Numerical Analysis of Nonlinear Directional Couplers for Optical Devices

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NUMERICAL ANALYSIS OF NONLINEAR DIRECTIONAL COUPLERS FOR OPTICAL DEVICES

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Chapter 1

Introduction

1.1 Backgrounds and Objective

The research on electromagnetic wave propagation in nonlinear dielectric materials has been developed to date. Electromagnetic waves in the material show interesting and widely applicable phenomena such as the optical bistability[1], the second harmonic generation, solitons, and the self focusing[2]. Recently, the applications of these nonlinear effects for a optical-waveguide device come into prominence so as to realize ultra-fast all-optical-signal-processing in integrated optical circuits. The nonlinear directional coupler (NLDC) which makes use of the directional coupling between Kerr-like nonlinear waveguides is one of the most promising devices. The function is governed by the linear coupling of two waveguides in close proximity and its nonlinear modulation due to the change of refractive index depending on the optical field intensity. During the past several years, a symmetric NLDC structure in which two linear planar waveguides separated by a common nonlinear gap layer [3]-[10] or the reversed situation [11]-[13] has been extensively investigated by using analytical and numerical approaches.

The coupled-mode theory is an analytical approximate approach which provides an
analytical closed description of the operating characteristics of NLDC such as the critical power for switching. Various formulations of the coupled-mode analysis of NLDC are possible depending on the choice of propagation model. Jensen [3] made a significant initial effort on the coupled-mode analysis, in which the modal fields of the isolated linear waveguides were employed as the basis of propagation model and the nonlinear effect in the coupling was accounted for only by the self-phase and the cross-phase modulation terms. The difficulties and the shortcomings of Jensen’s simple analysis has been recently criticized and several improvements have been reported [4]-[8]. Meng and Okamoto [7] made a substantial contribution in those improvements. They employed the modal fields of the isolated nonlinear waveguides as the basis and derived the improved coupled-mode equations in which all of the coefficients depend on the optical power. Dios et al. [8] followed up the work of Meng and Okamoto [7] with a further investigation, and developed the coupled-mode analysis based on the nonlinear supermodes of the composite waveguide system. Although these new treatments yield needed improvements, it is difficult, in general, to assert the validity and the accuracy of the solutions by the method itself. It is also noted that the coupled-mode analysis by Dios et al. [8] must resort to an involved numerical calculation to obtain the nonlinear supermodes.

One of the representative numerical approaches to the NLDC to verify the analytic coupled-mode solution is the direct solution method of nonlinear differential equation using the orthogonal collocation method[14][15]. In this method, the electromagnetic fields governing the NLDC structure are first expanded in terms of Hermite-Gaussian functions and the Helmholtz equation is transformed into a set of matrix differential equations which is solved efficiently by making use of the orthogonality of the coefficient matrix. Some important characteristics such as the coupling length and the critical power of the NLDC can be obtained also from this method [26]–[29] [37]–[39]. This method
is a reliable numerical technique for various guided wave problems, which allows the numerical computation of field distributions using the step size as small as required in both directions. However, the number of collocation points is limited because of the overflow in the computation of weighted function. In this sense, the use of orthogonal collocation method is restricted to relatively small structure model.

The more accurate numerical approach to NLDC is represented by the beam-propagation method (BPM), which provides a unified treatment of various guided wave structures within the paraxial approximation. Wabnitz et al. [9] and Thylén et al. [10] used the BPM based on the Fast-Fourier-Transform algorithm (FFT-BPM) [16] to simulate the propagation of nonlinear waves in the coupler. Their numerical examples are mainly concerned with the behavior of power transfer along the waveguides. The rigorous numerical evaluation of the coupling length and critical power is of significant importance not only to verify the validity of coupled-mode approximations but also to demonstrate the precise device parameters for optical switching.

The purpose of this thesis is to develop a rigorous numerical analysis of the symmetric and asymmetric NLDC and to provide the highly accurate numerical results of the coupling length and critical power. The coupling nature of NLDC near the critical power changes sensitively, depending on the change of nonlinear refractive index in the propagating and transverse directions. Consequently, its numerical study requires a precise treatment of the nonlinear refractive index that is a function of electric field intensity.

Here we shall apply the beam propagation method based on the finite difference scheme (FD-BPM) [17]. We can take larger number of the discrete points and finer spacing distance than the collocation method. In addition to the analysis of two-waveguide couplers, we extend the analysis to the three-waveguide couplers. The results are com-
pared with those of two-waveguide NLDC. We will find some important coupling characteristics such as the optical control, which can be applied as novel optical devices. The extension of the problem from two-waveguide couplers to three-waveguide couplers is very easily achieved in the numerical simulation than in the analytical approach.

1.2 Organization on the Thesis

This thesis is organized from five chapters. In chapter 2, the formulation of FD-BPM is explained in detail. To evaluate precisely the nonlinear refractive index change in one small propagation step size, we introduce the iteration procedure. As an index of the optical power transfer which is carried along the NLDC, the normalized transferred power is defined by using the discretized electromagnetic field.

In chapter 3, the power transfer characteristics of symmetric and asymmetric two-waveguide nonlinear directional couplers are numerically investigated in detail by FD-BPM. This method allows us to set the desired space discretization for the differenciation as fine as possible. Before starting the simulation, the discretization grid size is carefully determined. Two kinds of symmetric NLDC are first considered in the succeeding two sections. One is composed of two identical linear waveguide-cores and a common nonlinear cladding layer. The other is a NLDC with two identical nonlinear waveguide-cores situated in linear claddings. The numerical results for the power transfer characteristics are presented. In the third section, a novel asymmetric NLDC structure is proposed that consists of a nonlinear waveguide-core and a linear waveguide-core situated in a linear claddings. The power transfer characteristics are analyzed for two different excitation conditions; the excitation of one of two waveguides and the excitation of both waveguides with the combination of different input powers.
In chapter 4, we investigate two kinds of planar three-waveguide NLDC. One is composed of three identical nonlinear waveguide-cores in linear claddings. The other is a NLDC with two identical linear waveguides situated both sides of a center waveguide with nonlinear Kerr-like medium in linear claddings. The input/output characteristics of these three-waveguide NLDCs are analyzed numerically by FD-BPM for the linear TE<sub>0</sub> incident wave. We also investigate the case that the weak control light is simultaneously given into the center waveguide-core when one of the outer waveguide is illuminated by the signal light. Numerical results demonstrate that the proposed three-waveguide NLDCs are more useful for optical power filtering and switching devices than the two-waveguide NLDC.

In chapter 5, the results obtained throughout this study are briefly summarized. Some future subjects are also mentioned.
Chapter 2

Numerical Analysis Techniques

2.1 Wave Equation of Planar Waveguide

We consider the light wave propagating in the inhomogeneous medium with refractive index \( n(x, y, z) \) and the constant permeability \( \mu_0 \). From Maxwell's equations, we can derive following Helmholtz equations which the electric vector field \( \mathbf{E} \) and the magnetic vector field \( \mathbf{H} \) of the wave satisfies, respectively,

\[
\nabla^2 \mathbf{E} + k_0^2 n^2(x, y, z) \mathbf{E} + \nabla \left( \frac{\nabla(n^2)}{n^2} \cdot \mathbf{E} \right) = 0, \tag{2.1}
\]
\[
\nabla^2 \mathbf{H} + k_0^2 n^2(x, y, z) \mathbf{H} + \frac{\nabla(n^2)}{n^2} \times (\nabla \times \mathbf{H}) = 0, \tag{2.2}
\]

where

\[
\nabla = \frac{\partial}{\partial x} i_x + \frac{\partial}{\partial y} i_y + \frac{\partial}{\partial z} i_z, \tag{2.3}
\]

and \( k_0 \) is the wave number in free space. The time dependence of the monochromatic light wave \( \exp(j\omega t) \) is assumed and the expression is omitted throughout this thesis. In many optical waveguides, the variation of refractive index profile is very small, i.e. \( \nabla(n^2) \approx 0 \). Therefore the third terms of Eq.(2.1) and Eq.(2.2) are negligible, respectively.

