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<https://hdl.handle.net/2324/2344834>

出版情報 : 2019-10-07
バージョン :
権利関係 :

Note on a geometrical formula for the Hall conductivity in metals

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(October 7, 2019)

Abstract

The Tsuji formula for the Hall conductivity in metals is discussed in Haldane's framework.

The Tsuji formula [1] is widely known as a geometrical formula for the Hall conductivity in metals under weak magnetic field. Since it was derived under the assumption of the cubic symmetry, Haldane [2] tried to eliminate the assumption. Here we discuss the Tsuji formula using Haldane's framework. However, our conclusion is different from Haldane's. The details¹ are described in <http://hdl.handle.net/2324/1957531>.

In usual notation the weak-field DC Hall conductivity tensor σ^{xy} per spin is given by [1, 2]

$$\sigma^{xy} = e^3 B \int \frac{dS}{(2\pi)^3} (v^x, v^y) \begin{pmatrix} M_{yy}^{-1} & -M_{yx}^{-1} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} v^x \\ v^y \end{pmatrix} \frac{\tau^2}{|\vec{v}|}, \quad (1)$$

for the Fermi surface contribution in metals. Throughout this note we only consider the contribution from a single sheet of the Fermi surface. Here the magnetic field is chosen as $\vec{B} = (0, 0, B)$. The quasi-particle velocity $\vec{v} = (v^x, v^y, v^z)$ and the effective mass tensor $M_{\alpha\beta}$ are given by the derivative of the quasi-particle energy ε : $v^\alpha = \partial\varepsilon/\partial k^\alpha$ and $M_{\alpha\beta}^{-1} = \partial^2\varepsilon/\partial k^\alpha\partial k^\beta$. Since the contribution of the derivative of τ does not appear in the antisymmetric tensor $(\sigma^{xy} - \sigma^{yx})/2$, we have dropped it.

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¹This note is a nutshell of our previous note, <http://hdl.handle.net/2324/1957531>.

Experimentally σ^{xy} is obtained from the measurement where we measure the current in x -direction under the electric field in y -direction and the magnetic field in z -direction. If we measure the current in y -direction under the electric field in x -direction and the magnetic field in z -direction, we obtain σ^{yx} described as

$$\sigma^{yx} = e^3 B \int \frac{dS}{(2\pi)^3} (v^x, v^y) \begin{pmatrix} 0 & 0 \\ -M_{xy}^{-1} & M_{xx}^{-1} \end{pmatrix} \begin{pmatrix} v^x \\ v^y \end{pmatrix} \frac{\tau^2}{|\vec{v}|}. \quad (2)$$

Haldane [2] introduced the symmetric tensor $e^3 B \gamma_{zz} \equiv (\sigma^{xy} + \sigma^{yx})/2$. Eq. (1) and Eq. (2) lead to

$$\gamma_{zz} = \frac{1}{2} \int \frac{dS}{(2\pi)^3} (v^x, v^y) \begin{pmatrix} M_{yy}^{-1} & -M_{yx}^{-1} \\ -M_{xy}^{-1} & M_{xx}^{-1} \end{pmatrix} \begin{pmatrix} v^x \\ v^y \end{pmatrix} \frac{\tau^2}{|\vec{v}|}. \quad (3)$$

Other symmetric tensors are introduced in the same manner as $e^3 B \gamma_{xx} \equiv (\sigma^{yz} + \sigma^{zy})/2$ and $e^3 B \gamma_{yy} \equiv (\sigma^{zx} + \sigma^{xz})/2$. As shown in the following the geometrical nature is captured by these symmetric tensors. It should be noted that our result, Eq. (3), is different from Haldane's [2]. The difference arises from the following fact. While Eq. (3) contains $(\partial v^x / \partial k^y) / |\vec{v}|$, Haldane erroneously uses $\partial(v^x / |\vec{v}|) / \partial k^y$ instead.

The target of our geometrical description is the mean curvature H of the Fermi surface. It is given by

$$2H = \frac{1}{|\vec{v}|^3} \cdot \left[\varepsilon_x \varepsilon_x (\varepsilon_{yy} + \varepsilon_{zz}) + \varepsilon_y \varepsilon_y (\varepsilon_{zz} + \varepsilon_{xx}) + \varepsilon_z \varepsilon_z (\varepsilon_{xx} + \varepsilon_{yy}) \right. \\ \left. - \varepsilon_x (\varepsilon_y \varepsilon_{yx} + \varepsilon_z \varepsilon_{zx}) - \varepsilon_y (\varepsilon_x \varepsilon_{xy} + \varepsilon_z \varepsilon_{zy}) - \varepsilon_z (\varepsilon_x \varepsilon_{xz} + \varepsilon_y \varepsilon_{yz}) \right],$$

for any shape of the Fermi surface. Here we have used the notations $\varepsilon_\alpha \equiv v^\alpha$ and $\varepsilon_{\alpha\beta} \equiv M_{\alpha\beta}^{-1}$.

The geometrical information in our master equation, Eq. (3), is represented by h_{zz} as

$$\gamma_{zz} = \int \frac{dS}{(2\pi)^3} h_{zz} \tau^2,$$

with

$$h_{zz} = \frac{1}{2|\vec{v}|} (\varepsilon_x \varepsilon_x \varepsilon_{yy} + \varepsilon_y \varepsilon_y \varepsilon_{xx} - \varepsilon_x \varepsilon_y \varepsilon_{yx} - \varepsilon_y \varepsilon_x \varepsilon_{xy}).$$

Using

$$h_{xx} = \frac{1}{2|\vec{v}|} (\varepsilon_y \varepsilon_y \varepsilon_{zz} + \varepsilon_z \varepsilon_z \varepsilon_{yy} - \varepsilon_y \varepsilon_z \varepsilon_{zy} - \varepsilon_z \varepsilon_y \varepsilon_{yz}),$$

and

$$h_{yy} = \frac{1}{2|\vec{v}|} (\varepsilon_z \varepsilon_z \varepsilon_{xx} + \varepsilon_x \varepsilon_x \varepsilon_{zz} - \varepsilon_z \varepsilon_x \varepsilon_{xz} - \varepsilon_x \varepsilon_z \varepsilon_{zx}),$$

additionally, we obtain

$$\gamma_{zz} + \gamma_{xx} + \gamma_{yy} = \int \frac{dS}{(2\pi)^3} H l^2, \quad (4)$$

with $l^2 = |\vec{v}|^2 \tau^2$. Our result, Eq. (4), is applicable to any shape of the Fermi surface. In the case of cubic symmetry Eq. (4) is reduced to the Tsuji formula [1, 2]

$$\gamma_{zz} = \gamma_{xx} = \gamma_{yy} = \int \frac{dS}{(2\pi)^3} \frac{H}{3} l^2.$$

Experimentally γ_{cc} is obtained from the measurements of σ^{ab} and σ^{ba} where $(c, a, b) = (z, x, y), (x, y, z), (y, z, x)$. By summing six experimental results with different configurations we can use Eq. (4).

References

- [1] M. Tsuji, J. Phys. Soc. Jpn. **13**, 979 (1958).
- [2] F. D. M. Haldane, arXiv:cond-mat/0504227v2.