Hypothetical Extraction Method versus Betweenness Centrality for CO2 Emission Attribution in a Supply Chain

Tokito, Shohei
Kyushu University

出版情報：2019-06-27
バージョン：
権利関係：

Title: Hypothetical Extraction Method versus Betweenness Centrality for CO_2 Emission Attribution in a Supply Chain

Author affiliation: Shohei TOKITO

Faculty of Economics, Kyushu University, Fukuoka, Japan

Acknowledgement:

I thank the anonymous referees for their helpful comments on this manuscript. This research was supported by JSPS KAKENHI Grant Number JP17J03786. All errors are the authors'.

1. Introduction

The field of Input-Output Analysis (IOA) was developed as the empirical analysis of both the ripple effect induced by changes in the final demand and the interdependencies between different industrial sectors since the 1950s (Leontief, 1936, 1941; Rosenblatt, 1957; Chenery and Watanabe, 1958; Miller and Blair, 2009). Input-output structural analysis allows us to understand complex input-output networks. Previous studies have suggested indicators of key sectors and transactions that affect the whole network environmentally and economically using complex input-output networks.

With the underlying idea that sectors with strong linkages are in the position to induce the outputs expansion of other sectors, the linkage indicators as measuring interdependencies of sectors have been becoming a common tool. As key sector analysis using IOA, estimation of the "power of dispersion" and "sensitivity of dispersion" has been suggested (Rasmussen, 1956; Hirschman, 1958; Hazari, 1970; Nagashima *et al.,* 2017; Nakano *et al.,* 2017). These indicators focus on the linkage between sectors. The power of dispersion reflects the backward linkage effect, and the sensitivity of dispersion reflects the forward linkage effect. There are various methods for detecting the key sector by analyzing changes in the input structure (e.g., Casler and Hadlock, 1997; Wiebe, 2018). The hypothetical extraction method (Meller and Marfan, 1981; Cella, 1984; Dietzenbacher, 1993, 2013) is to quantify how much the total output of an economy would decrease if a particular sector were removed from that economy. The hypothetical extraction method can be used to assess the influence of backward and forward linkage for a sector.

Inter-industry transactions matrix can be interpreted as an adjacency matrix showing supply chain complexity of industries. Graph theoretic concepts have been widely used to highlight and visualize the important transactions in the supply chain complexity (e.g., Rosenblatt, 1957). Qualitative Input-Output Analysis (QIOA) has been proposed to visualize the relation between industrial sectors (Holub and Schnabl, 1985; Ghosh and Roy, 1998; Weber and Schnabl, 1998). In addition, key sector analysis using the centrality indicator (Freeman, 1977, 1978) from social network analysis (Friedkin and Johnsen, 1990; Muniz *et al*., 2008; Kagawa *et al.,* 2009; Brachert et al., 2016; Chen et al., 2017; Du et al., 2017; Duang and Jiang, 2018) and cluster analysis (Kagawa *et al.,* 2013a, 2013b, 2015; Tokito *et al.,* 2016; Tokito, 2018) has been applied to model the intermediate goods flow network.

Structural path analysis (Defourny and Thorbecke, 1984; Trelor, 1997; Lenzen, 2003; Suh, 2004; Peters and Hertwitch, 2006; Wood and Lenzen, 2009; Oshita, 2012; Nagashima *et al.,* 2017), betweenness-based emission analysis (Liang *et al.,* 2016) and edge betweenness centrality analysis (Hanaka *et al*.,2017) have been used to identify important sectors and transactions from the I-O network. Structural path analysis is based on economic influence and its transmission throughout the input-output system. Liang *et al*. (2016) suggested betweenness-based emission analysis by applying the concept of node betweenness (Freeman, 1977, 1978) into structural path analysis. Betweenness-based emissions represent both the positional and quantitative importance of a sector in the supply chain network. Hanaka *et al*. (2017) expanded the node betweenness-based emissions to edge betweenness-based emissions and suggested the use of the edge betweenness centrality.

Note that both the hypothetical extraction method and betweenness analysis focus on the output from all supply chain paths passing through the sector. However, betweenness centrality is weighted by the number of times the sector appears in the supply chain path. Thus, sectors which have higher betweenness centrality indicators will appear many times in a supply chain. Therefore, as in the policy discussions of Liang *et al.* (2016), Hanaka *et al.* (2017) and Tokito (2018), climate policies for the targeted sector and transaction which have higher betweenness centrality (e.g., reduction in emission intensity) can be implemented effectively using this information to reduce the emissions embedded in the supply chain network.

The novelty in this study comprises the following two points. First, I focused on the relationship between the various I-O structural analysis methods mentioned above and in particular, I derived an analytic expression for the relationship between hypothetical extraction analysis (Meller and Marfan, 1981; Cella, 1984; Dietzenbacher, 1993, 2013; Miller and Blair, 2009) and betweenness centrality analysis (Liang *et al.,* 2016; Hanaka *et al.,* 2017; Tokito, 2018). Second, using two widely used databases, Eora (Lenzen *et al.,* 2012, 2013) and WIOD (Dietzenbacher *et al.,* 2013; Timmer *et al.,* 2015), I analyzed how different the "important" sectors detected by two similar approaches, hypothetical extraction analysis and betweenness centrality analysis, are. When the results of these methods differ greatly, the importance of a sector that is high betweenness sector in the supply chains is ignored. Thus, we can say that betweenness centrality analysis is more appropriate for using the structure of a supply chain network to determine policies to reduce emissions.

The remainder of this paper is organized as follows: Sections 2 and 3 explain the methodology and data used here, Section 4 present the results, and Section 5 presents the discussion and conclusions.

2. Methodology

2.1 Leontief model

An intermediate input from industry *i* to industry *j* is denoted by $z_{ij}(i,j=1,\dots,N)$. The final demand for industry *i* is denoted by $f_i(i=1,\dots,N)$. Thus, the total output q_i of industry *i* is defined as $q_i=\sum_{j=1}^{N} z_{ij}+f_i$. If intermediate input coefficients $a_{ij}=z_{ij}/q_j$ are defined, the input coefficient matrix $\mathbf{A}=(a_{ij})$ is constructed as

$$\mathbf{A}=\begin{pmatrix} a_{11} & \cdots & a_{1i} & \cdots & a_{1N} \\ \vdots & \ddots & & & \vdots \\ a_{i1} & & a_{ii} & & a_{iN} \\ \vdots & & & \ddots & \vdots \\ a_{N1} & & a_{Ni} & & a_{NN} \end{pmatrix}=\begin{pmatrix}\mathbf{a}_1^c & \cdots & \mathbf{a}_i^c & \cdots & \mathbf{a}_N^c\end{pmatrix}=\begin{pmatrix}\mathbf{a}_1^r \\ \vdots \\ \mathbf{a}_i^r \\ \vdots \\ \mathbf{a}_N^r\end{pmatrix}$$

where $\mathbf{a}_i^c$ is the $(N\times 1)$ column vector representing the input coefficient from all sectors to sector i and $\mathbf{a}_i^r$ is the $(1\times N)$ row vector representing the input coefficient from sector i to all sectors. The first-order indirect economic influence induced by the final demand for industry *i* is calculated as $\sum_{u=1}^{N} a_{ui}f_i={}^t\,\mathbf{I}\mathbf{a}_i^c f_i$, in which $\mathbf{I}$ is the $(N\times 1)$ column vector whose all elements are 1. Similarly, the second economic influence induced by the final demand of country *s* from industry *i* in a country is calculated as $\sum_{v=1}^{N}\sum_{u=1}^{N} a_{vu}a_{ui}f_i=\mathbf{IAa}_i^c f_i$. The Leontief model, $x=\mathbf{e}(\mathbf{E}-\mathbf{A})^{-1}\mathbf{f}=\mathbf{eLf}$, can show the full extent of the final demand that directly and indirectly generates the industrial environmental burden *x*. Here, $\mathbf{e},\mathbf{E},\mathbf{f}$ are the emission intensity vector, the identity matrix and final demand vector, respectively, and the Leontief inverse, $\mathbf{L}=(\mathbf{E}-\mathbf{A})^{-1}$ is the direct and indirect requirement matrix. The Leontief inverse involves all ripple effects as