Generally, the configuration of many optical devices is three dimensional. The general
behavior of electromagnetic wave must be described by three-dimensional Eqs.(2.1) and (2.2). However, in most of optical devices, we can make use of the effective index method [21] and convert the three-dimensional problem to a pair of the two-dimensional (planar) one. Therefore, we are supposed to investigate the wave propagation in a planar optical device, such as a directional coupler depicted in Figure 2.1. Practically, we can reduce the dimension of Eqs.(2.1) and (2.2) by letting $\partial / \partial y = 0$.

The modes of the planar waveguide can be classified as TE (Transverse Electric) and TM (Transverse Magnetic) modes. The flow of NLDC analysis procedure for both modes is as follows. For TE mode, we choose the unique electric field of transverse direction as the leading field and calculate the value of nonlinear refractive index directly from this leading field. For TM mode, we choose the transverse magnetic field as the leading field and estimate the value of nonlinear refractive index utilizing the derivatives of the leading field which yield to a pair of electric field components. We refer to only TE mode in this thesis because the analysis of TM mode is essentially similar to that of TE mode in the analysis procedure. Using above conditions, we obtain the scalar Helmholtz equation for the transverse electric field $E_y$ of TE mode:

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial z^2} + k_0^2 n^2(x, z) E_y(x, z) = 0.$$\hspace{1cm} (2.4)

This is the starting equation of the numerical analysis of this thesis.

### 2.2 Finite Difference Beam Propagation Method

The transverse electric field component $E_y$ of the TE polarization satisfies the following nonlinear Helmholtz equation:

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial z^2} + k_0^2 n^2(x, z, E_y) E_y(x, z) = 0$$\hspace{1cm} (2.5)
Figure 2.1: Configuration of the planar waveguide.
where $k_0$ and $n(x, z, E_y)$ denote the wave number in free space and the refractive index profile, respectively. The refractive index in the Kerr-like nonlinear layer changes in proportion to the electric field intensity $|E_y|^2$ as follows;

$$n(x, z, E_y) = (n_1^2 + n_{NL}|E_y(x, z)|^2)^{1/2}, \quad (2.6)$$

where $n_1$ and $n_{NL}$ are the linear refractive index and the nonlinear coefficient, respectively.

In this section the finite-difference scheme is applied directly for the paraxial wave equation. The coupling between waveguides treated in this thesis is quite weak because the waveguide cores are well separated from each other. Therefore we can assume that the propagating wave has a slowly varying amplitude profile and the rapidly changing phase factor with a phase constant $\beta$. Applying this expression for the transverse electric field of TE wave, $E_y(x, z)$ is described as following form;

$$E_y(x, z) = E(x, z) \exp(-j\beta z) \quad (2.7)$$

where $E(x, z)$ is the amplitude envelope function. Substituting Eq.(2.7) into Eq.(2.5) and utilizing the fact that the term $\partial^2 E / \partial z^2$ is negligible compared with other terms, the paraxial wave equation is obtained,

$$2j\beta \frac{\partial E}{\partial z} = \frac{\partial^2 E}{\partial x^2} + [k_0^2 n_1^2(x, z) - \beta^2] E. \quad (2.8)$$

Following the scheme of finite-difference beam propagation method, the paraxial wave equation (2.8) with an arbitrary phase constant $\beta$ is discretized as follows:

$$2j\beta \frac{\partial E_m}{\partial z} = \frac{E_{m-1} - 2E_m + E_{m+1}}{\Delta x^2}$$

$$+ [k_0^2 n_1^2(x_m, |E_m|^2) - \beta^2] E_m \quad (2.9)$$

with

$$E_m = E_m(x, z) = E(x_m, z) \quad (m = 1, 2, 3, ..., M)$$

$$x_m = [m - 1 - (M - 1)/2] \Delta x \quad (2.10)$$
where \( x_m \) and \( \Delta z \) are the grid points and their spacing in the \( x \) direction. The unit \( M \) of grid points is taken to be an odd integer so that the center of grids coincides with that of the guiding structure. Using Crank-Nicolson's scheme \[17\] in the differentiation with respect to \( z \), Eq.(2.9) can be rewritten as follows:

\[
-E_{m-1}(z + \Delta z) + p_mE_m(z + \Delta z) - E_{m+1}(z + \Delta z) \\
= E_{m-1}(z) + q_mE_m(z) + E_{m+1}(z)
\] (2.11)

with

\[
p_m = 2 + 2\alpha \frac{\Delta x^2}{\Delta z} - b_m\Delta x^2
\] (2.12)

\[
q_m = -2 + 2\alpha \frac{\Delta x^2}{\Delta z} + b_m\Delta x^2
\] (2.13)

\[
a = 2j\beta
\] (2.14)

\[
b_m = k_0^2n^2(x_m, |E_m(z)|^2) - \beta^2
\] (2.15)

where \( \Delta z \) is the small step size in the \( z \) direction, and we have assumed that the nonlinear refractive index remains constant within a sufficiently small propagation length \( \Delta z \). The effect of a small change in the nonlinear refractive index on the field distribution can be taken into account by using the second-order iteration procedure as follows:

(a) Calculate the field \( E_m^{(0)}(z + \Delta z) \) with \( b_m = k_0^2n^2(x_m, |E_m^{(0)}(z)|^2) - \beta^2 \) by making use of the initial field \( E_m^{(0)}(z) \),

(b) Calculate the field \( E_m^{(1)}(z + \Delta z) \) with \( b_m = k_0^2\{n^2(x_m, |E_m^{(0)}(z)|^2) + n^2(x_m, |E_m^{(0)}(z + \Delta z)|^2)\}/2 - \beta^2 \),

(c) Calculate the field \( E_m^{(2)}(z + \Delta z) \) with \( b_m = k_0^2\{n^2(x_m, |E_m^{(0)}(z)|^2) + n^2(x_m, |E_m^{(1)}(z + \Delta z)|^2)\}/2 - \beta^2 \),

(d) Use \( E_m^{(2)}(z + \Delta z) \) as the initial field for the calculation in the next step,
where the superscripts (0), (1), and (2) represent the order of the iteration, respectively.

Equation (2.11) together with the second-order iteration results in a tridiagonal system of linear equations, which can be solved efficiently by Gaussian elimination under a suitable launching condition at the input end \( z = 0 \).

The piecewisely linear tridiagonal equations can be described by a matrix form as

\[
P x = Q
\]

(2.16)

where

\[
P = \begin{bmatrix}
  p_1 & -1 & & & \\
  -1 & p_2 & -1 & & \\
  & \cdots & \cdots & \cdots & \\
  0 & & & 0 & -1 \\
\end{bmatrix}
\]

(2.17)

\[
x = \begin{bmatrix}
  E_1(z + \Delta z) \\
  E_2(z + \Delta z) \\
  \vdots \\
  E_M(z + \Delta z)
\end{bmatrix}
\]

(2.18)

\[
Q = \begin{bmatrix}
  Q_1 \\
  Q_2 \\
  \vdots \\
  Q_{M-1} \\
  Q_M
\end{bmatrix} = \begin{bmatrix}
  q_1 E_1(z) + E_2(z) \\
  E_1(z) + q_2 E_2(z) + E_3(z) \\
  \vdots \\
  E_{M-2}(z) + q_{M-1} E_{M-1}(z) + E_M(z) \\
  E_{M-1}(z) + q_M E_M(z)
\end{bmatrix}
\]

(2.19)

