volume yield (m³/ha) |
|---|---|---|
| Standard error of estimate | 2.330 | 18.747 |
| Average bias | -1.155 | 4.983 |

## DISCUSSION

Clutter (1963) indicated that models were compatible when the algebraic form of the yield model can be derived by mathematical integration of the growth model.

Equations (6) and (7) are analytically compatible in this sense.

These provide also numerically consistent predictions of stand basal area and stand volume yield of thinned KARAMATSU plantations. Furthermore, the stand basal area and stand volume yield can be simultaneously computed by solving the system of differential equations (6) and (7). These equations are considered as a dynamic yield function revised periodically to incorporate results from new practices.

The method for simultaneously estimating stand basal area and stand volume yield illustrated in this paper should be exploited in the field of forest mensuration.

It should be emphasized, however, that the result presented here were derived from a small area in Kyushu University Experimental Forest of Hok-kaido. Due to the limited scope of the study, the findings must be viewed as suggestive rather than conclusive, although the methodology of derving the yield functions is general.

## ACKNOWLEDGEMENTS

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