Studies on the Cultivating Characteristics of the Knife Edge Curve of Rotary Blade (Part I) : The Calculation of Operational Rotating Angle

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Studies on the Cultivating Characteristics of the Knife Edge Curve of Rotary Blade (Part I)

The Calculation of Operational Rotating Angle

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When the effects of the edge curve of rotary vertical straight blade on the cultivating characteristics are studied, especially concerned with the cultivating resistance, it is necessary to know the operational rotating angle for the analysis of experimental data. The operational rotating angle means the rotating angle of a blade from the beginning to the end of the soil cutting during one rotation. In this paper, as a preliminary knowledge for these studies, the theoretical equations for the calculation of operational rotating angle of a blade were deduced from the geometrical characteristics of a rotating shaft under the fixed and traveled condition. Subsequently, the calculating values which are obtained by the substitution of certain values of the cultivating condition to these equations were compared with the measured values which were obtained from the drawings. Consequently, it was shown that there was not a large difference between both values even in the case of the rotating shaft under traveling condition. In other words, this approximation is reasonable in establishing the theoretical equations of the operational rotating angle.

INTRODUCTION

The edge curve of a rotary blade is one of the most important design factors, because this edge curve is influential to the entwining phenomenon of grass or straw on it and also to the tillage resistance characteristics. In the study on this edge curve, the operational rotating angle θ_{op} is very useful for the correction of the experimental data, and its accurate value is required. Because the elapsed time of the cultivating torque curve obtained by a rotary blade on recording paper is generally longer than the theoretical operating time.

This angle θ_{op} is the rotating angle of a blade from beginning to end of soil cutting during one rotation of a blade, and it has been measured conventionally from the drawing by hand (Matsuo, 1961). This method generally requires much labour and time for design, furthermore it is not so much accurate.

In order to avoid these situations, therefore, the authors introduced the new theoretical calculating equations of the operational rotating angle in two cases of the fixed and traveling rotational shaft of a cultivating blade.

THEORETICAL CALCULATING EQUATIONS

The methods for introducing the theoretical calculating equations of the operational rotating angle were displayed in the two cases of the fixed and traveling rotation. They are expounded as follows:

A. The case of fixed rotation (Circumference)

The rotating shaft is fixed forward apart only the distance equal to the cultivating pitch from the former center of rotation. Therefore, the locus of a tip of a rotary blade is a circle. In this condition, the three cases occur and the equations of the operational rotating angle are varied according to each case.

1) The edge curve angle a is smaller than angle $\theta_a (\alpha < \theta_a)$.

In Fig. 1, the angle θ_a is created by a straight line connected from the center of rotation O_1 to the first contact point A and a horizontal line across the center of rotation. This angle is calculated geometrically as follows:

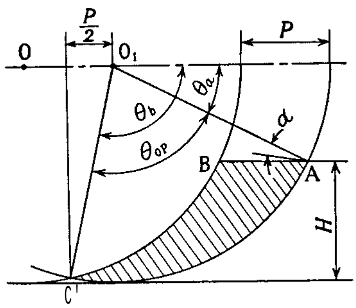

Fig. 1. The case of the fixed rotation and the edge curve angle $\alpha < \theta_a$.

$$\theta_a = \sin^{-1}\left(1 - \frac{H}{R}\right) \qquad (1)$$

where, H is the cultivating depth, R is the radius of rotation and equal to the length O_1A line.

When the angle θ_b is the rotating angle of the blade from a horizontal line to the point C' and P value is cultivating pitch, the following equation is obtained as

$$\cos(\pi - \theta_b) = \frac{P}{2R}$$

$$\therefore \ \theta_b = \cos^{-1}\left(-\frac{P}{2R}\right) \qquad (2)$$

In this case, from the definition of the operational rotating angle, the angle θ_{op} is calculated as follows:

$$\theta_{op} = \theta_b - \theta_a$$

$$=\cos^{-1}\left(-\frac{P}{2R}\right)-\sin^{-1}\left(1-\frac{H}{R}\right) \tag{3}$$

2) The edge curve angle α is larger than θ_a, besides smaller than θ_{r1} $(\theta_{r1}>\alpha>\theta_a)$.