In the case that a directional coupler is illuminated by the eigenmode of an isolated waveguide, the wave propagates along the coupler and well confined within the core region. The power is seldom radiated to the infinity. Therefore in the first and the last rows of Eqs.(2.17) and (2.19), we can approximate the electric fields on the outer-most grid points to be zero. However, the computation window must be set sufficiently wide so that the electric field becomes nearly zero at the edge of the window. We can solve Eq. (2.16) by applying the following procedure;

\[
E_M(z + \Delta z) = \iota_M
\]

(2.20)
\[ E_m(z + \Delta z) = t_m - s_mE_{m+1} \quad (m = 1, 2, \ldots, M - 1) \] (2.21)

where

\[ s_1 = -1/p_1 \] (2.22)

\[ t_1 = Q_1/p_1 \] (2.23)

\[ s_m = -1/(p_m + s_{m-1}) \quad (m = 2, 3, \ldots, M - 1) \] (2.24)

\[ t_m = (Q_m + t_{m-1})/(p_m + s_{m-1}) \quad (m = 2, 3, \ldots, M) \] (2.25)

### 2.3 Definition of Normalized Transferred Power

Various features of propagating waves along optical waveguides can be simulated by making use of the field distribution calculated by the numerical method given in the preceding section. For the nonlinear directional coupler which is investigated in this thesis, the evaluation of the coupling length and the power transfer characteristics between the coupled waveguides are of most importance. For this purpose, we define the normalized transferred powers \( \eta_i(z) \) in respective waveguides \( i \). Figure 2.2 shows an example of three-waveguide nonlinear directional coupler. For this case, \( \eta_i(z) \) is defined as follows;

\[ \eta_a(z) = \rho_a(z)/P_{i,in}, \] (2.26)

\[ \eta_b(z) = \rho_b(z)/P_{i,in}, \] (2.27)

\[ \eta_c(z) = \rho_c(z)/P_{i,in}, \] (2.28)

with

\[ \rho_a(z) = \frac{\beta_a}{2\omega\mu_0} \int_{x_i, x} \left| E(x, z) \right|^2 dx, \] (2.29)

\[ \rho_b(z) = \frac{\beta_b}{2\omega\mu_0} \int_{x_i, x} \left| E(x, z) \right|^2 dx, \] (2.30)

\[ \rho_c(z) = \frac{\beta_c}{2\omega\mu_0} \int_{x_i, x} \left| E(x, z) \right|^2 dx, \] (2.31)
where $\beta_a$, $\beta_b$, and $\beta_c$ are the linear propagation constants of each waveguide in the isolated situation. The starting and stopping points of above integrations are illustrated in Figure 2.2. $x_1$ and $x_M$ are the outer-most sampling points located in the substrate and cover layers. $x_{ab}$ and $x_{bc}$ are the mid-points between the waveguide $a$ and $b$ and between $b$ and $c$, respectively. The integrations in Eqs. (2.29) to (2.31) are performed by the trapezoidal formula in terms of the sampled fields $E_m(z)$. The coupling length $L_c$ of the coupler is given by the propagation distance $z$ at which either $\eta_a(z)$, $\eta_b(z)$, or $\eta_c(z)$ takes the first maximum depending on the launching condition at the input end $z = 0$.

To estimate the normalized transferred power of the two-waveguide coupler, as depicted in Figure 3.1, the waveguide $c$ is removed and the outer-most sampling points $x_1$ and $x_M$ are chosen so that they are located far enough from the waveguide in the substrate and cover region where the field intensity becomes sufficiently small.
Figure 2.2: The evaluation of the normalized transferred power; the case of three-waveguide NLDC.
Chapter 3

Analysis of Two-Waveguide Nonlinear Directional Couplers

3.1 Introduction

Recently, there are great interests in the possibility of optical integrated devices for ultra-high-speed all-optical data processing. Two-waveguide nonlinear directional couplers (NLDC) are the basic elements in many of those optical circuits such as switches, dividers and combiners. The power transfer characteristics of NLDC have been widely investigated by the analytical [3]-[8] and numerical [9]-[13] techniques for a couple of decades.

The refractive index of the third-order nonlinear Kerr material shows the dependence on the electric field intensity of light wave itself. For the purpose of the control of the coupling characteristics by the optical light intensity, Jensen [3] applied the material to the coupler for the first time. Since the presentation of his theoretical work, many researchers have engaged in the study on the power-dependent coupling by making use of the coupled-mode theory. The coupled-mode theory is an analytical approximate approach which provides an analytical closed description of the operating characteristics of NLDC such
as the critical power for switching. Various formulations of the coupled-mode analysis of NLDC are possible depending on the choice of propagation model. Although these formulations give some predictions of the characteristics, it is difficult to assert the validity and the accuracy of the solutions by the method itself.

On the other hand, one of the representative numerical approaches to the NLDC to verify the analytic coupled-mode solution is the direct solution method of nonlinear differential equation using the orthogonal collocation method [14]. This is a reliable numerical technique for various guided wave problems. However, the number of collocation points is limited because of the overflow in the computation of weighted function. In this sense, the use of orthogonal collocation method is restricted to relatively small structure model.

The more accurate numerical approach to NLDC is represented by the beam-propagation method (BPM), which provides a unified treatment of various guided wave structures within the paraxial approximation. Wabnitz et al. [9] and Thylen et al. [10] used the BPM based on the Fast-Fourier-Transform algorithm (FFT-BPM) [16] to simulate the propagation of nonlinear waves in the coupler. Their numerical examples are mainly concerned with the behavior of power transfer along the waveguides. The rigorous numerical evaluation of the coupling length and critical power is of significant importance not only to verify the validity of coupled-mode approximations but also to demonstrate the precise device parameters for optical switching.

In this chapter, the power transfer characteristics of symmetric and asymmetric two-waveguide nonlinear directional couplers are numerically investigated in detail by the finite difference beam propagation method (FD-BPM)[30][31]. This method allows us to set the desired space discretization for the differentiation as fine as possible. Before starting the simulation, the discretization grid size is carefully determined. Two kinds of
symmetric NLDC are first considered in the succeeding two sections. One is composed of two identical linear waveguide-cores and a common nonlinear cladding layer. The other is a NLDC with two identical nonlinear waveguide-cores situated in linear claddings. Some numerical results for the power transfer characteristics are presented using the FD-BPM.

In the fourth section, a novel asymmetric NLDC structure is proposed that consists of a nonlinear waveguide-core and a linear waveguide-core situated in linear claddings. The power transfer characteristics are analyzed for two different excitation conditions; the excitation of one of two waveguides and the excitation of both waveguides with a combination of different input powers.

Generally in the NLDC structure, following three stages of the output characteristic are observed as a function of the input optical power level.

1. Relatively low power level
   The coupling characteristics are very similar to the linear coupler because the nonlinear refractive index modulation is small enough. The optical wave couples perfectly from the input waveguide to the adjacent with increasing the coupling length as the input power level is increased to the critical power.

2. Critical power level
   Due to the close proximity of waveguides, the input power in one waveguide is divided into another waveguide. Once the power is divided to each waveguide, the optical wave never couples anymore and propagates keeping the split state. This input power level is referred to as a critical power. If the NLDC has two waveguides and the structure is symmetrical, the input power is divided into half to each waveguide.
3. Over the critical level

When the input power exceeds the critical level, the input wave couples imperfectly any longer. If the power is far over the critical, the input wave is trapped in the nonlinear region in which the refractive index is strongly modulated by the input power itself, and never couples to the adjacent waveguide.

The change of the coupling behaviour mentioned above is very important in the application for all-optical-signal-processing. We demonstrate these stages in the following chapter as results of numerical simulations.

