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<https://doi.org/10.5109/22834>

出版情報：九州大学大学院農学研究院紀要. 17 (3/4), pp.247-261, 1973-10. Kyushu University
バージョン：
権利関係：



Study on the Flow through the Sluices of Sea-Dike

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(Received April 18, 1973)

The purpose of this paper is to deal with the diffusion of the flow through the sluices of sea-dike.

As the flow through the sluices streams into the water basin which has uniform and steady flows, a law of conservation of momentum is applied to the flow to calculate the angle of the deflected jet and the velocity distribution. And also the propriety of their theoretical solutions is confirmed with hydraulic experiments.

As the practical applications of this study, the large scale sluices in Nagasaki Reclamation Project are taken up, and the change of the forces of the flow through the sluices and the influence which the tidal current exerts its flow are discussed.

INTRODUCTION

It is assumed that the flow through the sluices streams in normal to the tidal current or river, and that its flow is a turbulent jet in order to analyze motion of the flow easily, and it is considered that the flow is two-dimensional.

THEORETICAL ANALYSIS

1. The theory of the deflected jet

Up to this time, Morton (1961), Keffer and Baines (1963), Csanady (1965) and others have investigated the air jet directed normal to the uniform and steady cross-wind.

This paper deals with in more detail the mechanism of the deflection of a turbulent jet. The co-ordinate system is adapted in Fig. 1, and the analysis under the following assumptions is developed (Nizu and Kato, 1960),

(1) the turbulent jet and the uniform flow are ideal, irrotational and incompressible fluid, and an entrainment does not occur,

(2) the downstream of the jet has a constant pressure.

After the flow (velocity = V) which is kept uniform at infinity collides the jet, it changes the direction and streams along FEDC (see Fig. 1).

When the stream lines change at 90° , the velocity is zero and an uprising of the water level (an increase in pressure) occurs, thus a hodo-graph of the velocity (v) along FEDC is regarded as semicircular (see Fig. 2).

The pressure at D according to the uniform flow is obtained from Bernoulli's equation as follow,

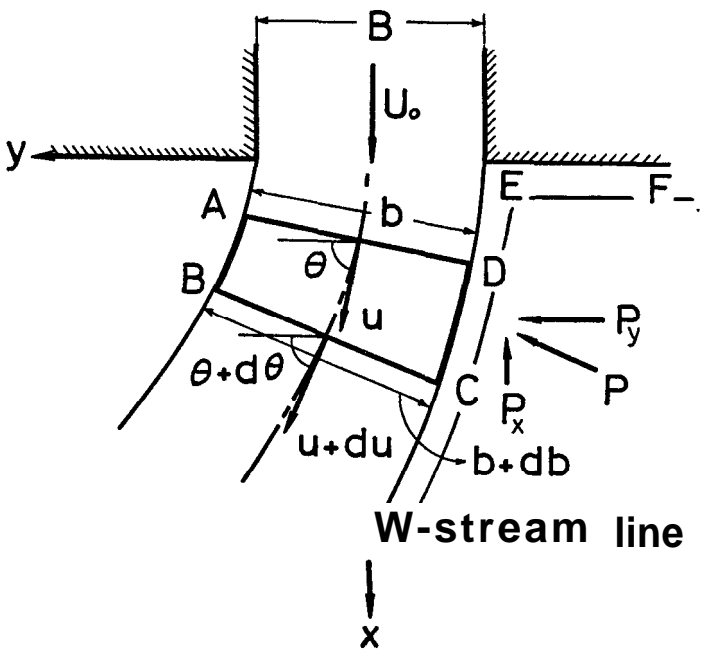


Fig. 1. Definition sketch of element of the jet.

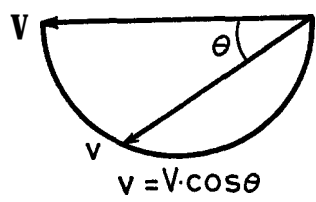


Fig. 2. Hodo-graph of velocity.

$$\frac{p}{w} = \frac{V^2}{2g} - \frac{v^2}{2g}$$

or

$$\begin{aligned} p_s &= \frac{\rho}{2} V^2 (1 - \cos^2 \theta) \\ &= - \frac{\rho}{2} V^2 \sin^2 \theta \end{aligned} \tag{1}$$

where p_s and v are a local static pressure and a local velocity at D respectively, V and θ are a velocity of the uniform flow and a deflected angle of the jet respectively, w is a specific weight of water. If a centrifugal force has been neglected, the mean static pressure through the A-D cross-section is obtained with the mean value of the static pressure at A (p_A) and at D (p_D), that is

