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Measurement of Force by Strain Gauges

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Among the existing methods to measure a force, the strain gauge method would provide the highest accuracy and the most economical way. In this paper, the simple and reasonable measurements of not only a magnitude and a direction but a position of the line of action of the resultant force by the strain gauge method are presented as to the case that forces applied to a body can be composed to a single resultant force.

INTRODUCTION

A measurement of force with high degree of accuracy is of great importance in the mechanical analysis of the engineering problem. In particular, the measurement of soil resistance to various working machinery such as plow and rotary is indispensable for our field of the agricultural machinery.

Among the existing approaches to this aim, the strain gauge method may provide the highest accuracy and the most economical and practical way. Then various measurements of the force by the strain gauge method have been utilized, but most of them use a roller bearing so that the measured values involve the some errors due to the frictional resistance (e. g. Blight and Carlow, 1968; Scholte, 1964). Against these, a special load ring which enables to measure the force in two-dimensional state without a bearing has been recently developed (Kitani *et al.*, 1971; Dean and Yoerger, 1974). This method, however, is rather complicated and does not enable to measure a position of the line of action of the force.

In this paper, a general measurement of the force in one to three dimensional states is elucidated, which enables to measure not only a magnitude and a direction of the force but also a position of the line of action.

A KNOWN CASE OF A LINE OF ACTION

We first consider the measurement of force in the case that a line of action is known. In this case, there exist following three kinds of measurement for the magnitude of force.

1) *Tension or Compression*

Simple tensional or compressional state occurs in the axis when the line of action coincides with the axis for measurement as shown in Fig. la Maximum principal strains are given by

$$\varepsilon_s^0 = \frac{\sigma_s^0}{E} = \frac{F}{EA} , \tag{1}$$

$$\varepsilon_s^{\frac{\pi}{2}} = -\mu\varepsilon_s^0 , \tag{2}$$

where

$\varepsilon_s^0, \varepsilon_s^{\frac{\pi}{2}}$ = normal strains at $\theta=0$ and $\frac{\pi}{2}$,

σ_s^0 = normal stress at $\theta=0$,

θ = angle measured from the center line of the axis to the direction of the normal strain ε in a counter-clockwise direction,

F = applied force,

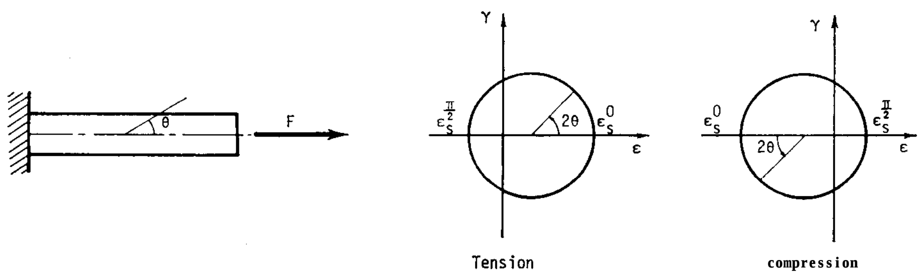
A = cross section area of the axis,

E = Young's modulus,

μ = Poisson's ratio.

Besides γ in Fig. lb designates a shear strain.

Throughout this paper, the superscript and the subscript are added to stresses and strains to indicate the directions of them and the conditions of acting force respectively.



(a) (b)

Fig. 1. State of strain in tension or compression.

Further refering to Fig. lb, the normal strain for an arbitrary angle θ is represented by

$$\varepsilon_s = \frac{\varepsilon_s^0 + \varepsilon_s^{\frac{\pi}{2}}}{2} + \frac{\varepsilon_s^0 - \varepsilon_s^{\frac{\pi}{2}}}{2} \cos 2\theta . \tag{3}$$

Substitution of Eqs. (1) and (2) to Eq. (3) leads to

$$\varepsilon_s = \frac{F}{2AE}\{(1-\mu) + (1+\mu)\cos 2\theta\} . \tag{4}$$

2) *Bending*

A bending state occurs in the axis when the line of action is perpendicular to the axis and intersects its center as shown in Fig. 2a. Maximum and