$$\begin{aligned} x &= \mathbf{e}(\mathbf{E}-\mathbf{A})^{-1}\mathbf{f} \\ &= \mathbf{e}(\mathbf{E}+\mathbf{A}+\mathbf{A}^2+\mathbf{A}^3+\dots)\mathbf{f} \\ &= \sum_{i=1}^{N} e_i f_i+\sum_{i=1}^{N}\sum_{j=1}^{N} e_i a_{ij} f_j+\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} e_i a_{ij}a_{jk} f_k+\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}\sum_{l=1}^{N} e_i a_{ij}a_{jk}a_{kl} f_l+\dots \end{aligned} \tag{1}$$

where e_i, f_i are the i^{th} element of the emission intensity vector **e** and the final demand vector **f**, respectively, and a_{ij} is the (i,j)th element of the technical coefficient matrix **A**. The Leontief inverse $\mathbf{L}=(l_{ij})$ is constructed as

$$\mathbf{L}=\begin{pmatrix} l_{11} & \cdots & l_{1j} & \cdots & l_{1N} \\ \vdots & \ddots & & & \vdots \\ l_{i1} & & l_{ij} & & l_{iN} \\ \vdots & & & \ddots & \vdots \\ l_{N1} & & l_{Nj} & & l_{NN} \end{pmatrix}=\begin{pmatrix}\mathbf{l}_1^c & \cdots & \mathbf{l}_j^c & \cdots & \mathbf{l}_N^c\end{pmatrix}=\begin{pmatrix}\mathbf{l}_1^r \\ \vdots \\ \mathbf{l}_i^r \\ \vdots \\ \mathbf{l}_N^r\end{pmatrix}$$

2.2 Hypothetical extraction method

2.2.1 Sector hypothetical extraction method

Using this hypothetical extraction method, we can calculate the impact arising from both the forward and backward direct linkage of a sector. In this paper, I calculate the environmental impact of the case that a specific sector i is extracted from the economy. The environmental sector extraction impact x^i can be calculated as follows:

$$x^i = x-\bar{x}^i \tag{2}$$

where, x is the total emission from the economy that all sectors exist in, and $\bar{x}^i$ is the total emission from the economy that a specific sector i is extracted from. $\bar{x}^i$ can be obtained using the "extracted" input coefficient matrix $\bar{\mathbf{A}}^i$ as

$$\begin{aligned} \bar{x}^i &= \mathbf{e}\left\{(\mathbf{E}-\bar{\mathbf{A}}^i)^{-1}-\mathbf{J}_{ii}\right\}\mathbf{f} \\ &= \mathbf{e}\bar{\mathbf{L}}^i\mathbf{f}-e_i f_i \end{aligned}$$

In which, $\bar{x}^i$ is the total emission from the supply chain paths not passing through sector *i*, and $\mathbf{J}_{uv}$ is the matrix whose (u,v)th element is 1 and the other elements are zero. An element of $\bar{\mathbf{A}}^i = (\bar{a}^i_{uv})$ is as:

$$\bar{a}^i_{uv} = \begin{cases} a_{uv} & u \neq i \wedge v \neq i \\ 0 & u = i \vee v = i \end{cases}$$

From the following equation (3), the environmental sector extraction impact x^i can be interpreted as the total emissions associated with the supply chain paths passing through sector *i*.

$$\begin{aligned} x^i &= x - \bar{x}^i \\ &= \mathbf{eLf} - \left(\mathbf{e}\bar{\mathbf{L}}^i\mathbf{f} - e_i f_i\right) \\ &= \mathbf{e}\left(\mathbf{L} - \bar{\mathbf{L}}^i\right)\mathbf{f} + e_i f_i \\ &= \mathbf{e}\left\{\left(\mathbf{E} + \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \ldots\right) - \left(\mathbf{E} + \bar{\mathbf{A}}^i + \left(\bar{\mathbf{A}}^i\right)^2 + \left(\bar{\mathbf{A}}^i\right)^3 + \ldots\right)\right\}\mathbf{f} + e_i f_i \\ &= e_i f_i + \sum_{u=1}^{N}\sum_{v=1}^{N}\left(e_u a_{uv} f_v - e_u \bar{a}^i_{uv} f_v\right) + \sum_{u=1}^{N}\sum_{v=1}^{N}\sum_{w=1}^{N}\left(e_u a_{uv} a_{vw} f_w - e_u \bar{a}^i_{uv}\bar{a}^i_{vw} f_w\right) + \ldots \end{aligned} \quad (3)$$

2.2.2 Edge hypothetical extraction method

Focusing on the direct linkage from sector i to sector j $(i \neq j)$, the environmental impact of the case that a specific transaction from sector i to sector j is extracted from the economy also can be calculated. The total impact of extracting the transaction from sector i to sector j, x^{ij} can be obtained alike sector extraction impact as follows:

$$\begin{aligned} x^{ij} &= x - \bar{x}^{ij} \\ &= \mathbf{eLf} - \mathbf{e}\bar{\mathbf{L}}^{ij}\mathbf{f} \\ &= \mathbf{e}\left(\mathbf{L} - \bar{\mathbf{L}}^{ij}\right)\mathbf{f} \\ &= \sum_{u=1}^{N}\sum_{v=1}^{N}\left(e_u a_{uv} f_v - e_u \bar{a}^{ij}_{uv} f_v\right) + \sum_{u=1}^{N}\sum_{v=1}^{N}\sum_{w=1}^{N}\left(e_u a_{uv} a_{vw} f_w - e_u \bar{a}^{ij}_{uv}\bar{a}^{ij}_{vw} f_w\right) + \ldots.. \end{aligned} \quad (4)$$

here $\bar{\mathbf{A}}^{ij} = (\bar{a}^{ij}_{uv})$ is the input coefficient matrix where the (i,j)th element is zero, and $\bar{\mathbf{L}}^{ij}$ is the "extracted" Leontief inverse calculated by using the "extracted" input coefficient $\bar{\mathbf{A}}^{ij}$. From this equation, the extraction impact of transaction from sector i to sector j, x^{ij} can be understood as the total emission from the supply chains that exclude the supply chain paths not passing through the transaction from sector i to sector j, and can be interpreted as the total emissions associated with the supply chain paths passing through the transaction from sector i to sector j.

2.3 Betweenness centrality

2.3.1 Node betweenness centrality

Liang *et al*. (2016) proposed the input-output node betweenness centrality, which is a measure of the betweenness of a specific sector that considers the production tiers in the supply chains. Sectors with higher betweenness centrality transmit larger amounts of CO_2 emissions throughout the supply chains. Using an input-output table, Liang *et al*. (2016) defined b_i as the input-output node betweenness centrality of a specific sector *i* as follows:

$$b_i = \sum_{s=1}^{n}\sum_{t=1}^{n}\sum_{r=1}^{\infty}\left(q_r \times w\left(s,t \middle| k_1,k_2,\ldots,k_r\right)\right). \quad (5)$$

Here, *s* and *t* are the start and end sectors of a supply chain path, respectively, q_r is the number of times that sector *i* appears in the supply chain path, *w* indicates the weight of the supply chain path starting from sector *s* and passing through *r* sectors (k_1, k_2, ..., k_r) to reach end sector *t*, and *w* is calculated as

$$w\left(s,t \middle| k_1,k_2,\ldots,k_r\right) = e_s a_{sk_1} a_{k_1k_2} \cdots a_{k_r t} f_t$$

Notice that a particular supply chain path passing through the same sector multiple times increases the sector betweenness. In other words, this definition allows the double-counting of the weight of the same supply chain path based on the number of times that sector *i* appears in this supply chain path.