Unlike the first case, the edge curve of a blade contacts with the soil surface at the point D which has to lie inside of the cultivating pitch AB (see Fig. 2). Then a segment of line from the center of rotation to the point B is a radius r_1 and is calculated as follows:

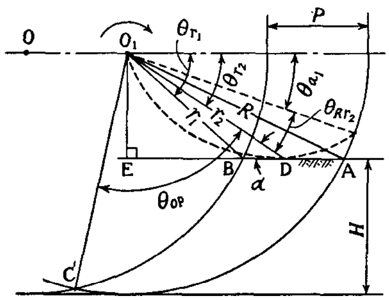

Fig. 2. The case of the fixed rotation and $\theta_{r1}>\alpha>\theta_a$.

$$\overline{AE}=R\cos\left\{\sin^{-1}\left(1-\frac{H}{R}\right)\right\}=\sqrt{2RH-H^2}$$

$$\overline{AB}=\frac{60v}{n}=\text{pitch},$$ v=forward speed of shaft,

n=rpm of shaft.

$$O_1E=R-H$$

$$\overline{EB}=\overline{AE}-\overline{AB}=\sqrt{2RH-H^2}-P$$

In the right-angled triangle $\triangle O_1EB$,

$$r_1=\{\overline{O_1E}^2+\overline{EB}^2\}^{1/2}$$

$$=\{R^2+P^2-2P\sqrt{H(2R-H)}\}^{1/2} \tag{4}$$

Then the angle θ_{r1} between a radius r_1 and a horizontal line across a point of the center of rotation can be calculated by the following equation :

$$\theta_{r1}=\sin^{-1}\left(\frac{R-H}{r_1}\right) \tag{5}$$

Conforming to argument which was explained at the introduction part, the edge curve of the vertical and straight blade portion was expressed by the spiral equation at the polar coordinate as follows: (Takahashi ***et al.,*** 1971).

$$r=r_0e^{\theta\cot\alpha}$$

$$r_0=\sqrt{(h_n)^2+\left(\frac{b}{2}\right)^2}$$

where r: The radius calculated by the variable θ in the above spiral function (see Fig. 3).

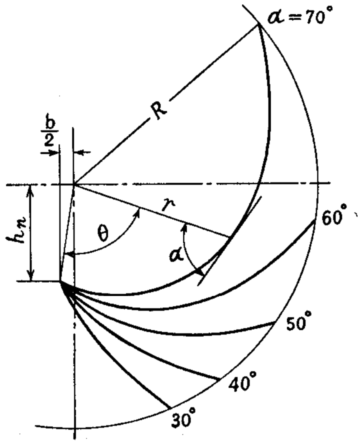

Fig. 3. The edge curves with edge curve angle α=30°~70°.

α: The edge curve angle (degree).
h_n: The radius of a gear box of rotational driving shaft (cm).
b: The blade width correspond to the distance between the cutting edge and the back edge of blade (cm).

When the radius r of a blade becomes the maximum radius R, the angle θ becomes $\theta_{\max}$, therefore, the above equation can be written as follows if these values are substituted into it.

$$R=r_0\theta^{\max\cdot\cot\alpha}$$

$$\therefore\ \theta_{\max}=\tan\alpha\cdot\log\frac{R}{(r_0)} \tag{6}$$

In Fig. 2, an intersection of the edge curve and the soil surface is the point D, then a segment of the line from this point to a center of rotation 0, is a radius r_2 and is calculated as follows:

$$r_2\ \sin\ \alpha=R-H$$

$$\therefore\ r_2=\frac{R-H}{\sin\alpha} \tag{7}$$

By the same method as the equation (6), we can obtain the following equation

$$\theta_{r2}=\tan\alpha\cdot\log\left(\frac{R-H}{r_0\sin\alpha}\right) \tag{8}$$

while

$$\theta_{Rr2}=\theta_{\max}-\theta_{r2}$$

$$=\tan\alpha\cdot\log\left(\frac{R\sin\alpha}{R-H}\right) \tag{9}$$

thus, the angle θ_{a1} (see Fig. 2) is shown as follows:

$$\theta_{a1}=\alpha-\theta_{Rr2}$$

$$\therefore\ \theta_{a1}=\alpha-\tan\alpha\cdot\log\left(\frac{R\sin\alpha}{R-H}\right) \tag{10}$$