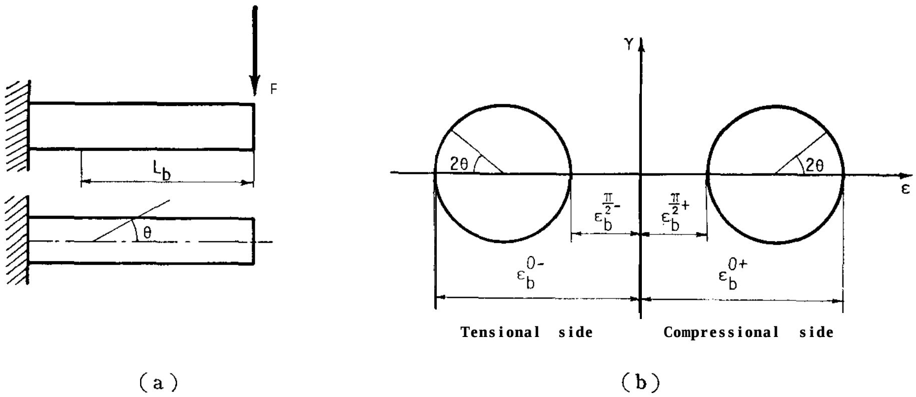


Fig. 2. State of strain in bending.

minimum strains in a position apart from the line of action with the distance L_b are given by

$$\varepsilon_b^{0\pm} = \pm\frac{\sigma_0}{E} = \pm\frac{M}{EZ_b} = \pm\frac{FL_b}{EZ_b}\ , \tag{5}$$

$$\varepsilon_b^{\frac{\pi}{2}\pm} = \mp\mu\varepsilon_b^{0\pm} = \mp\mu\frac{M}{EZ_b} = \mp\mu\frac{FL_b}{EZ_b} \tag{6}$$

where

$\varepsilon_b^{0\pm}$, $\varepsilon_b^{\frac{\pi}{2}\pm}$ = normal strains at $\theta=0$ and $\frac{\pi}{2}$ respectively, while the subscript + and – stands for a maximum tensional side and a maximum compressional side respectively.

Further, M $(=FL_b)$ is an applied moment and Z_b is a section modulus given by

$$Z_b = \frac{2I}{d}\ ,$$

where

I = second moment of area,

d = radius of the axis.

Refering to Fig. 2b, normal strain for an arbitrary angle θ is represented by

$$\varepsilon_b^{\pm} = \frac{\varepsilon_b^{0\pm} + \varepsilon_b^{\frac{\pi}{2}\pm}}{2} + \frac{\varepsilon_b^{0\pm} - \varepsilon_b^{\frac{\pi}{2}\pm}}{2}\cos 2\theta. \tag{7}$$

Substitution of Eqs. (5) and (6) to Eq. (7) leads to

$$\varepsilon_b^{\pm} = \pm\frac{FL_b}{2EZ_b}\{(1-\mu) + (1+\mu)\cos 2\theta\}\ . \tag{8}$$

3) *Torsion*

A torsional state occurs in the axis when the line of action does notintersect the axis and the scalar product of direction vectors of the line and

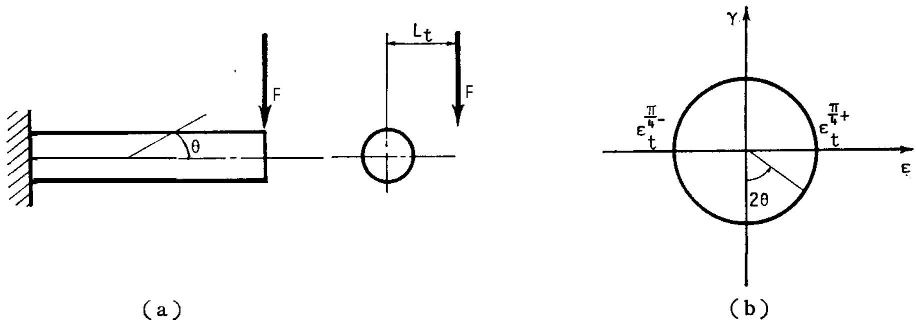


Fig. 3. State of strain in torsion.