Liang *et al*. (2016) formulated $b_i(l_1,l_2)$ as the total emissions associated with the supply chain paths that pass through sector *i* that has an industrial supply chain with l_1 upstream sectors and l_2 downstream sectors.

$$\begin{aligned} b_i(l_1,l_2) &= \sum_{1\le k_1,\ldots,k_{l_1}\le n}\ \sum_{1\le j_1,\ldots,j_{l_2}\le n}\left(e_{k_1} a_{k_1k_2}\cdots a_{k_{l_1}i} a_{ij_1}\cdots a_{j_{l_2-1}j_{l_2}} f_{j_{l_2}}\right) \\ &= \sum_{1\le k_1,\ldots,k_{l_1}\le n}\left(e_{k_1} a_{k_1k_2}\cdots a_{k_{l_1}i}\sum_{1\le j_1,\ldots,j_{l_2}\le n}\left(a_{ij_1}\cdots a_{j_{l_2-1}j_{l_2}} f_{j_{l_2}}\right)\right) \\ &= \left(\sum_{1\le k_1,\ldots,k_{l_1}\le n}\left(e_{k_1} a_{k_1k_2}\cdots a_{k_{l_1}i}\right)\right)\left(\sum_{1\le j_1,\ldots,j_{l_2}\le n}\left(a_{ij_1}\cdots a_{j_{l_2-1}j_{l_2}} f_{j_{l_2}}\right)\right) \\ &= \left(\mathbf{eA}^{l_1}\right)_i\left(\mathbf{A}^{l_2}\mathbf{f}\right)_i \\ &= \mathbf{eA}^{l_1}\mathbf{J}_{ii}\mathbf{A}^{l_2}\mathbf{f} \end{aligned} \quad (6)$$

Here, $(\mathbf{eA}^{l_1})_i$ and $(\mathbf{A}^{l_2}\mathbf{f})_i$ are the i^{th} element of the vector $\mathbf{eA}^{l_1}$ and $\mathbf{A}^{l_2}\mathbf{f}$, respectively. Using eq. (6), eq. (5) can be simplified as follows:

$$\begin{aligned} b_i &= \sum_{l_1=1}^{\infty}\sum_{l_2=1}^{\infty} b_i(l_1,l_2) \\ &= \sum_{l_1=1}^{\infty}\sum_{l_2=1}^{\infty}\left(\mathbf{eA}^{l_1}\mathbf{J}_{ii}\mathbf{A}^{l_2}\mathbf{f}\right) \\ &= \sum_{u=1}^{N}\sum_{v=1}^{N}\left(e_u a_{ui} a_{iv} f_v\right) + \sum_{u=1}^{N}\sum_{v=1}^{N}\sum_{w=1}^{N}\left(e_u a_{uv} a_{vi} a_{iw} f_w\right) + ... \\ &= \mathbf{eTJ}_{ii}\mathbf{Tf} = \mathbf{et}_i^c\mathbf{t}_i^r\mathbf{f} \end{aligned} \tag{7}$$

Where **T** is the indirect requirement matrix, and **T** is obtained with the following equations:

$$\begin{aligned} \mathbf{T} &= \mathbf{AL} \\ &= \mathbf{A} + \mathbf{A}^2 + \cdots \\ &= \{t_{uv}\} \end{aligned}$$

T is constructed as:

$$\mathbf{T} = \begin{pmatrix} t_{11} & \cdots & t_{1j} & \cdots & t_{1N} \\ \vdots & \ddots & & & \vdots \\ t_{i1} & & t_{ij} & & t_{iN} \\ \vdots & & & \ddots & \vdots \\ t_{N1} & & t_{Nj} & & t_{NN} \end{pmatrix} = \begin{pmatrix} \mathbf{t}_1^c & \cdots & \mathbf{t}_j^c & \cdots & \mathbf{t}_N^c \end{pmatrix} = \begin{pmatrix} \mathbf{t}_1^r \\ \vdots \\ \mathbf{t}_i^r \\ \vdots \\ \mathbf{t}_N^r \end{pmatrix}$$

It should be noted that the betweenness centrality in Liang *et al*. (2016) does not count the direct emission from sector *i*, $e_i f_i$ and the emission from the 1st supplier *i* triggered by final demand of sector *j*, $e_i a_{ij} f_j$, respectively.

In this paper, for comparison of the hypothetical extraction method and input-output betweenness centrality, I reformulated b_i as the environmental input-output node betweenness centrality of a specific sector *i*. Note that r can be 0 in eq. (5). It means that the betweenness centrality used in this paper counted the direct emission from sector *i* from final demand of sector *i*. Thus, eq. (7) can be reformulated as

$$\begin{aligned} b_i &= \sum_{l_1=0}^{\infty}\sum_{l_2=0}^{\infty} b_i(l_1,l_2) \\ &= \sum_{l_1=0}^{\infty}\sum_{l_2=0}^{\infty}\left(\mathbf{eA}^{l_1}\mathbf{J}_{ii}\mathbf{A}^{l_2}\mathbf{f}\right) \\ &= e_i f_i + \sum_{u=1}^{N}\left(e_u a_{ui} f_i\right) + \sum_{u=1}^{N}\sum_{v=1}^{N}\left(e_u a_{ui} a_{iv} f_v\right) + \sum_{u=1}^{N}\sum_{v=1}^{N}\sum_{w=1}^{N}\left(e_u a_{uv} a_{vi} a_{iw} f_w\right) + ... \\ &= \mathbf{eLJ}_{ii}\mathbf{Lf} = \mathbf{el}_i^c\mathbf{l}_i^r\mathbf{f} \end{aligned} \tag{9}$$

2.3.2 *Edge betweenness centrality*

Hanaka *et al*. (2017) proposed input-output edge betweenness centrality, which is a measure of the betweenness of transactions in the supply chains. Transactions with higher betweenness centrality transmit larger amounts of CO_2 emissions throughout the supply chains. Reformulating the methodology of Liang *et al*. (2016), Hanaka *et al*. (2017) defined b_{ij} as the input-output node betweenness centrality of a specific transaction from sector *i* to sector *j* ($i \neq j$) with a simple equation (see Hanaka *et al.*, 2017):

$$\begin{aligned} b_{ij} &= a_{ij}\mathbf{eLJ}_{ij}\mathbf{Lf} \\ &= \mathbf{el}_i^c a_{ij}\mathbf{l}_j^r\mathbf{f} \end{aligned} \tag{10}$$

2.4 *Differences between hypothetical extraction methods and betweenness centralities.*

In this paper, I address the question on what is the difference between the two methods that have a similar concept, the extraction impact x^i and x^{ij} and betweenness centrality b_i and b_{ij}. From the equation S6 and S9, b_i and b_{ij} are described by using x^i and x^{ij} respectively as follows (See Supplementary Information):

$$b_i = \left(1 + t_{ii}\right)x^i \tag{11}$$

$$b_{ij} = \left(1 + a_{ij} l_{ji}\right)x^{ij} \tag{12}$$

From equation (11) and (12), we can see that the value of the betweenness centrality is always higher than the value of the extraction impact. Figure 1 shows the difference between the sector hypothetical extraction method and node betweenness centrality. We can see that x^i and x^j are same but b_i and b_j are distinctly different. The value of betweenness centrality is weighted according to number of times that a sector appears in the supply chain path. From the perspective of betweenness centrality, a sector appearing more times in a supply chain is more important than sectors appearing fewer times for t_{ii} and $a_{ij}l_{ji}$ in the node betweenness and edge betweenness centrality, respectively. Hypothetical extraction method ignored the number of times that a sector appears in the supply chain path. From the perspective of policy implication, technical improvement in a sector appearing more times in supply chain are more effective than sectors appearing less times. When the results of these methods differ greatly, the number of appearing the sector in the supply chain is large, and the sector plays an important role in the supply chain.

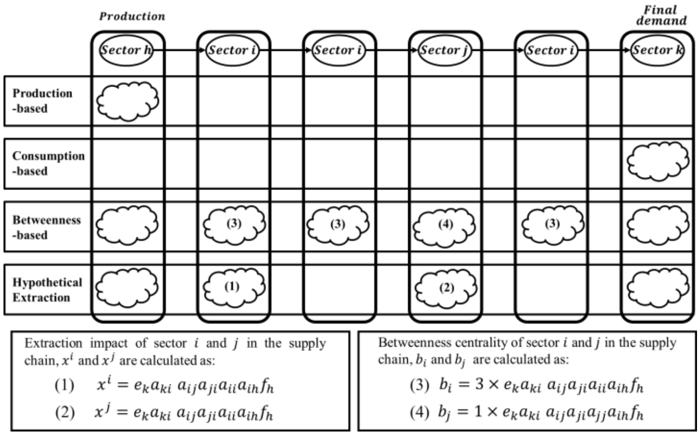

Figure 1. Difference between the major emission accountings of production-based, consumption-based, betweenness-based and hypothetical extraction methods.