$$p = \frac{1}{2} (p_A + p_D) . \quad (2)$$

Therefore, the following equation is obtained from $p_D = p_s$ and Eq. (1),

$$p = \frac{1}{2} p_A + \frac{\rho}{4} V^2 \sin^2 \theta . \quad (3)$$

At the outlet of the jet, $\theta = \frac{\pi}{2}$,

therefore

$$p_0 = \frac{1}{4} \rho V^2 + \frac{1}{2} p_A . \quad (4)$$

For the mean flux of the jet, the following equation is obtained from Bernoulli's theorem,

$$\frac{U_0^2}{2g} + \frac{p}{w} = \frac{u^2}{2g} + \frac{p}{w}$$

or

$$u^2 = U_0^2 + \frac{1}{2} V^2 \cos^2 \theta \quad (5)$$

where U_0 and p_0 are a local velocity and a local pressure at the outlet of the jet, u and p are a local velocity and a local pressure at any point on the jet respectively.

Letting R be

$$R = U_0/V , \quad (6)$$

Eq. (5) becomes

$$u = U_0 \sqrt{1 + \frac{1}{2} \left(\frac{1}{R} \right)^2 \cos^2 \theta} . \quad (7)$$

Considering the assumption (1), the equation of continuity is

$$Q = U_0 \cdot B = u \cdot b = (u + du) \cdot (b + db) \quad (8)$$

where B and b are a width of the outlet and a width at any cross-section of the jet respectively.

The following equation is obtained with applying the law of conservation of momentum to an element of the jet (ABCD), for the direction of x-axis,

$$\begin{aligned} P_x = & \rho \cdot Q \cdot u \cdot \cos \theta \cdot d\theta + \rho \cdot Q \cdot \sin \theta \cdot du + p \cdot b \cdot \cos \theta \cdot d\theta \\ & + p \cdot \sin \theta \cdot db + b \cdot \sin \theta \cdot dp , \end{aligned} \quad (9)$$

for the direction of y-axis,

$$\begin{aligned} P_y = & -p \cdot b \cdot \sin \theta \cdot d\theta + p \cdot \cos \theta \cdot db + b \cdot \cos \theta \cdot dp \\ & - \rho \cdot Q \cdot u \cdot \sin \theta \cdot d\theta + \rho \cdot Q \cdot \cos \theta \cdot du . \end{aligned} \quad (10)$$

From Eq. (1),

$$P_x = -p_s \cdot ds \cdot \cos \theta = -\frac{1}{2} \rho \cdot V^2 \cdot \sin \theta \cdot dy, \quad (11)$$

$$P_y = p_s \cdot ds \cdot \sin \theta = \frac{1}{2} \rho \cdot V^2 \cdot \sin \theta \cdot dx \quad (12)$$

where ds is a length of the element. The following equations are obtained from Eq. (3) and Eq. (7), respectively :

$$dp = \frac{\rho}{2} \cdot V^2 \cdot \sin \theta \cdot \cos \theta \cdot d\theta, \quad (13)$$

$$du = -\frac{U_0}{2R^2} \frac{\cos \theta \cdot \sin \theta}{\sqrt{1 + \frac{1}{2} \left(\frac{1}{R}\right)^2 \cos^2 \theta}} d\theta, \quad (14)$$

and from Eq. (7) and Eq. (8) :

$$b = \frac{B}{\sqrt{1 + \frac{1}{2} \left(\frac{1}{R}\right)^2 \cos^2 \theta}}, \quad (15)$$

$$db = \frac{B}{2R^2} \frac{\cos \theta \cdot \sin \theta}{\left\{1 + \frac{1}{2} \left(\frac{1}{R}\right)^2 \cos^2 \theta\right\}^{3/2}} d\theta. \quad (16)$$

Substituting Eq. (3), Eq. (7), Eq. (11) and Eq. (16) to Eq. (9),

$$\begin{aligned} dx = & \left\{ -\frac{B}{2} \frac{\sin \theta}{\sqrt{1 + \frac{1}{2} \left(\frac{1}{R}\right)^2 \cos^2 \theta}} + \frac{B}{4R^2} \frac{\cos^2 \theta \cdot \sin \theta}{\left\{1 + \frac{1}{2} \left(\frac{1}{R}\right)^2 \cos^2 \theta\right\}^{3/2}} \right. \\ & - 2R^2 B \frac{1}{\sin \theta} \sqrt{1 + \frac{1}{2} \left(\frac{1}{R}\right)^2 \cos^2 \theta} \\ & + \frac{p_A \cdot B}{2\rho U_0^2} \frac{1}{\left\{1 + \frac{1}{2} \left(\frac{1}{R}\right)^2 \cos^2 \theta\right\}^{3/2}} \cdot \frac{\cos^2 \theta}{\sin \theta} \\ & \left. - \frac{p_A \cdot B}{\rho V^2} \frac{1}{\sin \theta \sqrt{1 + \frac{1}{2} \left(\frac{1}{R}\right)^2 \cos^2 \theta}} \right\} d\theta, \quad (17) \\ dy = & -\left\{ 2R^2 B \sqrt{1 + \frac{1}{2} \left(\frac{1}{R}\right)^2 \cos^2 \theta} \cdot \frac{\cos \theta}{\sin^2 \theta} \right. \\ & + \frac{B}{2} \frac{\cos \theta}{\sqrt{1 + \frac{1}{2} \left(\frac{1}{R}\right)^2 \cos^2 \theta}} + \frac{B}{4R^2} \frac{\cos \theta \cdot \sin^2 \theta}{\left\{1 + \frac{1}{2} \left(\frac{1}{R}\right)^2 \cos^2 \theta\right\}^{3/2}} \\ & \left. + \frac{p_A \cdot B}{\rho V^2} \frac{1}{\sqrt{1 + \frac{1}{2} \left(\frac{1}{R}\right)^2 \cos^2 \theta}} \cdot \frac{\cos \theta}{\sin^2 \theta} \right\} d\theta \end{aligned}$$