3.2 Symmetric Nonlinear Directional Coupler with a Common Nonlinear Coupling Region

The geometry of a symmetric NLDC with a common nonlinear gap layer is shown in Figure 3.1. Two identical planar waveguides $a$ and $b$ with a linear medium of a refractive index $n_2$ and a width $d$ are situated parallel to each other with a separation distance $w$. The cover and substrate layers are composed of a linear medium with a refractive index $n_1$, and the intermediate layer of width $w$ is of a Kerr-like nonlinear medium with a refractive index

$$n_3(x, z) = (n_1^2 + n_{NL}|E(x, z)|^2)^{1/2}$$

(3.1)

where $n_{NL}$ denotes the nonlinear constant of the medium. The geometry is uniform in the $y$ and $z$ directions. We consider the light of wavelength $\lambda = 1.064 \, \mu m$ and take the values of parameters for NLDC in Figure 3.1 to be $n_2 = 1.57$, $n_1 = 1.55$, $n_{NL} = 6.377 \times 10^{-12} \, [m^2/V^2]$, and $d = 2.0 \, [\mu m]$. Those values of parameters are the same as in Ref.[7]. As the initial data for Eq. (2.11), we assumed that the linear $TE_0$ mode of waveguide $a$ alone with $n_{NL} = 0$ is incident at the input end $z = 0$. The amplitude of the
Figure 3.1: Geometry of a symmetrical directional coupler with a common nonlinear coupling region.

\[ n_3 = (n_1^2 + n_{NL} |E(x,z)|^2)^{1/2} \]
input wave \( E_{i,\text{max}} \) \([V/m]\) specified at \( x = \pm(d + w)/2 \) and \( z = 0 \) is related to the input power \( P_{i,\text{in}} \) \([W/m]\) as follows:

\[
E_{i,\text{max}} = \sqrt{\frac{2\omega \mu_0}{2\beta_0 d + \beta_0/\gamma_0}} P_{i,\text{in}}, \quad (i = a, b)
\]

where \( \omega = 2\pi c_0/\lambda \), \( c_0 \) is the velocity of light in free space, \( \mu_0 \) is the permeability in free space which is supposed to be constant in all the region, \( \gamma_0 = (\beta_0^2 - k_0^2 n_1^2)^{1/2} \), and \( \beta_0 \) is the propagation constant of the linear \( TE_0 \) mode in isolation.

Before starting the computation, we need to clarify the relation between the convergence of the numerical results and the small discretizing distances. We investigated the convergence of the coupling length in a linear directional coupler whose separation distance \( w = 2.0[\mu m] \), using the FD-BPM. The other parameters used here are the same as in Figure 3.1, except that \( n_{NL} = 0 \). For these parameters, the waveguide in isolation satisfies the single mode condition. The width of computation window is fixed to be \( |x| \leq 8.0[\mu m] \), while the number of grid points along \( x \) axis is variable. The waveguide \( a \) in Figure 3.1 in the linear limit is illuminated by a fundamental \( TE_0 \) mode of an isolated linear waveguide. In Figure 3.2, the coupling length is plotted as a function of a small propagation length for four typical spacing distances \( \Delta x \) along \( x \) axis. The coupling length is obtained as a propagation length at which the normalized transferred power defined by Eq. (2.27) takes the first maximum. It is shown that the coupling length converges very well when \( \Delta x \leq 0.05[\mu m] \) and \( \Delta z \leq 1.0[\mu m] \). For the numerical computation by the FD-BPM, therefore, the number of grid points and their spacing were chosen as \( M = 321 \) and \( \Delta x = 0.05[\mu m] \) in the \( x \) direction.

In the numerical analysis of a NLDC, we should carefully determine the small step size along \( z \) axis considering that the electromagnetic field changes slightly and continuously in a small distance \( \Delta z \). To determine the suitable step \( \Delta z \) in the \( z \) direction with or without
Figure 3.2: The linear coupling length for $w = 2.0[\mu m]$ as a function of small propagation step size $\Delta z$ for four typical spacing distance $\Delta z$. 
the iteration procedure mentioned in Section 2.2, we investigated the convergence of the critical power for a NLDC in Figure 3.1 with \( w = 2.4[\mu m] \). The result is shown in Figure 3.3 as a function of small propagation step \( \Delta z \). Consequently, the second order iteration procedure provides the needed accuracy even for \( \Delta z = 2.0[\mu m] \). If we do not use the second order iteration in the FD-BPM analysis, we must set \( \Delta z = 0.1[\mu m] \) as a reasonable choice from the result in Figure 3.3.

When we analyze the strongly coupled NLDC which is treated in Ref. [11] with the higher refractive index, the smaller difference between core and claddings, the narrower waveguide separation, the watchful attention must be paid. In such a strongly coupling case, the iterative procedure such as Adams-Moulton formula will play an important role in the convergence of numerical solutions.

We also examined carefully the energy conservation error of the numerical solutions, and we get \( \Delta z = 0.5[\mu m] \), \( \Delta z = 0.05[\mu m] \) and \( M = 321 \) with the second order iteration procedure, for which the energy conservation error less than \( 10^{-6} \) is attained.

Figure 3.4 shows the normalized transferred power \( \eta_b(z) \) obtained by the FD-BPM as the functions of propagation length \( z \) for \( w = 3.0[\mu m] \) and for different seven input power levels \( P_{a,in} \). The propagation length \( z \) at which \( \eta_b(z) \) takes the first maximum gives the coupling length \( L_c \). For \( P_{a,in} \leq 47.6[W/m] \), the full power transfer into the waveguide \( b \) is achieved and \( L_c \) increases with increasing \( P_{a,in} \). For \( P_{a,in} > 47.61[W/m] \), only a fraction of incident power is transferred and \( L_c \) decreases with increasing \( P_{a,in} \). The critical power \( P_c \) for switching from waveguide \( b \) to waveguide \( a \) is numerically estimated by the incident power which yields an extremely long coupling length. We have \( P_c = 47.61[W/m] \) for the present case.
Figure 3.3: The critical power $P_c$ as a function of $\Delta z$ for $w = 2.4[\mu m]$. The solid and dashed line indicate the result of second order iteration and of no iteration, respectively.
Figure 3.4: The normalized transferred power $\eta_b$ as a function of the propagating distance $z$ for $w = 3.0[\mu m]$ and for different seven input power levels $P_{a,\text{in}}$. 
Figures 3.5 shows the bird eyes' views of the evolutions of the normalized field distribution $|E(x, z)|/E_{a,\text{max}}$ along the coupler for $w = 3.0[\mu m]$ and for four typical input power levels; (a) the linear case, (b) $P_{a,\text{in}} = 47.61[W/m]$, (c) $P_{a,\text{in}} = 65.0[W/m]$, and (d) $P_{a,\text{in}} = 80.0[W/m]$. When $P_{a,\text{in}} = 80.0[W/m]$ that is far above the critical power $P_c = 47.61[W/m]$, the incident linear $TE_0$ mode is changed into the nonlinear surface-wave mode of the waveguide $a$ in isolation as the wave propagates. The peak of the amplitude $|E(x, z)|$ of the nonlinear surface wave is located in the nonlinear gap layer near the core boundary of the waveguide $a$.