that of the axis is zero as shown in Fig. 3a. Maximum and minimum princial strains are given by

$$\varepsilon_t^{\frac{\pi}{4}}=\sigma_t^{\frac{\pi}{4}}=\frac{\tau_t}{G}=\frac{T_t}{GZ_p}=\frac{FL_t}{GZ_p}, \tag{9}$$

$$\varepsilon_t^{\frac{\pi}{4}}=-\sigma_t^{\frac{\pi}{4}}=-\frac{\tau_t}{G}=-\frac{T}{GZ_p}=-\frac{FL_t}{GZ_p}, \tag{10}$$

where

$\varepsilon_t^{\frac{\pi}{4}-}, \varepsilon_t^{\frac{\pi}{4}+}$ = normal strains at $\theta=\pi/4$ and $-\pi/4$ respectively,

$\sigma_t^{\frac{\pi}{4}}$ = normal stress at $\theta=\pi/4$,

G = shearing modulus of elasticity,

T_t = torque applying to the axis,

L_t = distance between the line of action and the axis,

Z_p = polar moment of inertia of area.

Further, refering to Fig. 3b, a strain for an arbitrary angle θ is represented by

$$\varepsilon_t=\frac{FL_t}{GZ_p}\sin 2\theta. \tag{11}$$

A KNOWN CASE OF A DIRECTION OF LINE OF ACTION AND ONE OF PLANES INVOLVING IT.

We consider the measurement of force in the case that a direction of the line of action and one of planes involving it are known. In this case there are two measurements for a magnitude of force as follows.

1) *Bending*

Let the line of action be perpendicular to the axis as shown in Fig. 4, and let the axis also be involved in the known plane involving the line of action. The magnitude of force and the position of a line of action can be known by measuring the difference of strain values at two positions of the

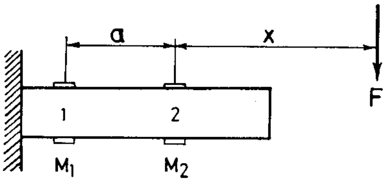


Fig. 4. Bending.

guages 1 and 2 (see Fig. 4). The relation of the applied force and the difference of bending moment at these positions are given as follows:

$$M_1 = F(a+x),$$
$$M_2 = Fx, \tag{12}$$

from which

$$F = \frac{M_1 - M_2}{a}, \tag{13}$$

where

M_1, M_2 = bending moment in the position of the gauge 1 and 2 respectively,

a = distance between the gauges 1 and 2.

x = distance between the gauge 1 and the line of action.

From Eq. (8), a relation of the normal strain and bending moment are given by

$$M_1 = F(a+x) = \pm\frac{2EZ_b}{(1-\mu)+(1+\mu)\cos 2\theta}\varepsilon_{b1}^{\pm} \tag{14}$$

and

$$M_2 = Fa = \pm\frac{2EZ_b}{(1-\mu)+(1+\mu)\cos 2\theta}\varepsilon_{b2}^{\pm}. \tag{15}$$

Then

$$M_1 - M_2 = Fx = \frac{2EZ_b}{(1-\mu)+(1+\mu)\cos 2\theta}(\varepsilon_{b1}^{\pm} - \varepsilon_{b2}^{\pm}). \tag{16}$$

2) ***Torsion***

Let the axis be perpendicular to the plane involving the line of action provided that there is an enough long distance between the line of action and the axis. Refering to Fig. 5, we can write

$$M = Fa, \tag{17}$$
$$T = Fx, \tag{18}$$

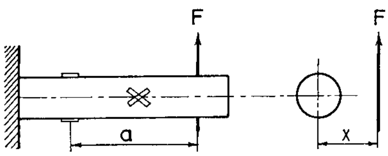


Fig. 5. Torsion.

from which

$$F = \frac{M}{a}, \tag{19}$$

$$x = \frac{Ta}{M}, \tag{20}$$

where

x = normal distance between the axis and the line of action,
a = normal distance between the gauge 1 and the known plane involving the line of action.