$$+ \frac{p_A \cdot B}{2\rho U^2} \left\{ \frac{\cos \theta}{1 + \frac{1}{2} \left(\frac{1}{R} \right)^2 \cos^2 \theta} \right\}^{3/2} d\theta . \quad (18)$$

Under the boundary conditions ;

$$\left. \begin{array}{ll} \mathbf{x}=\mathbf{y}=\mathbf{0} & \text{for } \theta=\pi/2 \\ \mathbf{x}=0 & \text{for } R=1 \end{array} \right\} , \quad (19)$$

the solutions of Eq. (17) and Eq. (18) are

$$\begin{aligned} \frac{x}{B} = & \frac{1}{2} \cos \theta \left\{ \frac{1}{\sqrt{1 + \frac{1}{2} \left(\frac{1}{R} \right)^2 \cos^2 \theta}} - \frac{1}{\sqrt{1 + \frac{1}{2} \cos^2 \theta}} \right\} \\ & - 2R^2 \sqrt{1 + \frac{1}{2R^2}} \cdot \ln \frac{\sqrt{1 + \frac{1}{2} \left(\frac{1}{R} \right)^2 \cos^2 \theta} - \sqrt{1 + \frac{1}{2} \left(\frac{1}{R} \right)^2} \cdot \cos \theta}{\sin e} \\ & + \sqrt{6} \cdot \ln \frac{\sqrt{1 + \frac{1}{2} \cos^2 \theta} - \sqrt{\frac{3}{2}} \cos \theta}{\sin e} \\ & - \sqrt{2} R \cdot \ln \frac{\cos \theta + \sqrt{2} R \sqrt{1 + \frac{1}{2} \left(\frac{1}{R} \right)^2 \cos^2 \theta}}{\sqrt{2} R \sqrt{1 + \frac{1}{2} \left(\frac{1}{R} \right)^2}} \\ & + \sqrt{2} \cdot \ln \frac{\cos \theta + \sqrt{2} \sqrt{1 + \frac{1}{2} \left(\frac{1}{R} \right)^2 \cos^2 \theta}}{\sqrt{3}} \\ & + \frac{p_A}{2\rho U_0^2} \left\{ \frac{\cos \theta}{\left(1 + \frac{1}{2R^2} \right) \sqrt{1 + \frac{1}{2} \left(\frac{1}{R} \right)^2 \cos^2 \theta}} \right. \\ & + \frac{1}{\left(1 + \frac{1}{2R^2} \right)^2} \cdot \ln \frac{\sin \theta}{\sqrt{1 + \frac{1}{2R^2}} \cdot \cos \theta + \sqrt{1 + \frac{1}{2} \left(\frac{1}{R} \right)^2 \cos^2 \theta}} \\ & - \frac{2 \cdot \cos e}{3 \sqrt{1 + \frac{1}{2} \cos^2 \theta}} - \left(\frac{2}{3} \right)^{3/2} \cdot \ln \frac{\sin e}{\sqrt{\frac{3}{2} \cos \theta} + \sqrt{1 + \frac{1}{2} \cos^2 \theta}} \Big\} \\ & - \frac{p_A}{\rho V^2} \left\{ \frac{1}{\sqrt{1 + \frac{1}{2R^2}}} \cdot \ln \frac{\sin e}{\sqrt{1 + \frac{1}{2R^2}} \cdot \cos \theta + \sqrt{1 + \frac{1}{2} \left(\frac{1}{R} \right)^2 \cos^2 \theta}} \right. \\ & \left. - \sqrt{\frac{2}{3}} \cdot \ln \frac{\sin e}{\sqrt{\frac{3}{2} \cos \theta} + \sqrt{1 + \frac{1}{2} \cos^2 \theta}} \right\} , \quad (20) \end{aligned}$$