Figure 3.6 shows the coupling length $L_c$ as a function of incident power $P_{a,\text{in}}$ for $w = 2.4[\mu m]$. The results of the simple coupled-mode theory [3] and the improved coupled-mode theory [7] are also plotted for the comparison. The critical power $P_c$ at which $L_c$ increases abruptly to infinity is given by 69.4[W/m], 127[W/m], and 65[W/m] for the FD-BPM, the simple coupled-mode theory, and the improved coupled-mode theory, respectively. We can see that the coupling length obtained by the improved theory [7] is in good agreement with that of the present numerical analysis, except for the small difference near the critical power. Figure 3.7 shows the critical power $P_c$ as a function of the gap layer width $w$, compared with the results of Ref.[3] and Ref.[7]. The critical power obtained from the improved coupled-mode theory [7] agrees well with that of the present analyses over the wide range of $w$, whereas the simple coupled-mode theory [3] leads to the much larger critical power in the strongly coupling regime with $w$ smaller than 3.0[\mu m].
Figure 3.5: (a) Bird eyes’ views of the evolutions of normalized electric field distribution $|E(x, z)|/E_{a,max}$ along the linear directional coupler for $w = 3.0{\mu m}$. 
Figure 3.5: (b) Bird eyes' views of the evolutions of normalized electric field distribution $|E(x, z)|/E_{a,max}$ for $w = 3.0[\mu m]$ and for the input power level $P_{a,in} = 47.61[W/m]$. 
Figure 3.5: (c) Bird eyes’ views of the evolutions of normalized electric field distribution $|E(x, z)|/E_{a,max}$ for $w = 3.0[\mu m]$ and for the input power level $P_{a,in} = 65.0[W/m]$. 
Figure 3.5: (d) Bird eyes' views of the evolutions of normalized electric field distribution \( |E(x, z)|/E_{a,\text{max}} \) for \( w = 3.0[\mu m] \) and for the input power level \( P_{a,\text{in}} = 80.0[W/m] \).
Figure 3.6: The coupling length $L_c$ as a function of the incident power $P_{in}$ for $w = 2.4[\mu m]$. The solid, dashed-dotted, and dashed lines indicate the results of the FD-BPM, Ref.[3], and Ref.[7], respectively.
Figure 3.7: The critical power $P_c$ as a function of the gap layer width $w$. The solid, dashed-dotted, and dashed lines indicate the results of the FD-BPM, Ref.[3], and Ref.[7], respectively.
3.3 Symmetric Nonlinear Directional Coupler with Nonlinear Waveguide Cores

The other configuration of a symmetric two-waveguide NLDC is depicted in Figure 3.8. This NLDC also provides a critical state. However, the critical power level is expected to be relatively small compared with the NLDC which was analyzed in the previous section because the refractive index of nonlinear waveguides is very strongly modulated due to the concentration of the optical field in the nonlinear waveguide core.

A set of the small step size $\Delta x$ and $\Delta z$ are chosen as $0.05\, [\mu m]$ and $0.1\, [\mu m]$ without the iteration procedure, respectively. The number of grid points along $x$ axis $M$ is 401. For the parameters $n_1 = 1.53$, $n_2 = 1.55$, $n_{NL} = 6.38 \times 10^{-12}\, [m^2/V^2]$, $d = 2.0\, [\mu m]$, $\lambda = 1.064\, [\mu m]$, the critical power level is depicted in Figure 3.9 as a function of the waveguide separation $w$. The solid and the dashed lines indicate the result of the FD-BPM and the coupled-mode analysis using a singular perturbation scheme [32], respectively. Both curves agree well in very wide range of $w$. 

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Figure 3.8: The configuration of the nonlinear directional coupler with two identical nonlinear waveguide cores situated in the linear background.
Figure 3.9: The critical power $P_c$ as a function of the waveguide separation $w$. The solid and dashed lines indicate the results of the FD-BPM and the coupled-mode theory based on the singular perturbation scheme, respectively.
3.4 Asymmetric Nonlinear Directional Couplers

Two-waveguide asymmetric nonlinear directional couplers, which have a nonlinear and a linear waveguide core, are numerically analyzed by FD-BPM in this section. These NLDCs are designed so as to be the phase-mismatch state in the low input power level. As the input optical power increases, the phase of two waveguides matches due to the optical refractive index modulation of the nonlinear Kerr material and the perfect power transfer between two waveguides is attained at a suitable power level. This power-dependent coupling characteristic is applicable for an optical switch or an optical power filter [27]-[31].

It is possible to realize the asymmetry of the coupler by two means; one is to change the linear part of refractive index of each waveguide core and the other is to change the thickness of each one. At first, we analyze the asymmetric nonlinear directional coupler with different waveguide thickness. Next, the asymmetric nonlinear directional coupler with different waveguide refractive index is analyzed. Several power-dependent coupling phenomena are observed [31].

3.4.1 Asymmetric Nonlinear Directional Coupler with Different Waveguide Thickness

The side view of an asymmetric NLDC is shown in Figure 3.10. A linear waveguide $a$ of width $2d_a$ and a nonlinear waveguide $b$ of width $2d_b$ are situated parallel to each other with a separation distance $w$. The refractive index of the core of linear waveguide $a$ is $n_2$ and that of the cover, gap, and substrate layers occupied by the same linear medium is $n_1$. The core of the nonlinear waveguide $b$ is composed of a Kerr-like medium, which is
Figure 3.10: Side view of an asymmetric nonlinear directional coupler composed of a linear waveguide $a$ and a nonlinear waveguide $b$. 

\[ n_4 = \left( n_3^2 + n_{NL} |E(x,z)|^2 \right)^{1/2} \]
the same expression as what we treated in the previous sections, with a refractive index

\[ n_4 = \left( n_3^2 + n_{NL}|E(x, z)|^2 \right)^{1/2}, \tag{3.3} \]

where \( n_3 \) and \( n_{NL} \) denote the linear refractive index and the nonlinear coefficient of the medium, and \( |E(x, z)|^2 \) is the intensity of electric field. The geometry is uniform in the \( y \) and \( z \) directions. We examine the propagation of a two dimensional TE wave. The behaviour of the transverse electric field amplitude \( E \) is described by Eq.(2.8). Under the paraxial approximation, we can obtain the same finite difference formulation as Eq. (2.11).

The fixed parameters used throughout this section are the wavelength \( \lambda = 1.0[\mu m] \), the nonlinear coefficient \( n_{NL} = 6.377 \times 10^{-12}[m^2/V^2] \), and the linear refractive index \( n_1 = 1.53 \), which is common in the cladding, gap, and substrate layers. As the initial data for Eq.(2.11), we assumed that the linear TE_0 mode of the isolated waveguide \( a \) or \( b \) is incident into the waveguide \( a \) or the waveguide \( b \) of the NLDC at the input end \( z = 0 \), respectively. When the initial launching condition is specified, the amplitude \( E_{\text{max}} \) of the incident TE_0 mode defined at the core center \( x = w + d_a \) or \( x = -w - d_b \) is related to the input power \( P_{i,\text{in}} \) as follows:

\[ E_{\text{max}} = \sqrt{\frac{2\omega \mu_0}{2\beta_i d_i + \beta_i / \gamma_i}} P_{i,\text{in}} \quad (i = a \text{ or } b), \tag{3.4} \]

where \( \omega \) and \( \mu_0 \) are the angular frequency of light and the permeability in free space, \( \gamma_i = \sqrt{\beta_i^2 - k_0^2 n_1^2} \), and \( \beta_i \) is the propagation constant of the linear TE_0 mode of waveguide \( i \). For the numerical computation, we have taken the reference wavenumber \( \beta \) in Eq.(2.11) to be \( (\beta_a + \beta_b)/2 \), and chosen the computation area in the \( x \) direction, the number of grid points, and their spacing as \( |x| \leq 10.5[\mu m] \), \( M = 421 \), and \( \Delta x = 0.05[\mu m] \), respectively. Under this discretisation, the error in power of incident TE_0 mode at the input end \( z = 0 \) is attained to be less than \( 10^{-4} \). The small step size in the \( z \) direction was
chosen as $\Delta z = 0.1[\mu m]$ without the iteration, after having confirmed that the error in power conservation with propagation remains less than $10^{-6}$ over the propagation length $0 < z \leq 2[mm]$.