Therefore, the operational rotating angle θ_{op} can be written as:

$$\theta_{op}=\theta_b-\theta_{a1} \tag{11}$$

By substituting the equations (2) and (10) into the equation (11),

$$\theta_{op}=\cos^{-1}\left(-\frac{P}{2R}\right)-\alpha+\tan\alpha\cdot\log\left(\frac{R\sin\alpha}{R-H}\right) \tag{12}$$

3) The edge curve angle a is larger than $\theta_{r1}(\alpha>\theta_{r1})$.

In this case, the first contact point of the edge curve of a blade with the soil surface is the point B (see Fig. 4), and we can obtain the following equation in the same manner as the case of the equation (9).

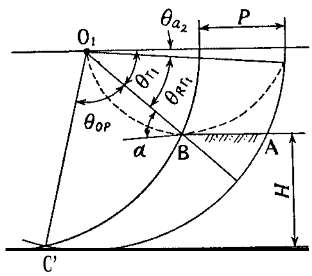

Fig. 4. The case of the fixed rotation and the edge curve angle $\alpha>\theta_{r1}$.

$$\theta_{Rr1}=\tan\alpha\cdot\log\left(\frac{R}{r_1}\right) \tag{13}$$

and

$$\theta_{a2}=\theta_{r1}-\theta_{Rr1} \tag{14}$$

By the substitution of equations (5) and (13) into the above equation,

$$\theta_{a2}=\sin^{-1}\left(\frac{R-H}{r_1}\right)-\tan\alpha\cdot\log\left(\frac{R}{r_1}\right) \tag{15}$$

therefore, the operational rotating angle θ_{op} is calculated as:

$$\theta_{op}=\theta_b-\theta_{a2} \tag{16}$$

$$=\cos^{-1}\left(-\frac{P}{2R}\right)-\sin^{-1}\left(\frac{R-H}{r_1}\right)+\tan\alpha\cdot\log\left(\frac{R}{r_1}\right) \tag{16$'$}$$

B. The case of the rotating shaft with traveling (Trocoid curve)

Usually the locus of the tip of a rotary blade draws a trocoid curve during operation. The determination of an intersection of the edge curve and the soil surface is impossible. Therefore, the calculating equation of the operational rotating angle has to be found on the assumption that the radius r_1 is the same as the case of fixed rotation, because the calculating results of these equations are approximated the values measured from the drawing by hand as described below.

The angle θ_b is one of the most important factors for the calculating equation of the operational rotating angle. In this case, it's shown as follows (see Fig. 5) :

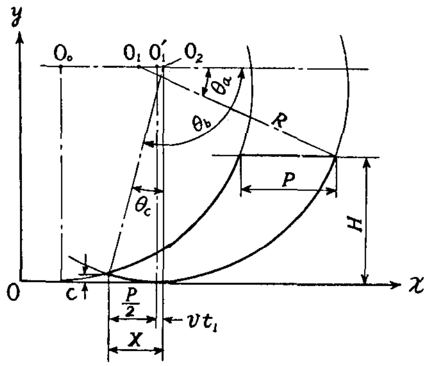

Fig. 5. The case of the traveling rotation with a trocoid curve.

$$\theta_b=\frac{\pi}{2}+\theta_c \tag{17}$$

From the past report (Lukyanov, 1971)

$$\mathrm{x}=\frac{P}{2}+vt_1 \tag{18}$$

and

$$\theta_c=\cos^{-1}\left(\text{ i - f}\right) \tag{19}$$

The required rotating time for the angle π is $\frac{30}{n}$ seconds. Therefore, for the angle θ_c it is

$$t_1=\frac{30}{n\pi}\cos^{-1}\left(1-\frac{c}{R}\right)$$

$$\therefore\ vt_1=\frac{30v}{n\pi}\cos^{-1}\left(1-\frac{c}{R}\right) \tag{20}$$