$$\begin{aligned}
\frac{y}{B} = & \frac{\sqrt{2}}{2} R \left\{ \arcsin \left(\frac{\sin \theta}{\sqrt{1+2R^2}} \right) - \arcsin \left(\frac{1}{\sqrt{1+2R^2}} \right) \right\} \\
& + 2R^2 \left(\frac{\sqrt{1 + \frac{1}{2} \left(\frac{1}{R} \right)^2 \cos^2 \theta}}{\sin \theta} - 1 \right) - \frac{1}{2} \left(\frac{\sin \theta}{\sqrt{1 + \frac{1}{2} \left(\frac{1}{R} \right)^2 \cos^2 \theta}} - 1 \right) \\
& + \frac{\sqrt{2} R}{4} \left\{ \arcsin \left(\frac{1-2R^2-2\cos^2 \theta}{1+2R^2} \right) - \arcsin \left(\frac{1-2R^2}{1+2R^2} \right) \right\} \\
& - \frac{p_A}{\rho V^2} \left\{ \frac{\sqrt{1 + \frac{1}{2} \left(\frac{1}{R} \right)^2 \cos^2 \theta}}{\left(1 + \frac{1}{2R^2} \right) \sin \theta} - \frac{1}{1 + \frac{1}{2R^2}} \right\} \\
& + \frac{p_A}{2\rho U_0^2} \left\{ \frac{\sin \theta}{\left(1 + \frac{1}{2R^2} \right) \sqrt{1 + \frac{1}{2} \left(\frac{1}{R} \right)^2 \cos^2 \theta}} - \frac{1}{1 + \frac{1}{2R^2}} \right\} . \quad (21)
\end{aligned}$$

According to the Eq. (20) and Eq. (21), any co-ordinates of the deflected jet are obtained through the parameter, θ (the deflected angle).

2. The horizontal velocity distribution

There are the zone of flow establishment and the zone of established flow (Tsuchiya, 1962; Kato, 1969) not only in a free jet which streams into the water basin at rest, but also in a jet that streams in normal to the water basin which has the uniform and steady flow.

In the zone of flow establishment, it satisfies to consider that there is no effect of the uniform flow.

Therefore, in its zone the jet axis is not deflected and the horizontal velocity distribution is similar to that of the free jet as follows (Kato, 1969),

$$u = U_0(1 + \operatorname{erf}(\xi))/2, \quad (22)$$

$$\xi = \sigma \left(A - \frac{y}{x} \right), \quad (23)$$

where σ and A are free constants, U_0 is the outlet velocity, erf is error function, that is,

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

(see Fig. 3).

While the horizontal velocity distribution in the zone of established flow is obtained under the following assumptions,

(1) the velocity profiles are similar, so that the velocities are functions of $\zeta (= \sigma \cdot r/s)$, where s is a length along the jet axis ($\xi = \xi_0$) from the outlet, r is a length along the η -axis normal to $\xi = \xi_0$ (see Fig. 4),

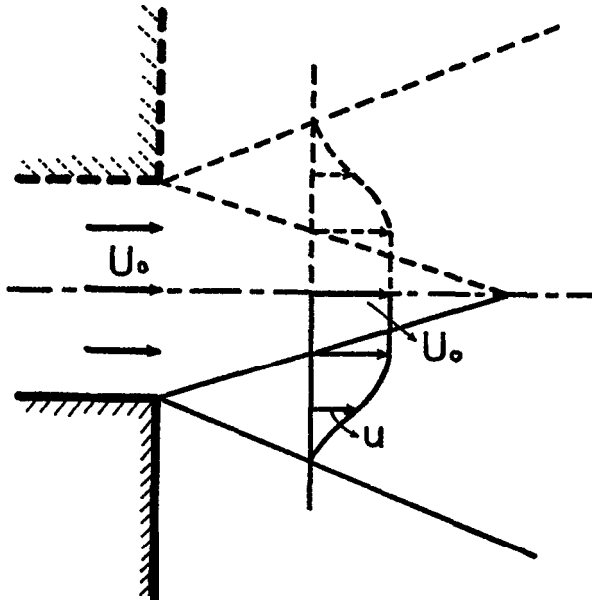


Fig. 3. Definition sketch for zone of flow establishment in the free jet.

(2) the function of the horizontal velocity distribution is obtained with the hyperbolic-function similar to the free jet.

$(U_0 - V)$, $(U - V)$ and $(u - V)$ are adapted as the quantities considering a uniform flow velocity (V), where U_0 is a velocity of the outlet, U is a velocity on the jet axis and u is a velocity of the jet direction at any point.

From the assumptions (1) and (2), the velocity distribution of the jet direction in a cross-section normal to $\xi = \xi_0$ is

$$\frac{u - V}{U_0 - V} = 1 - \tanh^2 \zeta, \quad (24)$$

$$\text{where } \zeta = \sigma \cdot r / s. \quad (25)$$

A method to obtain values of s and r is shown in chapter 4.