2.5 Analyzing correlation between hypothetical extraction methods and betweenness centralities

I analyzed the substitution of x^i and b_i, x^{ij} and b_{ij}, and calculated the Spearman rank correlation coefficient to see the consistency of the ranks assigned to different sectors by the hypothetical extraction method and betweenness centrality.

3. Data

In this study, I used the Eora MRIO table for 2015 covering 26 industrial sectors and 189 regions, which is publicly available at http://www.worldmrio.com/ (Lenzen *et al.*, 2012, 2013) and the WIOD MRIO table for 2008 covering 35 industrial sectors and 40 regions, which is publicly available at http://www.wiod.org/home (Dietzenbacher *et al.*, 2013; Timmer *et al.*, 2015).

4. Results

4.1 The result of hypothetical extraction method and betweenness centrality analysis

From WIOD for 2008, the emissions from industries in 41 countries total 25598 Mt CO_2. From the production-based CO_2 emissions obtained from WIOD, the largest emitters were China (5923 Mt CO_2), followed by the United States (4550 Mt CO_2), Russia (1515 Mt CO_2), India (1367 Mt CO_2), and Japan (1021 Mt CO_2). The emissions in these five countries accounts for about 70% of the total emissions.

Some studies have reported the production of trade goods in developing countries has also contributed greatly to the increase in CO_2 emissions in recent decades (e.g., Peters, 2011). In this situation, considering the consumption-based emissions is important when assessing the emission responsibility of developed and developing countries (Wiedmann, 2009; Peters, 2011). The emission responsible countries should reduce the CO_2 emission through the climate policy as technology investment to key sectors.

The hypothetical extraction method can be used for detecting key sectors. Using the hypothetical extraction method, I can calculate the magnitude of the linkage between sectors, and the emissions of the supply chain paths passing through a sector or a transaction.

Applying the sector hypothetical extraction method and the edge hypothetical extraction method outlined in Sections 2.2.1 and 2.2.2 to the WIOD, I calculated two indicators and ranks (Tables 5 and 6). From Table 5, the highest extraction impact sector is Electricity, Gas and Water Supply (CHN)(3250Mt-CO_2).

From Table 6, we can also see the Chinese sectors have a large extraction impact. Especially, transaction from the Electricity, Gas and Water Supply (CHN) sector or the transaction to the Construction (CHN) sector affect the total emissions throughout the supply chain network.

Table 5. Top 10 sectors by extraction impact: WIOD

	Sector_name (WIOD)	x^i (Mt-CO_2)
1	CHN _ Electricity, Gas and Water Supply	3250
2	USA _ Electricity, Gas and Water Supply	2318
3	CHN _ Construction	1973
4	RoW _ Electricity, Gas and Water Supply	1861
5	CHN _ Basic Metals and Fabricated Metal	1737
6	CHN _ Other Non-Metallic Mineral	1219
7	CHN _ Electrical and Optical Equipment	1085
8	CHN _ Chemicals and Chemical Products	1078
9	RoW _ Construction	938
10	RoW _ Mining and Quarrying	878

Table 6. Top 10 transactions by extraction impact: WIOD

	Source_sector		Target_sector	x^{ij} (Mt-CO_2)
1	CHN _ Other Non-Metallic Mineral	→	CHN _ Construction	853
2	CHN _ Electricity, Gas and Water Supply	→	CHN _ Basic Metals and Fabricated Metal	608
3	CHN _ Basic Metals and Fabricated Metal	→	CHN _ Construction	425
4	CHN _ Electricity, Gas and Water Supply	→	CHN _ Chemicals and Chemical Products	417
5	RoW _ Electricity, Gas and Water Supply	→	RoW _ Mining and Quarrying	407
6	CHN _ Basic Metals and Fabricated Metal	→	CHN _ Electrical and Optical Equipment	393
7	CHN _ Electricity, Gas and Water Supply	→	CHN _ Mining and Quarrying	337
8	CHN _ Basic Metals and Fabricated Metal	→	CHN _ Machinery, Nec	306
9	CHN _ Electricity, Gas and Water Supply	→	CHN _ Other Non-Metallic Mineral	235
10	RoW _ Other Non-Metallic Mineral	→	RoW _ Construction	222

Hypothetical extraction method analysis allows us to calculate the CO_2 emissions throughout a sector. In the actual supply chain, however, even if the CO_2 emissions are the same, if a sector appears in a supply chain multiple times, then the sector and the transaction will differ in importance. For climate policy, this perspective has important implications because the technical improvement in a sector that appears in a supply chain many times may reduce the more emission from the supply chain than that in a sector that has same extraction impact and appears in the same supply chain only once (see Figure 1). Therefore, I analyze the results of the betweenness centrality in the next paragraph.

Applying the node betweenness centrality and edge betweenness centrality outlined in Sections 2.3.1 and 2.3.2 to the WIOD, I calculated two indicators and sets of ranks (Tables 7 and 8). From Table 7, the sector with the highest node betweenness centrality is also Electricity, Gas and Water Supply (CHN)(4806Mt-CO2). Similar to the results for the extraction impact, the betweenness centrality of Chinese sectors are highest. In the list of the top 10 sectors, the ranks in Table 7 are similar to those in Table 5. Focusing on the Basic Metals and Fabricated Metal (CHN) sector, this sector is the 5th highest in extraction impact but 2nd highest in betweenness centrality. Thus, I can say the supply chain paths in this sector appear multiple times, and are induced by global final demand more than the supply chains with the 2nd to 4th highest extraction impact. The betweenness centrality reflects the size of the number of times that the sector appears in the supply chain path. Policy makers should focus on Basic Metals and Fabricated Metal (CHN) sector rather than sectors of 2nd to 4th highest extraction impact.

Then, from Table 8, we can see the highest edge betweenness centrality transaction is Other Non-Metallic Mineral (CHN) -> Construction (CHN). Similar to the results for the extraction impact, the betweenness centrality of the transactions between the Chinese sectors are highest. In comparison to Hanaka *et al*. (2017), the size of both the node betweenness and edge betweenness of the Chinese sectors and transactions are induced by Chinese final demand. Note that the value and rank of the results of the edge betweenness and edge hypothetical extraction method are almost the same. This is apparently attributable to self-loop exclusion resulting in differences in both the value and rank being far smaller than those obtained in the results for the nodes. It may therefore be seen that in the Chinese domestic supply chain, particularly in the input from Electricity, Gas and Water Supply (CHN) and the input to Construction (CHN), not only is the intermediate emission rate high but the number of occurrences of the Transaction is large and is important from a graph theory perspective.

Table 7. Top 10 sectors by node betweenness centrality: WIOD

	Sector_name (WIOD)	b_i (Mt-CO_2)
1	CHN _ Electricity, Gas and Water Supply	4806
2	CHN _ Basic Metals and Fabricated Metal	2747
3	USA _ Electricity, Gas and Water Supply	2332
4	RoW _ Electricity, Gas and Water Supply	2190
5	CHN _ Construction	1999
6	CHN _ Electrical and Optical Equipment	1567
7	CHN _ Chemicals and Chemical Products	1524
8	CHN _ Other Non-Metallic Mineral	1449
9	RoW _ Mining and Quarrying	1172
10	RoW _ Construction	960

Table 8. Top 10 transactions by edge betweenness centrality: WIOD

	Source_sector		Target_sector	b_{ij} (Mt-CO_2)
1	CHN _ Other Non-Metallic Mineral	→	CHN _ Construction	853
2	CHN _ Electricity, Gas and Water Supply	→	CHN _ Basic Metals and Fabricated Metal	610
3	CHN _ Basic Metals and Fabricated Metal	→	CHN _ Construction	425
4	CHN _ Electricity, Gas and Water Supply	→	CHN _ Chemicals and Chemical Products	419
5	RoW _ Electricity, Gas and Water Supply	→	RoW _ Mining and Quarrying	416
6	CHN _ Basic Metals and Fabricated Metal	→	CHN _ Electrical and Optical Equipment	396
7	CHN _ Electricity, Gas and Water Supply	→	CHN _ Mining and Quarrying	343
8	CHN _ Basic Metals and Fabricated Metal	→	CHN _ Machinery, Nec	312
9	CHN _ Electricity, Gas and Water Supply	→	CHN _ Other Non-Metallic Mineral	235
10	RoW _ Other Non-Metallic Mineral	→	RoW _ Construction	222

4.2 Correlation between the sector hypothetical extraction method and node betweenness centrality, and the edge hypothetical extraction method and edge betweenness centrality

Applying the sector hypothetical extraction method and node betweenness centrality outlined in Sections 2.2.1 and 2.3.1 (see Figs. 2) and Sections 2.2.2 and 2.3.2 (see Figs. 3) to the Eora dataset, I calculated two correlation coefficients (see Table 9). These tables show that both correlation coefficients are positive and significant.