The above equation (20) is substituted in the equation (18) and is simplified as

$$X=\frac{30v}{n\pi}\cos^{-1}\left(1-\frac{c}{R}\right)+\frac{30v}{n} \tag{21}$$

Further, refering to Fig. 4

$$X=\{R^2-(R-c)^2\}^{1/2} \tag{22}$$

Combining equations (21) and (22) yields:

$$30\frac{v}{n}\left\{1+\frac{\cos^{1}\left(1-\frac{c}{R}\right)}{\pi}\right\}=\{2Rc-c^2\}^{1/2}$$

if $\qquad \frac{c}{R}=M_c$ and $\frac{v}{n}=\mu$

$$\therefore\ \mu=\frac{R\pi\{M_c(2-M_c)\}^{1/2}}{30\{\pi+\cos^{-1}(1-M_c)\}} \tag{23}$$

According to the equation (23), the relations between M_c and μ are easily obtained. For example, the calculating result under the following temporary cultivating condition is shown in Fig. 6. The forward speed v is 9.6 cm/s, the revolutionper minute of shaft n is 100 rpm, the maximum radius R is 22 cm and the cultivating depth H is 10 cm. The length of c may be obtained from the graph of Fig. 6. Therefore, the value θ_b of the equation (17) is shown as follows:

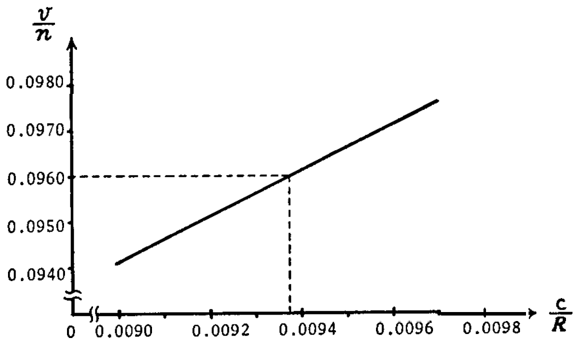

Fig. 6. The relation of $\frac{v}{n}$ and $\frac{c}{R}$.

$$\theta_b=\frac{\pi}{2}+\cos^{-1}\left(1-\frac{c}{R}\right) \tag{24}$$

The operational rotating angle θ_{op} of the case of traveling rotation is also shown as the equation (3), but the angle θ_b is calculated from the equation (24), so that the theoretical equations for calculation of the operational rotating angle θ_{op} are established in the three cases as follows:

1) The edge curve angle a is smaller than the angle θ_a $(\alpha<\theta_a)$.

According to the equations (3) and (24), the angle θ_{op} can be written as follows :

$$\theta_{op}=\frac{\pi}{2}+\cos^{-1}\left(1-\frac{c}{R}\right)-\sin^{-1}\left(1-\frac{H}{R}\right)$$

$$=\cos^{-1}\left(1-\frac{c}{R}\right)+\cos^{-1}\left(1-\frac{H}{R}\right) \tag{25}$$

2) The edge curve angle a is larger than θ_a, besides smaller than θ_{r1}

$(\theta_{r1}>\alpha>\theta_a)$.

By substituting the equations (10) and (24) into the equation (11), the angle θ_{op} is expressed as follows:

$$\theta_{op}=\frac{\pi}{2}+\cos^{-1}\left(1-\frac{c}{R}\right)-\alpha+\tan\alpha\cdot\log\left(\frac{R\sin\alpha}{R-H}\right) \tag{26}$$

3) The edge curve angle α is larger than θ_{r1} $(\alpha>\theta_{r1})$.

By the substitution of the equations (15) and (24) into the equation (16), the angle θ_{op} is

$$\theta_{op}=\cos^{-1}\left(1-\frac{c}{R}\right)+\cos^{-1}\left(\frac{R-H}{r_1}\right)+\tan\alpha\cdot\log\left(\frac{R}{r_1}\right) \tag{27}$$

C. The rearranging results

The previous theoretical equations are arranged as follows:

(I). The case of fixed rotation

1) $\alpha<\theta_a$

$$\theta_{op}=\cos^{-1}\left(-\frac{P}{2R}\right)-\sin^{-1}\left(1-\frac{H}{R}\right)$$

2) $\theta_{r1}>\alpha>\theta_a$

$$\theta_{op}=\cos^{-1}\left(-\frac{P}{2R}\right)-\alpha+\tan\alpha\cdot\log\left(\frac{R\sin\alpha}{R-H}\right)$$