3. The decrease of the maximum velocity

It may be considered that the maximum velocity (U) in the zone of established flow decreases in inverse proportion to a power of the length (s) along the jet axis ($\xi = \xi_0$) similarly to the free jet, therefore it is obtained as the following equation,

$$\frac{U - V}{U_0 - V} = \frac{\lambda}{s^m}, \quad (26)$$

where λ and m are free constants (see Fig. 4).

$$y = C \cdot (\cosh \xi_0 + \cosh \xi \cdot \cos \eta), \quad (28)$$

or

$$\frac{(y - C \cosh \xi_0)^2}{C^2 \cosh^2 \xi_0} + \frac{x^2}{C^2 \sinh^2 \xi_0} = 1. \quad (29)$$

In Fig. 6, if the co-ordinates of O, P and Q are (ξ_0, π) , (ξ_0, θ) and (ξ, θ) respectively, ds and dr are the length elements of s and r respectively, and they are obtained from Eq. (27), Eq. (28), h_ξ and h_η , where h_ξ and h_η are linearizing factors, that is,

$$ds = h_\eta \cdot d\eta, \quad (30)$$

$$dr = h_\xi \cdot d\xi, \quad (31)$$

$$h_\xi = \left\{ \left(\frac{\partial x}{\partial \xi} \right)^2 + \left(\frac{\partial y}{\partial \xi} \right)^2 \right\}^{1/2}, \quad (32)$$

$$h_\eta = \left\{ \left(\frac{\partial x}{\partial \eta} \right)^2 + \left(\frac{\partial y}{\partial \eta} \right)^2 \right\}^{1/2}. \quad (33)$$

Therefore, s and r become

$$s = C \cdot \int_0^\pi (\sinh^2 \xi_0 + \sin^2 \eta)^{1/2} d\eta, \quad (34)$$

$$r = C \cdot \int_{\xi_0}^\xi (\sinh^2 \xi + \sin^2 \theta)^{1/2} d\xi. \quad (35)$$

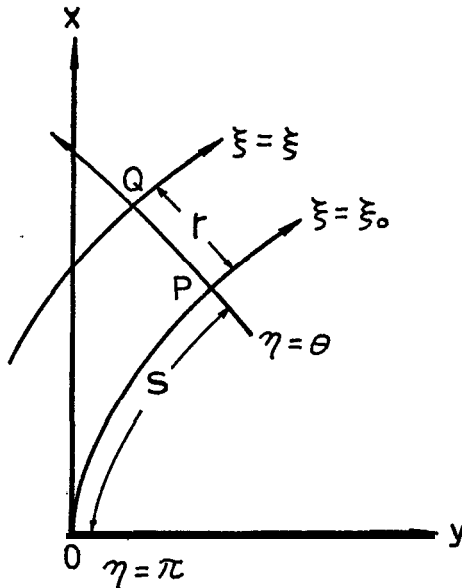


Fig. 6. Notation used in the analysis.

EXPERIMENTS

1. The equipments and methods of an experiment

The channel using the experiment is shown in Fig. 7.

The velocity is measured with pitot tubes, the direction of stream lines is obtained with spilling ink pouring into the flow from an injection syringe, and the appearance of whole flows is required with woolen yarns.

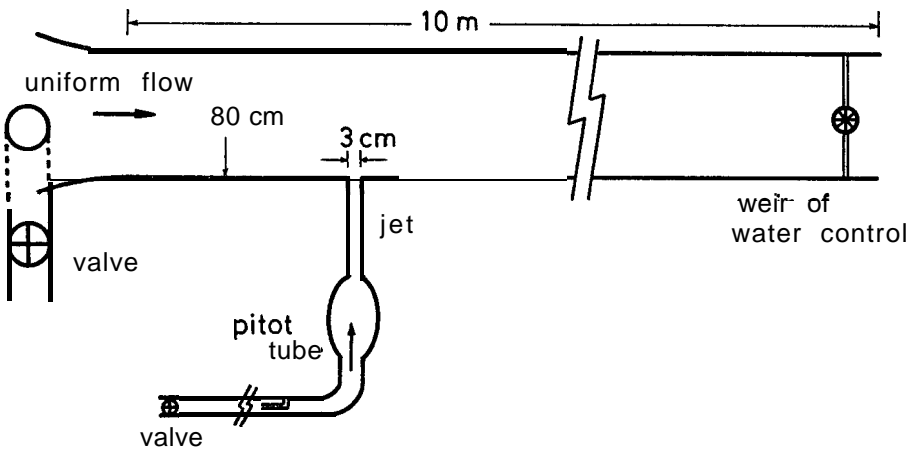


Fig. 7. Experimental apparatus.

2. The deflected jet

A difference of the deflection in virtue of ratio of the outlet jet velocity (U_0) to the uniform flow velocity (V), that is, $R=U_0/V$, is shown in Fig. 8.