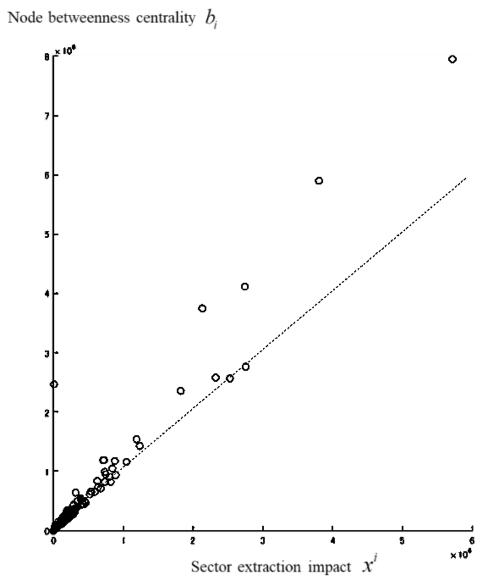

Figure 2. Sector extraction impact values versus node betweenness centrality: Eora

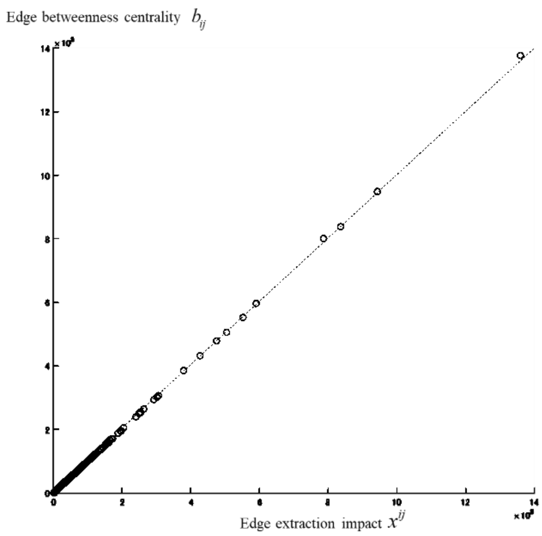

Figure 3. Edge extraction impact versus edge betweenness centrality: Eora

Table 9. Correlation between the sector hypothetical extraction method and betweenness centrality

	WIOD	Eora
Rank correlation (x^i and b_i)	0.999142	0.999996
Rank correlation (x^{ij} and b_{ij})	0.999998	0.999995

5. Discussion and Conclusion

In this study, I detected and analyzed the key sectors and transactions in a supply chain by applying the hypothetical extraction method and betweenness centrality to WIOD and Eora. From the results, Electricity, Gas and Water Supply (CHN) was identified as a key sector by both indicators and 12% of the total emissions accompanied the supply chain passing through this sector. On the other hand, Other Non-Metallic Mineral (CHN) -> Construction (CHN) had the highest values for both indicators, and of the CO_2 emissions accompanying the supply chain via Other Non-Metallic Mineral (CHN) and Construction (CHN), these sectors account for 43% and 69%, respectively. Moreover, in the CO_2 emissions via the transactions including Electricity, Gas and Water Supply (CHN), Electricity, Gas and Water Supply (CHN) -> Basic Metals and Fabricated Metal (CHN) is largest, accounting for 18% of the total.

Results from the extraction impact, which is the emissions actually passing through the sector and may be called the sector reduction potential, and the betweenness centrality analysis, which is the value representing the importance of a sector including a weighting based on the number of occurrences, were very similar. Furthermore, the rank correlation of the results for these two indicators is large and positive. The double-counting of transactions did not have a particularly large effect on the results of the edge betweenness, and the results for the two methods were similar.

In the hypothetical extraction method computation, for the calculation of one sector and edge impact, setting the input coefficients and calculating the inverse matrix would take an extremely long time for a large matrix such as the Eora or WIOD datasets. In contrast, computation of the betweenness centrality can be performed using a fixed Leontief inverse matrix and can therefore be accomplished in a very short time. For a large dataset such as WIOD or Eora, an extremely large calculation is necessary and big data can be more readily treated. For analysis of I-O networks that are more global, the computing-volume problem is important. In this paper, I showed that the extraction impact can be calculated from the *less computationally-expensive* betweenness centrality obtained using the equations (11) and (12).

The extraction impacts show the magnitude of influencing outputs of other industries along the supply chains related to transactions of an industry in question, whereas the betweenness centrality shows the importance of networking industries through a node of an industry in question as well as a transaction between the industry in question and another industry. The hypothetical extraction method is widely used to assess inter-industry linkages and the economic importance of industries (e.g., Dietzenbacher *et al*., 2019). Thus, the both methods have different advantages. Therefore, I propose that researchers firstly use betweenness centrality that is less computationally-expensive and secondly estimate the extraction impacts using equations (11) and (12) developed in this study.

Reference

Brachert, M., Brautzschb, H., Titzec, M. (2016) Mapping potentials for input–output-based innovation flows in industrial clusters – an application to Germany. *Economic Systems Research* 28(4): 450–466.

Casler, S., Hadlock, D. (1997) Contributions to change in the input-output model: The search for inverse important coefficients. *Journal of Regional Science* 37 (2): 175 – 193.

Cella, G. (1984) The Input-Output Measurement of Interindustry Linkages. *Oxford Bulletin of Economics and Statistics* (46): 73 - 84.

Chen, B., Lib, J., Wu, X., Han, M., Zeng, L., Li, Z., Chen, G. (2017) Global energy flows embodied in international trade: A combination of environmentally extended input–output analysis and complex network analysis. *Applied Energy* 210: 98–107.

Chenery, B., Watanabe, T. (1958) International comparisons of the structure of production. *Econometrica* 26 (4): 487 – 521.

Dietzenbacher, E., Linden, J., Steenge, A. (1993) The Regional Hypothetical extraction method: EC Input–Output Comparisons. *Economic Systems Research* 5(2): 185–206

Dietzenbacher, E., Lahr, M. (2013) Expanding Extractions *Economic Systems Research* 25(3): 341–360.

Dietzenbacher, E., Los, B., Stehrer, R., Timmer, M., de Vries, G. (2013) The Construction of World Input-Output Tables in the WIOD Project *Economic Systems Research* 25: 71 – 98.

Dietzenbacher, E., Burken, B., Kondo, Y. (2019) Hypothetical extractions from a global perspective. *Economic Systems Research* Online

Du, R., Wang, Y., Dong, G., Tian, L., Liu, Y., Wang, M., Fang, G., (2017) A complex network perspective on interrelations and evolution features of international oil trade, 2002 - 2013. *Applied Energy* 196 (12): 142 - 151.

Duang, Y., Jiang, X. (2018) Visualizing the change of embodied CO2 emissions along global production chains. *Journal of Cleaner Production* 194: 499 – 514.

Freeman, L. C. (1977) A Set of Measures of Centrality Based on Betweenness. *Sociometry* 40(1): 3541.

Freeman, L. C. (1978) Centrality in social networks conceptual clarification. *Social Networks* 1 (3): 215 – 239.

Friedkin, N., Johnsen, E. (1990) Social Influence and Opinions. *Journal of Mathematical Sociology* 15(3-4):193-206

Ghosh, S., Roy, J. (1998) Qualitative input-output analysis of the Indian economic structure. *Economic systems Reseach* 10(3): 263 – 274.

Hazari, B. (1970) Empirical identification of key-sectors in the Indian economy. *Review of Economics and Statistics* 52(3): 301 – 305.