We consider first the case in which the two cores of NLDC have the same refractive index in the linear limit but different widths. Figure 3.11 shows the normalized transferred power $\eta_a(z) = \rho_a(z)/P_{a,in}$ in the linear waveguide $a$ defined by Eq.(2.26) as functions of propagation length $z$ for $d_a = 2.2[\mu m]$, $d_b = 2.0[\mu m]$, $w = 2.0[\mu m]$, $n_2 = 1.55$, and for different several input power levels $P_{a,in}$ launched into the linear waveguide $a$. In the linear limit with $n_{NL} = 0$ or $n_3 = n_2$, only a fraction of input power is transferred between the two waveguides. The rate of power transfer increases with the increment of input power, and a nearly complete power transfer into the waveguide $b$ is attained when $P_{a,in} = 4.0[W/m]$. For further increase of input power, the rate of power transfer is decreased rapidly. The propagation length for the complete power transfer is estimated as $z = 1.0[mm]$, which is about twice the coupling length in the linear limit. Figure 3.12 shows the normalized transferred power $\eta_a(z) = \rho_a(z)/P_{b,in}$ as functions of propagation length $z$ for different several input power levels $P_{b,in}$ initially launched into the nonlinear waveguide $b$ with the same parameters as in Figure 3.11. For the input power around $P_{b,in} = 4.0[W/m]$, nearly complete power transfer from the waveguide $b$ to the waveguide $a$ is obtained at the propagation length $z = 1.0[mm]$, which is also about twice the coupling length in linear limit. Taking into account the features of power transfer depicted in Figures 3.11 and 3.12, we assumed the device length of NLDC to be $1.0[mm]$ and set the output end in the linear waveguide $a$. Figure 3.13 shows the normalized output power $\eta_{a,out} = \rho_a(z)/P_{a,in}$ at $z = 1.0[mm]$ as a function of the input power $P_{a,in}$ launched into the linear waveguide $a$ with the same parameters as in Figure 3.11. It can be seen that the output power is strongly suppressed for the input optical signal with the power level.
around 3.8$[W/m]$. Figure 3.14 shows the normalized output power $\eta_{a,\text{out}} = \rho_a(z)/P_{b,\text{in}}$ at the same output end as a function of the input power $P_{b,\text{in}}$ launched into the nonlinear waveguide $b$ with the same parameters as in Figure 3.11. In this case, a full output is obtained only for the input optical signal with the power level around 3.8$[W/m]$. When the output end of NLDC is set in the nonlinear waveguide $b$, we have the input/output characteristics opposite to those depicted in Figures 3.13 and 3.14 because the energy conservation is well satisfied in the present numerical computation.
Figure 3.11: Normalized transferred power $\eta_a$ in the linear waveguide $a$ as functions of the propagating distance $z$ for $d_a = 2.2[\mu m]$, $d_b = 2.0[\mu m]$, $w = 2.0[\mu m]$, and $n_1 = n_3 = 1.55$, and for several different input power levels $P_{a,\text{in}}$ launched into the linear waveguide $a$. 
Figure 3.12: Normalized transferred power $\eta_a$ as functions of the propagating distance $z$ for different several input power levels $P_{b,\text{in}}$ launched into the nonlinear waveguide $b$. The values of parameters are the same as in Figure 3.11.
Figure 3.13: Normalized output power $\eta_{a,\text{out}}$ at $z = 1.0[\text{mm}]$ as a function of input power $P_{a,\text{in}}$ launched into the linear waveguide $a$. The values of the parameters are the same as in Figure 3.11.
Figure 3.14: Normalized output power $\eta_{a,\text{out}}$ at $z = 1.0[\text{mm}]$ as a function of input power $P_{b,\text{in}}$ launched into the linear waveguide b. The values of the parameters are the same as in Figure 3.11.
3.4.2 Asymmetric Nonlinear Directional Coupler with Different Waveguide Refractive Indices

We consider next the asymmetric NLDC in which the two cores of NLDC have a same width but different refractive indices in the linear limit. Figure 3.15 shows the normalized transferred power $\eta_a(z)$ for $d_a = d_b = 2.0[\mu m]$, $w = 2.4[\mu m]$, $n_2 = 1.551$, $n_3 = 1.550$, and for different several input power levels $P_{a,in}$. A complete power transfer to the nonlinear waveguide $b$ is expected to occur at the propagation length around $z = 2.0[mm]$ for a proper input power level within $5.0[W/m] < P_{a,in} < 6.0[W/m]$. This propagation length is nearly four times as large as the coupling length in the linear limit. Figure 3.16 shows the similar results when the input power $P_{b,in}$ is launched into the nonlinear waveguide $b$. It can be seen that nearly complete power transfer to the linear waveguide $a$ is attained at around $z = 2.0[mm]$ for the input power level slightly less than $4.0[W/m]$. For this configuration of NLDC, therefore, we assumed the device length of $2.0[mm]$ and set the output end in the linear waveguide $a$. The input/output power characteristics are shown in Figure 3.17 for the input into the linear waveguide $a$ and in Figure 3.18 for the input into the nonlinear waveguide $b$. Compared with Figures 3.13 and 3.14, the input/output characteristics in the main response become much sharper, but on the other side, a stringent adjustment of device length is required to realize a complete power transfer between the two waveguides.

From the input/output characteristics discussed above, it can be seen that the asymmetric NLDC in Figure 3.11 is useful for constructing a power band-pass filter or a power band-reject filter, by setting properly the input and output ends and the device length. Before concluding, it is worth to mention the effect of the waveguide separation $w$ on the power transfer characteristics. When the values of other parameters are fixed, there
exists an optimum separation distance for which the complete power transfer for filtering is obtained within a propagation length less than a few millimeters. As the separation distance increases, it becomes rather difficult to achieve the phase-matching between two waveguides within a realistic propagation length, the rate of maximum power transfer is remarkably reduced, and eventually the power filtering function of NLDC is diminished.
Figure 3.15: Normalized transferred power $\eta_a$ as functions of the propagating distance $z$ for $d_a = d_b = 2.0[\mu m]$, $w = 2.4[\mu m]$, $n_2 = 1.551$, $n_3 = 1.550$, and for different several input power levels $P_{a,in}$ launched in the linear waveguide $a$. 
Figure 3.16: Normalized transferred power $\eta_a$ as functions of the propagating distance $z$ for several different input power levels $P_{b,\text{in}}$ launched into the nonlinear waveguide $b$. The values of parameters are the same as in Figure 3.15.
Figure 3.17: Normalized output power $\eta_{a,\text{out}}$ at $z = 2.0\text{[mm]}$ as a function of input power $P_{a,\text{in}}$. The values of parameters are the same as in Figure 3.15.
Figure 3.18: Normalized output power $\eta_{a,\text{out}}$ at $z = 2.0[\text{mm}]$ as a function of input power $P_{b,\text{in}}$. The values of parameters are the same as in Figure 3.15.
3.4.3 Optical Control of Coupling Characteristics in Two-Waveguide Asymmetric Nonlinear Directional Coupler

We consider the case that the two waveguides of an asymmetric nonlinear directional coupler are illuminated simultaneously by the each TE\(_0\) mode of isolated situation in the linear limit. The NLDC structure is same as shown in Figure 3.10. The asymmetry is achieved by the difference between the core thicknesses, where \(d_a = d_b + 0.2[\mu m] = 2.2[\mu m]\). The other parameters used are the same as Section 3.4.1.

Firstly, in Figure 3.19, the bird eyes' view of the electric field intensity of propagating wave without control light is plotted. The input power in the waveguide \(a\) is \(P_{a,\text{in}} = 2.38[W/m]\). It is clearly shown that the input wave couples imperfectly to the adjacent nonlinear waveguide. Under this situation, the power transfer characteristics can be drastically changed by launching additional small input power into waveguide \(b\). Figure 3.20 shows the result for \(P_{a,\text{in}} = 2.38[W/m]\) and \(P_{b,\text{in}} = 0.40[W/m]\). We can see that this combination of input power excites the nonlinear eigenmode of the NLDC and there occurs no power transfer between the element waveguides. This result suggests that we can control the power coupling between two waveguides using an additional light beam with relatively small amount of power, and is of particular importance in the application of optical switching by making use of control light.
Figure 3.19: Bird eyes' view of the electric field intensity without control light. The input power of signal light $P_{a,in} = 2.38 \, [W/m]$. 
Figure 3.20: Bird eyes’ view of the electric field intensity. An example of the eigenmode of asymmetric nonlinear directional coupler which is achieved by the superposition of TE$_0$ mode of isolated linear waveguides. The input power of the signal light $P_{a,in} = 2.38$ [W/m] and that of the control light $P_{b,in} = 0.40$ [W/m], respectively.
3.5 Conclusions

In This chapter, the power transfer characteristics of symmetric and asymmetric two-waveguide planar NLDCs have been analyzed numerically by making use of the FD-BPM.