Compared the experimental results with Eq. (20) and Eq. (21), R and B are obtained from initial conditions, where B is a width of the outlet, but p_A is not known. So that its value must be obtained from the measured values. Already Eq. (20) and Eq. (21) were led under the assumption which the jet and the uniform flow were irrotational fluid, but in real phenomenon eddies occur (see Fig. 9). So p_A contains the influence of eddies.

Considering the fact described above, the following equation is assumed,

$$\frac{p}{w} = \beta' \cdot \frac{U_0^2}{2g}$$

or

$$PA = \beta \cdot U_0^2, \tag{36}$$

where β is a function of R , and it is obtained from experiments as shown in Fig. 10.

By the aid of Eq. (36), the comparison of experimental values with Eq. (20) and Eq. (21) is shown in Fig. 11, where dotted lines are obtained from Eq. (20)

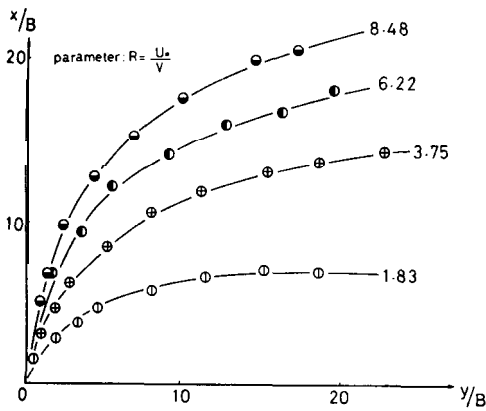


Fig. 8. Locations of the jet center-line.

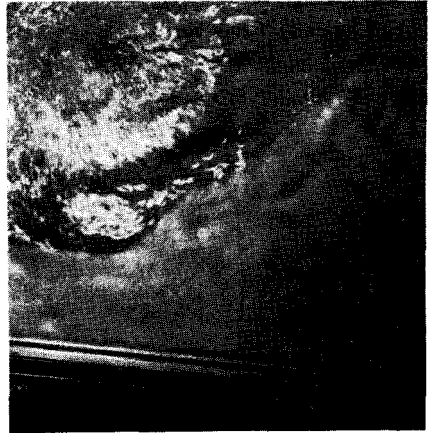


Fig. 9. Eddies occurring in down stream of the jet.

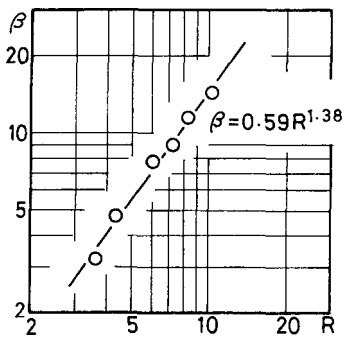
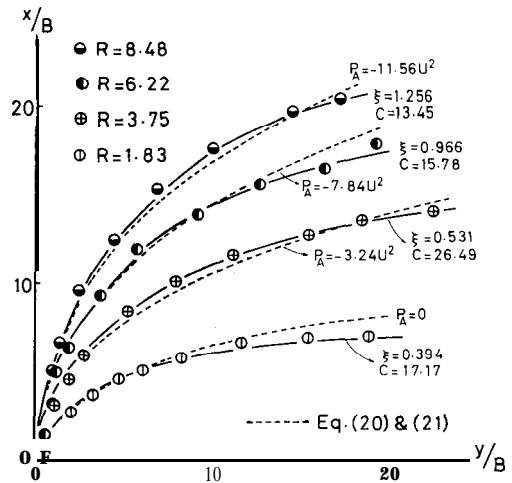
Fig. 10. Relation between R and β .

Fig. 11. Locations of the jet center-line compared the experimental results with the calculated results.

and Eq. (21).

3. The horizontal velocity distribution

In the zone of established flow, the velocity distribution is obtained from Eq. (24) and Eq. (25).

But as described in the last chapter, owing to the occurrence of eddies, the velocity distribution is not symmetric with respect to the jet axis, however it is solved from making change of the values of σ in Eq. (25).

The relation between σ and R in upper and down stream sides respectively is shown in Fig. 12 and Fig. 13.

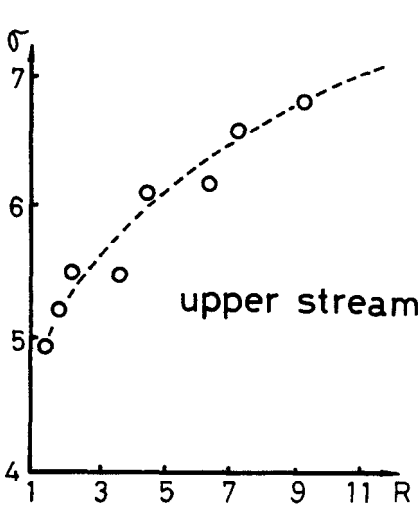


Fig. 12. Between R and σ in upper stream.