Further, according to Eq. (11), the relationship of the torque and the normal strain is expressed by following equation:

$$T = \frac{\varepsilon_t G Z_p}{\sin 2\theta}. \tag{21}$$

AN UNKNOWN CASE OF A LINE OF ACTION

We consider the measurement of force in the case that a direction of the line of action and its position are unknown. The measurement is explained separately in two and three-dimensional states as follows.

A) Two-dimensional state

1) Bending

In this case, the magnitude of force and the position of the line of action can be known by measuring moments at the positions of two gauges as shown in Fig. 6. The relations between the applied force and the moments are given by the following equations:

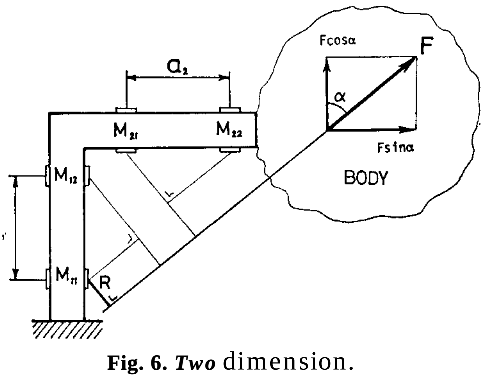


Fig. 6. ***Two*** dimension.

$$M_{11} - M_{21} = F\,a_1 \sin\alpha, \tag{22}$$

$$M_{21} - M_{22} = F\,a_2 \cos\alpha, \tag{23}$$

$$M_{11} = FR, \tag{24}$$

where

a_1, a_2 = distances between two gauges attached parallely to two axes which are perpendicular to each other,

α = angle between the direction of force F and the axis shown in Fig. 6,

R = distance between the line of action and the gauge 1.

From Eqs. (22), (23) and (24), we have

$$F=\sqrt{\left(\frac{M_{11}-M_{12}}{a_1}\right)^2+\left(\frac{M_{21}-M_{22}}{a_2}\right)^2}\,, \tag{25}$$

$$\alpha=\sin^{-1}\left(\frac{M_{11}-M_{12}}{a_1 F}\right)=\sin^{-1}\left\{\frac{1}{\sqrt{1+\left(\frac{a_1}{a_2}\cdot\frac{M_{11}-M_{12}}{M_{21}-M_{22}}\right)^2}}\right\}. \tag{26}$$

and

$$R=\frac{M_{11}}{F}=\frac{M_{11}}{\sqrt{\left(\frac{M_{11}-M_{12}}{a_1}\right)^2+\left(\frac{M_{21}-M_{22}}{a_2}\right)^2}}\,. \tag{27}$$

Differences of moments $M_{11}-M_{12}$ and $M_{21}-M_{22}$ are determined by the method explained in the known case of a line of action. Therefore, the magnitude of force F and its line of action can be determined easily by Eqs. (25), (26) and (27).

2) ***Bending and Torsion***

By combining the bending and torsion methods as shown in Fig. 7, we can measure the magnitude of force and the position of the line of action. This method has been reported by Matsuo (1961). Let bending moments produced by the components of force in the direction of x and y which are perpendicular to each other be denoted by Mx and My respectively. Further, let an angle measured from the x-direction to the direction of force in anticlockwise direction be denoted by β, and the distance between the axis and the line of action by a. Then we can write

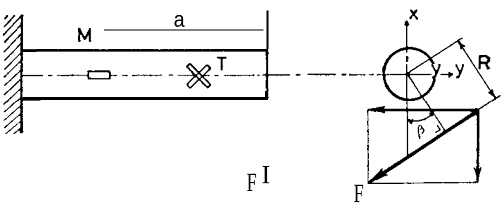


Fig. 7. Torsion and bending.