For the symmetric NLDC composed of identical two linear waveguide-cores and a common nonlinear gap layer, the numerical results such as critical power level agreed well with those of the improved coupled-mode theory [7] in the wide range of gap layer width [30]. The intensity of the propagating electric field was also illustrated and the interesting behaviour of the field depending on the input power level was observed. The analysis of the other symmetric NLDC, composed of two identical nonlinear waveguide-cores situated in linear claddings, provided the very close numerical results of critical power level compared to the coupled-mode theory using singular perturbation method [33]. These facts suggest that the FD-BPM and these coupled-mode analysis techniques assert the reliability on the study of NLDC each other.

In the third section, a novel asymmetric NLDC structure was proposed that consists of a nonlinear waveguide-core and a linear waveguide-core situated in the linear claddings. The numerical demonstration indicated that the proposed asymmetric NLDC structure is useful for constructing a power band-pass or a power band-reject filter of light wave, by setting the waveguide parameters properly.

In the last section, we simulated the case that the asymmetric NLDC is excited initially by a signal light and a control light with a fractional power level. The results suggest that we can control the power coupling between two waveguides using an additional light beam. It is possible to apply the asymmetric NLDC as an optically controlled switch.
Chapter 4

Analysis of Three-Waveguide Nonlinear Directional Couplers

4.1 Introduction

The basic nonlinear directional coupler (NLDC) consists of two parallel nonlinear optical waveguides placed in close proximity [30]. During the past decade, the characteristics of power transfer of optical waves along a two-waveguide NLDC have been extensively investigated using the coupled-mode theory [3]-[8][32][33]. Recently the theoretical investigations have been extended to a multiple-waveguide NLDC with the expectation to produce a variety of function of NLDC. Several authors [22]-[24] studied the power switching characteristics in a three-waveguide NLDC and indicated that the presence of a parasitic waveguide, placed between the two outermost waveguides, makes the switching curves remarkably sharper than in the conventional two-waveguide NLDC. However there is some controversy[24] on the validity of the results, because they have used a simple coupled-mode approach by Jensen[3]. Yasumoto presented the coupled-mode formulation based on the singular perturbation scheme[25] and developed the self-consistent analysis of three-waveguide NLDC[33]. It is also noted that the rigorous numerical analysis of the

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three-waveguide NLDC to assert the validity of the coupled-mode analysis have not been presented yet.

In this chapter, we investigate two kinds of planar three-waveguide NLDC. One is composed of three identical nonlinear waveguide-cores in linear claddings. The other is a NLDC with two identical linear waveguides situated both sides of a center waveguide with nonlinear Kerr like medium in linear claddings. The input/output characteristics of these three-waveguide NLDCs are analyzed numerically by the FD-BPM for the linear TE$_0$ incident wave. We also investigate the case that the weak control light is simultaneously given into the center waveguide-core when one of the outer waveguide is illuminated by a signal light. Numerical results demonstrate that the proposed three-waveguide NLDCs are more useful for optical power filtering and switching devices than two-waveguide NLDCs.

4.2 Three-Waveguide Nonlinear Directional Couplers with Identical Nonlinear Waveguide Cores

The side view of a planar three-waveguide NLDC with three identical waveguide cores is illustrated in Figure 4.1. The small step size for FD-BPM was determined carefully observing the energy conservation error and selected to be $\Delta x = 0.05[\mu m]$ and $\Delta z = 0.1[\mu m]$ without the iteration. The linear coupling length is $L_c = 2.73[\text{mm}]$ in the following case. The first and perfect power transfer is observed at $z = L_c$ in waveguide c when waveguide a is initially illuminated at $z = 0$. For the parameters $n_1 = 1.53$, $n_2 = 1.55$, $n_{NL} = 6.38 \times 10^{-12}[m^2/V^2]$, $d = 2.0[\mu m]$, $w = 3.0[\mu m]$, and $\lambda = 1.064[\mu m]$, the power fraction in each waveguide is shown in Figure 4.2 (a) to (c) as functions of the propagating length $z$ for four typical input powers when waveguide a is excited by linear TE$_0$ mode at $z = 0$. Figure 4.2 (a) shows the power fraction in waveguide a. The input power
Figure 4.1: The side view of a three-waveguide nonlinear directional coupler.
is perfectly transferred to the adjacent waveguides at low input power levels. However, when the waveguide is excited by the higher input levels over $P_{a,\text{in}} = 2.5[W/m]$, the power transfer characteristic becomes imperfect because of the phase mismatching due to the self phase modulation effect. Figure 4.2 (b) shows the power fraction in waveguide $b$. The unique power transmission is observed in waveguide $b$ when $P_{a,\text{in}} = 2.5[W/m]$ whereas the other curves are periodically changing with the increment of $z$. Figure 4.2 (c) shows the power fraction in waveguide $c$. The input light wave is transferred to the opposite outer waveguide $c$ for $P_{a,\text{in}} \leq 2.0[W/m]$. When the input power level exceeds 2.5[W/m], the power transfer becomes incomplete. We can seldom find the guided wave in waveguide $c$ over $P_{a,\text{in}} = 3.5[W/m]$.

Figure 4.3 shows the power fraction in waveguide $b$ when the center waveguide $b$ is initially excited. The power transfer characteristics in two outer waveguides are the same each other because of the symmetry of the configuration. The linear coupler provides the perfect power transfer from waveguide $b$ to $a$ and $c$. As the increment of the input power level, the coupled waveguide system becomes phase mismatch and the self-focusing effect governs the power transfer. For the case that $P_{b,\text{in}} = 8.0[W/m]$ is given into waveguide $b$, more than 95% of the input power is concentrated in it.

The power filtering characteristics in each waveguide at $z = 2.73[mm]$ which is normalized by the input power is depicted in Figure 4.4 (a) to (c) as a function of the normalized input power level[33][36]. The coupling length of this three-waveguide directional coupler for the above parameters in the linear limit yields to $L_c = 2.73[mm]$, as mentioned before. The input power is normalized by some characteristic input power level $P_0$, at which the output power in waveguide $a$ takes the first maximum. For the FD-BPM analysis, $P_0 = 1.75[W/m]$ and for the coupled-mode analysis based on the singular per-
turbation scheme, $P_0 = 1.54\,[\text{W/m}]$. For the comparison with the coupled-mode analysis based on the singular perturbation scheme, the result is depicted with dashed line. It is clear that the both curves agree in the wide range of input power $P_{a,in}$ in all of the figures.