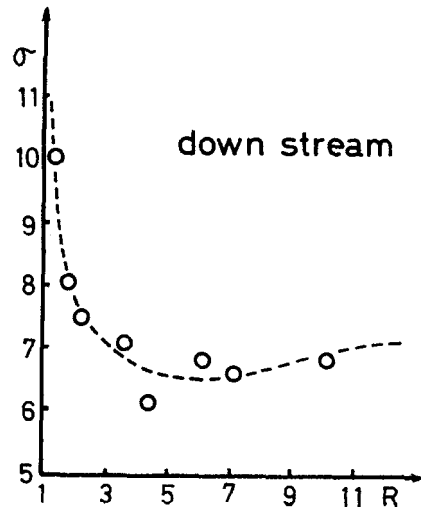


Fig. 13. Between R and σ in down stream.

The value of σ converges to that of the free jet (about 7.15) with increase of R . By the aid of Eq. (24), Eq. (25), Fig. 12 and Fig. 13, the horizontal velocity distributions are shown in Fig. 14.

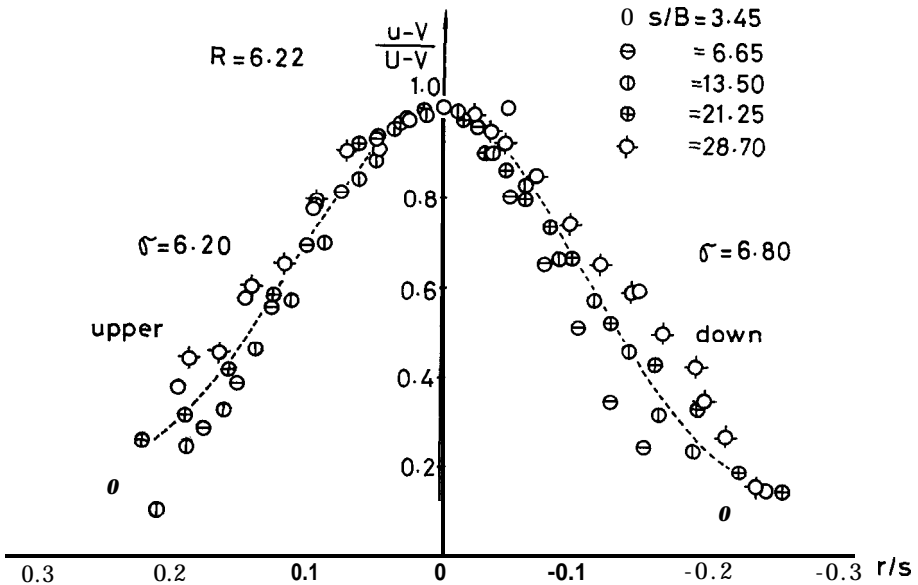


Fig. 14. Horizontal velocity distribution in jet ($R=6.22$).

4. The decrease of maximum velocity

The decrease of maximum velocity along the jet axis is obtained, and from using of non-dimensional quantities, t and s/B , the following equation is obtained,

$$t = \frac{U-V}{U_0-V} = \alpha \cdot \left(\frac{1}{s/B} \right)^m, \quad (37)$$

where α and m are constants, B is a width of the jet outlet. Its results are shown in Fig. 15.

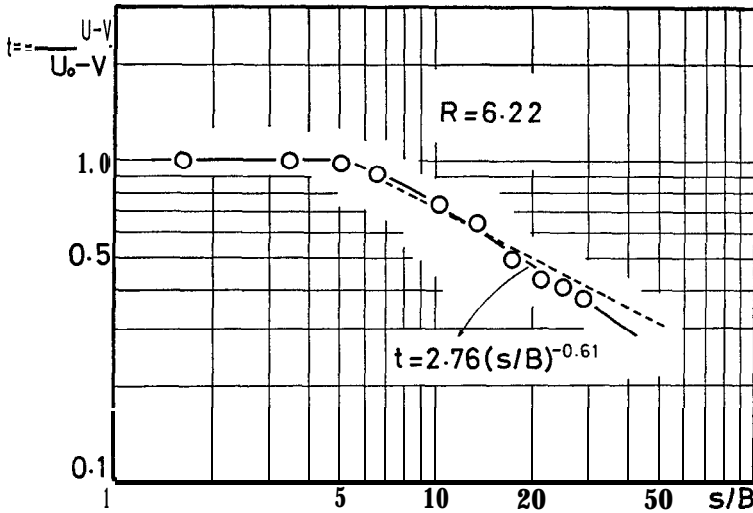


Fig. 15. Decrease of maximum velocity ($R=6.22$).

APPLICATIONS

Recently, flux and strength of the stream which discharges from the sluices of large scale reclaimed lands and estuary reservoirs have been increasing.