3) $\alpha>\theta_{r1}$

$$\theta_{op}=\cos^{-1}\left(-\frac{P}{2R}\right)-\sin^{-1}\left(\frac{R-H}{r_1}\right)+\tan\alpha\cdot\log\left(\frac{R}{r_1}\right)$$

(II). The case of the traveling rotation

1) $\alpha<\theta_a$

$$\theta_{op}=\cos^{-1}\left(1-\frac{c}{R}\right)+\cos^{-1}\left(1-\frac{H}{R}\right)$$

2) $\theta_{r1}>\alpha>\theta_a$

$$\theta_{op}=\frac{\pi}{2}+\cos^{-1}\left(1-\frac{c}{R}\right)-\alpha+\tan\alpha\cdot\log\left(\frac{R\sin\alpha}{R-H}\right)$$

3) $\alpha>\theta_{r1}$

$$\theta_{op}=\cos^{-1}\left(1-\frac{c}{R}\right)+\cos^{-1}\left(\frac{R-H}{r_1}\right)+\tan\alpha\cdot\log\left(\frac{R}{r_1}\right)$$

DISCUSSION

In the foregoing calculating equations, there was not a special assumption in the case of fixed rotation, so that the calculating values were expected to be accurate. However, in the case of the traveling rotation, it was assumed, as mentioned above, that a radius r_1 was the same as the case of fixed rota-

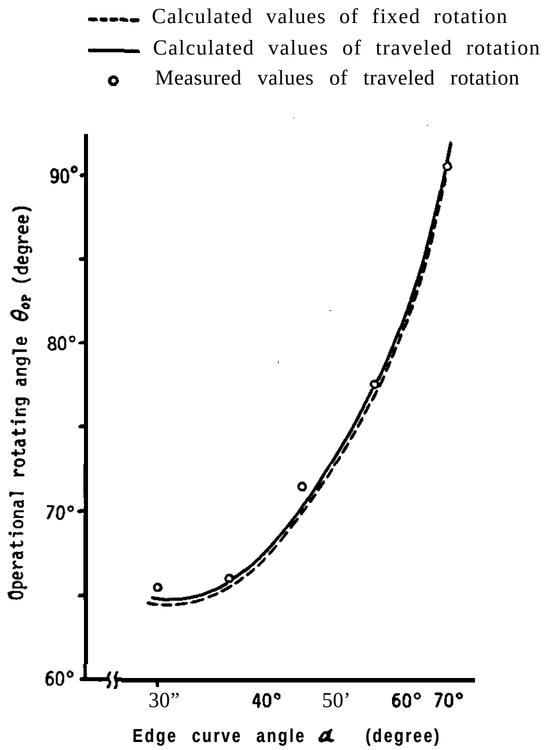

Fig. 7. The comparison of calculated and measured values at the fixed and traveled rotation.

tion. The calculated results were compared with the measured results, and its accuracy was inspected.

In order to prove the appropriateness of this assumption, the equations of the operational rotating angle θ_{op} were calculated with the various edge curve angles from 30° to 70’ under the foregoing conditions: $R=22$ cm, $v=9.6$ cm/s, $n=100$ rpm, $H=10$ cm, $h_n=10$ cm and $b=2.4$ cm. These calculating results were indicated in Table 1.

Table 1. The comparison of calculated and measured values of operational rotating angle.

Edge curve angle α	Operational rotating angle θ_{op}			
	Fixed rotation (Circle)		Traveling rotation (Trocoid curve)	
	Calculated values	Measured values	Calculated values	Measured values
30”	64.5”	64.5”	64.8’	65.5”
40”	65.4”	65.4”	65.8’	66.0’
50°	69.9”	69.9”	70.2”	71.5’
60°	77.0’	77.0”	77.4”	77.5”
70’	90.4”	90.4’	90.8”	90.5”

In addition, according to Fig. 7, it was concluded that the calculated and measured values of the operational rotating angle θ_{op} were naturally equal in the case of fixed rotation, but there were little difference between the both values of angle θ_{op} in the case of traveling rotation, so that this preceding assumption is adequate for the establishing theoretical equations of the operational rotating angle θ_{op}.

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