Hanaka, T., Kagawa, S., Ono, H., Kanemoto, K. (2017) Finding Environmental Critical Transmission Sectors, Transactions, and Path in Global Supply Chain Networks. *Energy Ecconomics* 68: 44 – 52.

Hirschman, A. (1958) The Strategy of Economic Development. *Yale University Press, New Haven, CT, USA.*

Holub, H., Schnabl, H. (1985) Qualitative Input-Output Analysis and Structural Information. *Economic Modelling* 2(1): 67 – 73.

Kagawa, S., Oshita, Y., Nansai, K., Suh, S. (2009) How has dematerialization contributed to reducing oil price pressure? A qualitative Input-output analysis for the Japanese economy during 1990 – 2000. *Environmental Science and Technology* 43(2): 245 – 252.

Kagawa, S., Okamoto, S., Suh, S., Kondo, Y., Nansai, K. (2013a) Finding Environmentally Important Industry Clusters: Multiway Cut Approach Using Nonnegative Matrix Factorization. *Social Networks* 35: 423–438.

Kagawa, S., Suh, S., Kondo, Y., Nansai, K. (2013b) Identifying Environmentally-Important Supply-Chain Clusters in the Automobile Industry. *Economic Systems Research* 25: 265–286.

Kagawa, S., Suh, S., Hubacek, K., Wiedmann, T., Nansai, K., Minx, J. (2015) CO2 Emission Clusters within Global Supply Chain Networks: Implications for Climate Change Mitigation. *Global Environmental Change* 35: 486–496.

Lenzen, M. (2003) Environmentally important paths, linkages and key sectors in the Australian economy. *Structural Change and Economic Dynamics* 14: 1–34.

Lenzen, M., Kanemoto, K., Moran, D., Geschke, A. (2012) Mapping the Structure of the World Economy. *Environmental Science & Technology* 46(15): 8374–8381.

Lenzen, M., Kanemoto, K., Moran, D., Geschke, A. (2013) Building Eora: a Global Multi-Region Input-Output Database at High Country and Sector Resolution. *Economic Systems Research* 25(1): 20-49.

Leontief, W. (1936) Quantitative Input-Output Relations in the Economic System of the United States Review of *Economics and Statistics* 18: 105–125.

Leontief, W. (1941) The Structure of American Economy 1919–1939. New York: *Oxford University Press*.

Liang, S., Qu, S., Xu, M. (2016) Betweeness-Based Method to Identify Critical Transmission Sectors for Supply Chain Environmental Pressure Mitigation. *Environmental Science & Technology* 50: 1330-1337.

Miller, R., Blair, P. (2009) Input-Output Analysis: Foundations and Extensions. Cambridge University Press.

Meller, P., Marfan, M. (1981) Small and large industry: employment generation, linkages and key sectors.*Economic Development and Cultural Change* 29: 263 - 274.

Muniz, A., Raya, A., Carvajal, C. (2008) Key sectors: a new proposal from network theory. *Regional Studies* 42(7) 1 – 18.

Nagashima, F., Kagawa, S., Suh, S., Nansai, K., Moran, D. (2017) Identifying Critical Supply Chain Paths and Key Sectors for Mitigating Primary Carbonaceous PM 2.5 Mortality in Asia. *Economic Systems Research* 29 (1): 105 – 123.

Nakano, S., Arai, S., Washizu, A. (2017) Economic Impacts of Japan's Renewable Energy Sector and the Feed-In Tariff System: Using an Input-Output Table to Analyze a Next-Generation Energy System. *Environmental Economics and Policy Studies* 19 (3): 555 – 580.

Oshita, Y (2012) Identifying critical supply chain paths that drive changes in CO2 emissions. *Energy Economics* 34: 1041–1050

Peters, G., Hertwich, E. (2006) Structural Analysis of International Trade: Environmental Impacts of Norway. *Economic Systems Research* 18(2): 155–181.

Peters, G., Minx, J. C., Weber, C. L., Edenhofer, O. (2011) Growth in emission transfers via international trade from 1990 to 2008. *PNAS* 108(21): 8903–8908.

Rasmussen, P. (1956) Studies in intersectional relations, *København Amsterdam Einar Harcks North-Holland.*

Rosenblatt, D. (1957) On Linear Models and the Graphs of Minkowski-Leontief Matrices. *Econometrica* 25(2):325 – 338.

Suh, S. (2004) Functions, commodities and environmental impacts in an ecological-economic model. *Ecological Economics* 48(4):451 – 467.

Timmer, M., Dietzenbacher, E., Los, B., Stehrer, R., de Vries, G. (2015) An Illustrated User Guide to the World Input–Output Database: the Case of Global Automotive Production. *Review of International Economics* (23) 575–605.

Tokito, S., Kagawa, S., Nansai, K. (2016) (2016) Understanding International Trade Network Complexity of Platinum: Case of Japan. *Resources Policy* 49: 415-421.

Tokito, S. (2018) Environmentally-Targeted Sectors and Linkages in the Global Supply-Chain Complexity of Transport Equipment. *Ecological Economics* 150: 177 – 183.

Treolar, G. (1997), Extracting embodied energy paths from input-output tables: Towards an input-output based hybrid energy analysis method. *Economic Systems Research* 9(4): 375 – 391.

Weber and Schnabl, (1998) Environmentally important Intersectoral flows: insights from main contributions identification and minimal flow analysis. *Economic Systems Research* 10(4) 337 – 355.

Wiedmann, T. (2009) A review of recent multi-region input–output models used for consumption-based emission and resource accounting. *Ecological Economics* 69 (2): 211-222.

Wiebe, S. (2018) Identifying emission hotspots for low carbon technology transfers. *Journal of Cleaner Production* 1 (194): 243 – 252.

Wood, R., Lenzen, M. (2009) Structural Path Decomposition. *Energy Economics* 31: 335-341

<u>**Supplementary Information**</u>

S2.4 Differences between extraction methods and betweenness centralities.

First, the input coefficient matrix can be decomposed to the "sector i -extracted" input coefficient matrix $\bar{\mathbf{A}}^i = (\bar{a}_{uv}^i)$ and the input coefficient matrix that has the element associated with sector i $\mathbf{A}^i = (a_{uv}^i)$ as follows:

$$\mathbf{A} = \bar{\mathbf{A}}^i + \mathbf{A}^i \quad \text{(S1)}$$

where

$$\bar{a}_{uv}^i = \begin{cases} a_{uv} & u \neq i \wedge v \neq i \\ 0 & u = i \vee v = i \end{cases}, \quad a_{uv}^i = \begin{cases} a_{uv} & u = i \vee v = i \\ 0 & u \neq i \wedge v \neq i \end{cases}.$$

Here, $\mathbf{x}$ can be evaluated using eq. (S1) as follows:

$$\begin{aligned} \mathbf{x} &= \mathbf{L}\mathbf{f} \\ &= \mathbf{A}\mathbf{x} + \mathbf{f} \\ &= \left(\bar{\mathbf{A}}^i + \mathbf{A}^i\right)\mathbf{x} + \mathbf{f} \\ \mathbf{x} - \bar{\mathbf{A}}^i\mathbf{x} &= \mathbf{A}^i\mathbf{x} + \mathbf{f} \\ \left(\mathbf{E} - \bar{\mathbf{A}}^i\right)\mathbf{x} &= \mathbf{A}^i\mathbf{L}\mathbf{f} + \mathbf{f} \\ \mathbf{x} &= \left(\mathbf{E} - \bar{\mathbf{A}}^i\right)^{-1}\left(\mathbf{A}^i\mathbf{L} + \mathbf{E}\right)\mathbf{f} \end{aligned}$$

Here, $\left(\mathbf{E} - \bar{\mathbf{A}}^i\right)^{-1}$ can be replaced with $\bar{\mathbf{L}}^i$ as:

$$\mathbf{x} = \left(\bar{\mathbf{L}}^i\mathbf{A}^i\mathbf{L} + \bar{\mathbf{L}}^i\right)\mathbf{f}$$

Thus, the Leontief inverse can be represented as

$$\mathbf{L} = \left(\bar{\mathbf{L}}^i\mathbf{A}^i\mathbf{L} + \bar{\mathbf{L}}^i\right). \quad \text{(S2)}$$