So, since its flow has a large momentum, it influences facilities around the outlet of the sluices and the opposite shore where water basin is narrow. The fact described above is considered in the Project of Reclamation in Ariake Bay. In Fig. 16, a tentative plan of the reclamations in Ariake Bay is shown (Takata, 1959).

The quantities for calculations are as follows, the width of the outlet (B) = 300 m, the maximum velocity of the outlet (U_0) = 5 m/sec (calculated from the maximum flux = 9000 m³/sec), the any velocity of the outlet (U_0) = 2.5 m/sec, the maximum tidal current (the uniform flow velocity (V)) = 1.2 m/sec (estimated from the model experiments by Tohara (1959)).

For convenience, it is considered that tidal current is uniform (1.2 m/sec) along the reclamation dike (PQ). The result is shown in Fig. 17. The result that Tohara (1959) investigated experimentally the appearance of tidal flow at

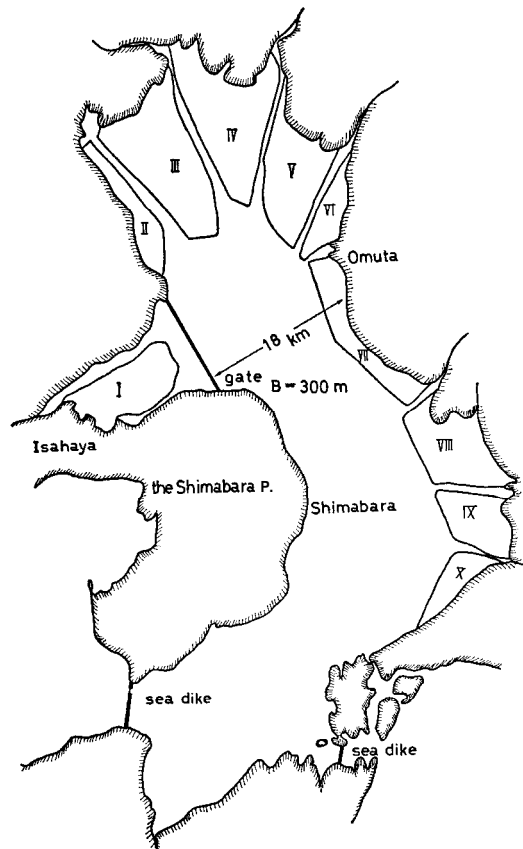


Fig. 16. A tentative plan of reclamations in Ariake Bay (by Takata, 1959).

final closure according to the Construction Plan of Nagasaki-Reclamation is shown in Fig. 18.

In his experiments the width of dike gap is assumed about 1 km, and the appearance of tidal current is estimated with the photograph, therefore Fig. 17 can not be compared with Fig. 18 quantitatively owing to differences of condition, but both can be done qualitatively.

CONCLUSIONS

(1) The theory that the jet axis is deflected with a uniform flow is obtained from the law of conservation of momentum through a parameter (0).

(2) The equations that obtain the deflected axis include p_A , which is dependent on eddies that occur at the downstream of the jet, and the following equation is obtained,

$$\beta = 0.59 \cdot R^{1.38} ,$$

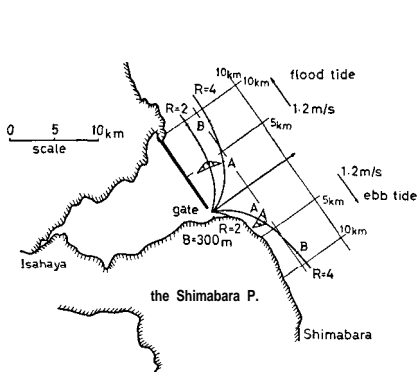


Fig. 17. Result calculated with Eq. (20) and Eq. (21).

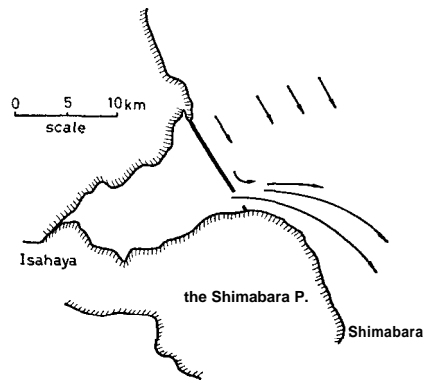


Fig. 18. Appearance of tidal flow at final closure according to the Construction Plan of Nagasaki-Reclamation (by Tohara, 1959).

where $R = U_0 / V$.

(3) The horizontal velocity distribution is obtained with the hyperbolic function similar to the free jet.

(4) The decrease of maximum velocity is obedient to a power of the distance along the jet axis from the outlet.

(5) The results described above agree qualitatively with the model experiments.

ACKNOWLEDGEMENTS

The author thanks to honorary Prof. Takata and assistant Prof. Tohara for many helpful discussions and suggestions during this work.

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