$$Mx=a\,F\sin\beta,$$
$$My=a\,F\cos\beta,$$
$$T=FR. \tag{28}$$

from which

$$F=\frac{1}{a}\sqrt{Mx^2+My^2}\,, \tag{29}$$

$$R = \frac{aT}{\sqrt{Mx^2 + My^2}}, \tag{30}$$

$$\beta = \sin^{-1}\left(\frac{Mx}{\sqrt{Mx^2 + My^2}}\right). \tag{31}$$

B) Three-dimensional state

The magnitude of force and the position of the line of action can be measured by the distributed gauges as shown in Fig. 8. Let the force vector and the position vector of the line of action be denoted by $\vec{F}$ and $\vec{R}$ respectively, which are perpendicular to each other.

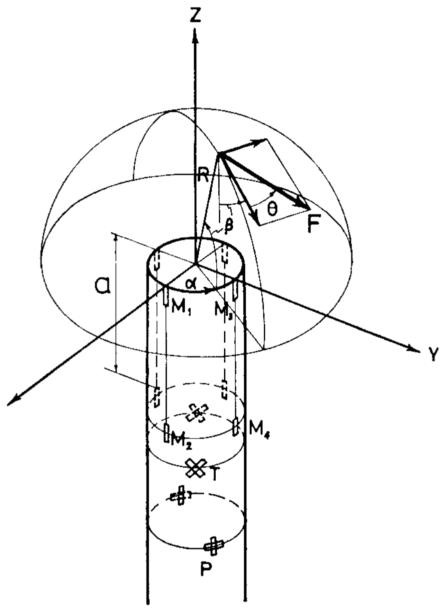


Fig. 8. Three demension.

Therefore, $\vec{F} \cdot \vec{R} = 0$

or $$FxRx + FyRy + FzRz = 0, \tag{32}$$

where Fx, Fy, Fz and Rx, Ry, Rz are components of $\vec{F}$ and $\vec{R}$ in direction of the co-ordinate axis x, y and z respectively.
From Eq. (32), the component Rx is given by

$$Rx = -\frac{FxRx + FyRy}{Fz}. \tag{33}$$

By the similar way to that in Eqs. (22) and (23), we have

$$Fx = \frac{M_1x - M_2x}{a}, \tag{34}$$

$$Fy = \frac{M_1 y - M_2 y}{a}, \tag{35}$$

where $M_1 x$, $M_2 x$ and $M_1 y$, $M_2 y$ are the bending moments produced in the positions 1 and 2 by the force components Fx and Fy respectively.
Further, it holds that

$$M_1 x = Fx\, Rz$$
$$= -\frac{Fx}{Rz}(Fx\, Rx + Fy\, Ry), \tag{36}$$

$$P \equiv Fz, \tag{37}$$
$$T \equiv -Fx\, Rx + Fy\, Ry, \tag{38}$$

where P is a mean normal stress applied in a cross section of the axis and T is a torsional moment occuring in the axis.

From Eqs. (36), (37), (38), components of $\vec{R}$ are given by

$$Rx = \frac{T\, Fy - M_1 x\, Fz}{Fx^2 + Fy^2},$$

$$Ry = -\frac{M_1 x\, Fy\, Fz + T\, Fx^2}{(Fx^2 + Fy^2)\, Fx}$$

$$Rz = \frac{T\, Fy - M_1 x\, Fz}{Fx^2 + Fy^2}. \tag{39}$$

The measurement of a force described in the above enables to measure not only a magnitude and a direction of force but a position of the line of action in a general and rational way without an error due to frictional resistance. When distributed loads act on a body in one or two-dimensional state, they can be composed to a single resultant force which can be measured by the method elucidated in this paper. In the three-dimensional state, however, they cannot be composed to a single resultant force, andt here fore it is necessary to measure not only three components of a force but those of a moment. A more general measurement of the force and the moment in the three-dimensional application of the distributed loads will be reported in another paper.

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