From eq. (S2) and (3), the extraction impact of sector i x^i can be reformulated as

$$\begin{aligned} x^i &= \mathbf{e}\mathbf{L}\mathbf{f} - \left(\mathbf{e}\bar{\mathbf{L}}^i\mathbf{f} - e_i f_i\right) \\ &= \mathbf{e}\left(\mathbf{L} - \bar{\mathbf{L}}^i\right)\mathbf{f} + e_i f_i \\ &= \mathbf{e}\left(\bar{\mathbf{L}}^i\mathbf{A}^i\mathbf{L} + \bar{\mathbf{L}}^i - \bar{\mathbf{L}}^i\right)\mathbf{f} + e_i f_i \\ &= \mathbf{e}\bar{\mathbf{L}}^i\mathbf{A}^i\mathbf{L}\mathbf{f} + e_i f_i \end{aligned} \quad \text{(S3)}$$

Here, the left term in eq. (S3) can be decomposed as:

$$\begin{aligned} \mathbf{e}\bar{\mathbf{L}}^i\mathbf{A}^i\mathbf{L}\mathbf{f} &= \mathbf{e}\left(\bar{\mathbf{L}}^i + \mathbf{J}_{ii} - \mathbf{J}_{ii}\right)\mathbf{A}^i\mathbf{L}\mathbf{f} \\ &= \mathbf{e}\bar{\mathbf{L}}^i\mathbf{A}^i\mathbf{L}\mathbf{f} + \mathbf{e}\mathbf{J}_{ii}\mathbf{A}^i\mathbf{L}\mathbf{f} - \mathbf{e}\mathbf{J}_{ii}\mathbf{A}^i\mathbf{L}\mathbf{f} \\ &= \mathbf{e}\bar{\mathbf{L}}^i\mathbf{A}^i\left(\mathbf{E} + \mathbf{J}_{ii} - \mathbf{J}_{ii}\right)\mathbf{L}\mathbf{f} + \mathbf{e}\mathbf{J}_{ii}\mathbf{A}^i\mathbf{L}\mathbf{f} - \mathbf{e}\mathbf{J}_{ii}\mathbf{A}^i\mathbf{L}\mathbf{f} \\ &= \mathbf{e}\bar{\mathbf{L}}^i\mathbf{A}^i\mathbf{L}\mathbf{f} + \mathbf{e}\bar{\mathbf{L}}^i\mathbf{A}^i\mathbf{J}_{ii}\mathbf{L}\mathbf{f} - \mathbf{e}\left(\bar{\mathbf{L}}^i + \mathbf{J}_{ii} - \mathbf{J}_{ii}\right)\mathbf{A}^i\mathbf{J}_{ii}\mathbf{L}\mathbf{f} + \mathbf{e}\mathbf{J}_{ii}\mathbf{A}^i\mathbf{L}\mathbf{f} - \mathbf{e}\mathbf{J}_{ii}\mathbf{A}^i\mathbf{L}\mathbf{f} \\ &= \left\{\mathbf{e}\bar{\mathbf{L}}^i\mathbf{A}^i\mathbf{L}\mathbf{f} + \mathbf{e}\mathbf{J}_{ii}\mathbf{A}^i\mathbf{J}_{ii}\mathbf{L}\mathbf{f} - \mathbf{e}\mathbf{J}_{ii}\mathbf{A}^i\mathbf{L}\mathbf{f}\right\} - \mathbf{e}\mathbf{J}_{ii}\mathbf{A}^i\mathbf{J}_{ii}\mathbf{L}\mathbf{f} + \mathbf{e}\mathbf{J}_{ii}\mathbf{A}^i\mathbf{L}\mathbf{f} \\ &= \left\{\mathbf{e}\left(\bar{\mathbf{L}}^i\mathbf{A}^i + \mathbf{J}_{ii}\mathbf{A}^i\mathbf{J}_{ii} - \mathbf{J}_{ii}\mathbf{A}^i\right)\mathbf{L}\mathbf{f}\right\} - \mathbf{e}\mathbf{J}_{ii}\mathbf{A}^i\mathbf{J}_{ii}\mathbf{L}\mathbf{f} + \mathbf{e}\mathbf{J}_{ii}\mathbf{A}^i\mathbf{L}\mathbf{f} \\ &= \left\{\mathbf{e}\left(\bar{\mathbf{L}}^i\mathbf{A}^i + a_{ii}\mathbf{J}_{ii} - \mathbf{J}_{ii}\mathbf{A}^i\right)\mathbf{L}\mathbf{f}\right\} - \mathbf{e}\mathbf{J}_{ii}\mathbf{A}^i\mathbf{J}_{ii}\mathbf{L}\mathbf{f} + \mathbf{e}\mathbf{J}_{ii}\mathbf{A}^i\mathbf{L}\mathbf{f} \end{aligned}$$

(S4)

Here, the i th row elements of $\bar{\mathbf{L}}^i\mathbf{A}^i$ are i th row elements of the input coefficient matrix, and $\mathbf{J}_{ii}\mathbf{A}^i$ is the matrix whose i th row elements are i th row elements of the input coefficient matrix. Thus, eq. (S4) can be reformulated as:

$$\begin{aligned} \text{Eq.(S4)} &= \left\{\mathbf{e}\left(\bar{\mathbf{L}}^i\mathbf{A}^i\mathbf{J}_{ii}\right)\mathbf{L}\mathbf{f}\right\} - \mathbf{e}\mathbf{J}_{ii}\mathbf{A}^i\mathbf{J}_{ii}\mathbf{L}\mathbf{f} + \mathbf{e}\mathbf{J}_{ii}\mathbf{A}^i\mathbf{L}\mathbf{f} \\ &= \mathbf{e}\bar{\mathbf{L}}^i\mathbf{A}^i\mathbf{J}_{ii}\mathbf{L}\mathbf{f} + \mathbf{e}\mathbf{J}_{ii}\mathbf{A}^i\mathbf{L}\mathbf{f} - \mathbf{e}\mathbf{J}_{ii}\mathbf{A}^i\mathbf{J}_{ii}\mathbf{L}\mathbf{f} \\ &= \mathbf{e}\bar{\mathbf{L}}^i\mathbf{a}_i^c\mathbf{l}_i^r\mathbf{f} + e_i\mathbf{a}_i^r\mathbf{L}\mathbf{f} - e_i a_{ii}\mathbf{l}_i^r\mathbf{f} \\ &= \mathbf{e}\bar{\mathbf{L}}^i\mathbf{a}_i^c\mathbf{l}_i^r\mathbf{f} + e_i\mathbf{t}_i^r\mathbf{f} - e_i a_{ii}\mathbf{l}_i^r\mathbf{f} \end{aligned} \quad \text{(S5)}$$

Using eq. (S2) and (9), the betweenness centrality b_i associated with a sector i can be reformulated as:

$$\begin{aligned} b_i &= \mathbf{e}\mathbf{L}\mathbf{J}_{ii}\mathbf{L}\mathbf{f} \\ &= \mathbf{e}\left(\bar{\mathbf{L}}^i + \bar{\mathbf{L}}^i\mathbf{A}^i\mathbf{L}\right)\mathbf{J}_{ii}\mathbf{L}\mathbf{f} \\ &= \mathbf{e}\bar{\mathbf{L}}^i\mathbf{J}_{ii}\mathbf{L}\mathbf{f} + \mathbf{e}\bar{\mathbf{L}}^i\mathbf{A}^i\mathbf{L}\mathbf{J}_{ii}\mathbf{L}\mathbf{f} \end{aligned}$$

Here, the i th row elements and the i th column elements of $\bar{\mathbf{L}}^i$ are 0 except (i, i)th element, 1. Therefore, $\bar{\mathbf{L}}^i\mathbf{J}_{ii} = \mathbf{J}_{ii}$. Thus, using eq. (S5), b_i can be represent as:

$$\begin{aligned} b_i &= \mathbf{e}\bar{\mathbf{L}}^i\mathbf{J}_{ii}\mathbf{L}\mathbf{f} + \mathbf{e}\bar{\mathbf{L}}^i\mathbf{A}^i\mathbf{L}\mathbf{J}_{ii}\mathbf{L}\mathbf{f} \\ &= \mathbf{e}\mathbf{J}_{ii}\mathbf{L}\mathbf{f} + \mathbf{e}\left(\bar{\mathbf{L}}^i\mathbf{A}^i\mathbf{J}_{ii}\mathbf{L} + \mathbf{J}_{ii}\mathbf{A}^i\mathbf{L} - \mathbf{J}_{ii}\mathbf{A}^i\mathbf{J}_{ii}\mathbf{L}\right)\mathbf{J}_{ii}\mathbf{L}\mathbf{f} \\ &= \mathbf{e}\mathbf{J}_{ii}\mathbf{L}\mathbf{f} + \mathbf{e}\bar{\mathbf{L}}^i\mathbf{A}^i\mathbf{J}_{ii}\mathbf{L}\mathbf{J}_{ii}\mathbf{L}\mathbf{f} + \mathbf{e}\mathbf{J}_{ii}\mathbf{A}^i\mathbf{L}\mathbf{J}_{ii}\mathbf{L}\mathbf{f} - \mathbf{e}\mathbf{J}_{ii}\mathbf{A}^i\mathbf{J}_{ii}\mathbf{L}\mathbf{J}_{ii}\mathbf{L}\mathbf{f} \\ &= e_i\mathbf{l}_i^r\mathbf{f} + \mathbf{e}\bar{\mathbf{L}}^i\mathbf{a}_i^c l_{ii}\mathbf{l}_i^r\mathbf{f} + \mathbf{e}\mathbf{J}_{ii}\mathbf{T}\mathbf{J}_{ii}\mathbf{L}\mathbf{f} - e_i a_{ii} l_{ii}\mathbf{l}_i^r\mathbf{f} \\ &= e_i\mathbf{l}_i^r\mathbf{f} + l_{ii}\left(\mathbf{e}\bar{\mathbf{L}}^i\mathbf{a}_i^c\mathbf{l}_i^r\mathbf{f} - e_i a_{ii}\mathbf{l}_i^r\mathbf{f}\right) + e_i t_{ii}\mathbf{l}_i^r\mathbf{f} \\ &= \left(1 + t_{ii}\right)\left(\mathbf{e}\bar{\mathbf{L}}^i\mathbf{a}_i^c\mathbf{l}_i^r\mathbf{f} - e_i a_{ii}\mathbf{l}_i^r\mathbf{f}\right) + \left(1 + t_{ii}\right)e_i\mathbf{l}_i^r\mathbf{f} \\ &= \left(1 + t_{ii}\right)\left(\mathbf{e}\bar{\mathbf{L}}^i\mathbf{a}_i^c\mathbf{l}_i^r\mathbf{f} - e_i a_{ii}\mathbf{l}_i^r\mathbf{f} + e_i\mathbf{t}_i^r\mathbf{f} + e_i f_i\right) \\ &= \left(1 + t_{ii}\right)x^i \end{aligned} \quad \text{(S6)}$$

From this equation, in the betweenness centrality analysis, the total emission from supply chains passing through sector i is over calculated for t_{ii}.

Similarly, the edge betweenness centrality b_{ij} can be obtained by the environmental edge extraction impact x^{ij}. First, the input coefficient matrix can be decomposed to the "transaction from sector

i to sector j-extracted" input coefficient matrix $\bar{\mathbf{A}}^{ij} = (\bar{a}_{uv}^{ij})$ and the input coefficient matrix whose (i, j)th element is a_{ij} and others are 0, $a_{ij}\mathbf{J}_{ij}$ as follows:

$$\mathbf{A} = \bar{\mathbf{A}}^{ij} + a_{ij}\mathbf{J}_{ij} \tag{S7}$$

where

$$\bar{a}_{uv}^{ij} = \begin{cases} a_{uv} & u \neq i \vee v \neq j \\ 0 & u = i \wedge v = j \end{cases}$$

Here, $\mathbf{x}$ can be evaluated using eq. (S7) as follows:

$$\begin{aligned} \mathbf{x} &= \mathbf{Lf} \\ &= \mathbf{Ax} + \mathbf{f} \\ &= \left(\bar{\mathbf{A}}^{ij} + a_{ij}\mathbf{J}_{ij}\right)\mathbf{x} + \mathbf{f} \\ \mathbf{x} - \bar{\mathbf{A}}^{ij}\mathbf{x} &= a_{ij}\mathbf{J}_{ij}\mathbf{x} + \mathbf{f} \\ \left(\mathbf{E} - \bar{\mathbf{A}}^{ij}\right)\mathbf{x} &= a_{ij}\mathbf{J}_{ij}\mathbf{Lf} + \mathbf{f} \\ \mathbf{x} &= \left(\mathbf{E} - \bar{\mathbf{A}}^{ij}\right)^{-1}\left(a_{ij}\mathbf{J}_{ij}\mathbf{L} + \mathbf{E}\right)\mathbf{f} \end{aligned}$$

Here, $\left(\mathbf{E} - \bar{\mathbf{A}}^{ij}\right)^{-1}$ can be replaced with $\bar{\mathbf{L}}^{ij}$ as:

$$\mathbf{x} = \left(\bar{\mathbf{L}}^{ij} a_{ij}\mathbf{J}_{ij}\mathbf{L} + \bar{\mathbf{L}}^{ij}\right)\mathbf{f} \ .$$

Thus, the Leontief inverse can be represented as $\mathbf{L} = \left(\bar{\mathbf{L}}^{ij} a_{ij}\mathbf{J}_{ij}\mathbf{L} + \bar{\mathbf{L}}^{ij}\right)$, and x^{ij} as the environmental extraction impact of a specific transaction between sector i and sector j can be reformulated using equation (4) as

$$\begin{aligned} x^{ij} &= \mathbf{e}\left(\mathbf{L} - \bar{\mathbf{L}}^{ij}\right)\mathbf{f} \\ &= \mathbf{e}\left(\bar{\mathbf{L}}^{ij} a_{ij}\mathbf{J}_{ij}\mathbf{L} + \bar{\mathbf{L}}^{ij} - \bar{\mathbf{L}}^{ij}\right)\mathbf{f} \\ &= \mathbf{e}\bar{\mathbf{L}}^{ij} a_{ij}\mathbf{J}_{ij}\mathbf{Lf} \\ &= \mathbf{e}\bar{\mathbf{l}}_i^{ij\,c} a_{ij}\mathbf{l}_j^r\mathbf{f} \end{aligned} \tag{S8}$$

Using eq. (S8) and (10), the edge betweenness centrality b_{ij} associated with a transaction from sector i to sector j can be reformulated as:

$$\begin{aligned} b_{ij} &= \mathbf{eL}a_{ij}\mathbf{J}_{ij}\mathbf{Lf} \\ &= \mathbf{e}\left(\bar{\mathbf{L}}^{ij} a_{ij}\mathbf{J}_{ij}\mathbf{L} + \bar{\mathbf{L}}^{ij}\right)a_{ij}\mathbf{J}_{ij}\mathbf{Lf} \\ &= \mathbf{e}\bar{\mathbf{L}}^{ij} a_{ij}\mathbf{J}_{ij}\mathbf{L}a_{ij}\mathbf{J}_{ij}\mathbf{Lf} + \mathbf{e}\bar{\mathbf{L}}^{ij} a_{ij}\mathbf{J}_{ij}\mathbf{Lf} \\ &= \mathbf{e}\bar{\mathbf{l}}_i^{ij\,c} a_{ij} l_{ji} a_{ij}\mathbf{l}_j^r\mathbf{f} + \mathbf{e}\bar{\mathbf{l}}_i^{ij\,c} a_{ij}\mathbf{l}_j^r\mathbf{f} \\ &= \left(1 + a_{ij} l_{ji}\right)\left(\mathbf{e}\bar{\mathbf{l}}_i^{ij\,c} a_{ij}\mathbf{l}_j^r\mathbf{f}\right) = \left(1 + a_{ij} l_{ji}\right)x^{ij} \end{aligned} \tag{S9}$$

From this equation, in the edge betweenness centrality analysis, the total emission from supply chains passing through the transaction from sector i to sector j is over calculated for $a_{ij}l_{ji}$.