The power filtering characteristics of three-waveguide NLDC in the waveguide $a$ is compared with two-waveguide NLDC in Figure 4.5. The parameters are same as Figure 4.4 (a) to (c) except the device length. The device length is $1.93\,[\text{mm}]$ for the two-waveguide NLDC and $z = 2.73\,[\text{mm}]$ for the three-waveguide NLDC. These device lengths correspond to the linear coupling length of two-waveguide and three-waveguide directional coupler, respectively. It is shown from Figure 4.5 that the filtering characteristic of the three-waveguide NLDC indicates sharper standing curve than that of the two-waveguide NLDC. We can conclude that three-waveguide NLDC is more preferable as an optical switch or a high-pass filter of the optical power than the two-waveguide NLDC except that the device length becomes a little longer. This characteristic is applicable for a high pass filter of the optical power or a power-dependent optical switch.
Figure 4.2: (a) Power fraction in the waveguide $a$ as functions of the propagating length $z$ for typical four input powers when waveguide $a$ is initially excited.
Figure 4.2: (b) Power fraction in the waveguide $b$ as functions of the propagating length $z$ for typical four input powers when waveguide $a$ is initially excited.
Figure 4.2: (c) Power fraction in the waveguide as functions of the propagating length for typical four input powers when waveguide \( a \) is initially excited.
Figure 4.3: Power fraction in the waveguide $b$ as functions of the propagating length $z$ for typical four input powers when waveguide $b$ is initially excited.
Figure 4.4: (a) The power filtering characteristics of three-waveguide NLDC in the waveguide $a$ as a function of the normalized input power $P_{a,in}/P_0$. 

\[ \frac{P_{a}}{P_{a,in}} \]

- **FD-BPM**
  \( (P_0=1.75 \ [W/m]) \)

- **Coupled Mode Theory [36]**
  \( (P_0=1.54 \ [W/m]) \)

\[ z=2.73 \ [mm] \]
Figure 4.4: (b) The power filtering characteristics of three-waveguide NLDC in the waveguide $b$ as a function of the normalized input power $P_{a,\text{in}}/P_0$. 

- FD-BPM
  $(P_0=1.75 \text{ [W/m]})$
- Coupled Mode
  Theory [36]
  $(P_0=1.54 \text{ [W/m]})$
- $z=2.73 \text{ [mm]}$
Figure 4.4: (c) The power filtering characteristics of three-waveguide NLDC in the waveguide $c$ as a function of the normalized input power $P_{a,in}/P_0$. 

$P_a/P_{a,in}$

$P_{a,in}/P_0$

FD-BPM
$\left(P_0=1.75 \, \text{[W/m]} \right)$

Coupled Mode Theory [36]
$\left(P_c=1.54 \, \text{[W/m]} \right)$

$z=2.73 \, \text{[mm]}$
Figure 4.5: The power filtering characteristics of two-waveguide and three-waveguide NLDC as functions of the normalized input power level. The solid and dashed lines indicate the results of the FD-BPM and the coupled-mode theory based on the singular perturbation technique, respectively. The device length is 1.93 [mm] for the two-waveguide NLDC and 2.73 [mm] for the three-waveguide NLDC, respectively.
4.3 Optical Control of Coupling Characteristics of Identical Three-Waveguide Nonlinear Directional Coupler

In this section we will show that the coupling characteristics of an identical three-waveguide nonlinear directional coupler can be controllable by adding a fraction of control light into another waveguide [36]. The three-waveguide NLDC considered here has three identical waveguide cores, which is the same configuration as Figure 4.1. The used parameters for the simulation are the same as the previous section. The device length is set to be the linear coupling length $z = L_c = 2.73[\text{mm}]$. The input power is given into the waveguide $a$ and the weak control light is applied to waveguide $b$. The wavelength of the control light is assumed to be the same as that of the signal light. Figure 4.6 shows the normalized output powers $P_a/P_{a,\text{in}}$ as functions of the input signal power $P_{a,\text{in}}$ under three different input conditions for the waveguide $b$, where $P_{b,\text{in}}$ is the input power of the control light and $\theta$ denotes the phase difference between the signal and control lights. It is seen that the switching power changes depending on the phase of the weak control light. The control light in the same phase with the signal light intensifies the coupling between two outermost waveguides and decreases the switching power, whereas the control light in the opposite phase reduces the coupling and hence increases the switching power. Figure 4.6 suggests that the output state of the strong input signal can be controlled by the phase of the weak input light into the center waveguide. Wabnitz et al. [9] have reported such a phase-controlled switching characteristics in the two-waveguide NLDC. However the three-waveguide NLDC has an advantage such that the output signal channels can be separated from the channel of weak control beam [36].
Figure 4.6: Normalized output power $\rho_a/P_a, in$ at the coupling length $z = 2.73[\text{mm}]$ as functions of the input signal power $P_{a,\text{in}}$ when a weak control light was applied to the center waveguide b. $P_{b,\text{in}}$ is the power of the control light and $\theta$ is the phase difference between the signal and control lights.
4.4 Three-Waveguide Nonlinear Directional Couplers with Nonidentical Waveguide cores

We consider a three-waveguide NLDC that consists of two identical outer linear waveguides and a center waveguide with Kerr-like nonlinear material, depicted in Figure 4.7 [35]. The refractive index of the center waveguide in the linear limit is assumed to be slightly smaller than that of the outer waveguides. For this configuration, there exists some characteristic power for which the center waveguide becomes phase matched with the two outer linear waveguides through the effect of power-induced self-phase modulation. When the power trapped in the center waveguide is near the characteristic power, the NLDC behaves as a phase-matched directional coupler and a complete power transfer among three waveguides is attained. However, when the trapped power is somewhat smaller or larger, only a small fraction of power is transferred because of the phase mismatching between the center and outer waveguides. The purpose in this section is to analyze such a power-band selective transmission of the three-waveguide NLDC with nonidentical waveguide elements. The power-transfer characteristics along the coupler are discussed for the excitation of the center waveguide and for the excitation of both two outermost waveguides. It is shown that the proposed three-waveguide NLDC behaves as a power-dependent power divider, a power-dependent power combiner, and an optical logic element under suitable excitation conditions.

The parameters of a planar three-waveguide NLDC depicted in Figure 4.7 are \( n_1 = 1.53, n_2 = 1.55, \alpha = 6.377 \times 10^{-11}[m^2/V^2], \) and \( d_a = d_c = 2.0[\mu m], d_b = 1.6[\mu m], \)

\[ w = 2.0[\mu m], \] and the wavelength \( \lambda = 1.064[\mu m]. \) For this situation, only the fundamental TE_0 mode is guided by each isolated waveguide in the linear limit, and the respective propagation constants are given by \( \beta_a/k_0 = \beta_c/k_0 = 1.54240 \) and \( \beta_b/k_0 = 1.54050. \) As the
initial data, we assumed the linear $TE_0$ mode of the isolated single waveguides. For the numerical computation, we have chosen $\Delta x = 0.05[\mu m], \Delta z = 0.1[\mu m]$ without the iteration, and $\beta = (\beta_a + \beta_b + \beta_c)/3$ as the reference wavenumber, and used a numerical boundary condition [19] that the electric field should vanish at the edge of the computational window $|x| \leq 25[\mu m]$. Increasing the width of computational window and decreasing the step sizes in the $x$ and $z$ direction made negligible changes in the results. This is because the electric field is well confined in the core regions of the NLDC. The numerical solutions were obtained with the error in power conservation less than $10^{-6}$ over the propagation length $0 \leq z \leq 2.5[mm]$.

We consider first the case of excitation of the center waveguide $b$. Figure 4.8 shows the normalized power $\eta_b(z)$ in waveguide $b$ as the functions of propagation length $z$ when the waveguide $b$ is illuminated by different several input powers $P_{b,in}$. Since the three-waveguide NLDC is symmetric, the power fraction $[1 - \eta_b(z)]/2$ is equally transferred into the outermost waveguides $a$ and $c$. At low input powers, the NLDC behaves like a conventional inefficient coupler because of the phase-mismatching between the center and outermost waveguides. The rate of power transfer increases with the increment of input powers, and an almost complete power transfer into waveguides $a$ and $c$ is obtained for $P_{b,in} \approx 9.0[W/m]$. The curve of the power fraction for this case agrees with that of the linear three-waveguide coupler with $\alpha = 0$ and $d_a = d_b = d_c = 2.0[\mu m]$. This implies that $P_{b,in} \approx 9.0[W/m]$ gives a characteristic power for which the center waveguide becomes phase-matched with the two outermost waveguides. We see also that there exists some critical power for which the coupling length tends to be infinity. The behavior of the NLDC at powers close to the critical power is very sensitive to the input power. The value $P_{b,in} \approx 10.407[W/m]$ in Figure 4.8 actually corresponds to an input power very slightly below the critical power. At the critical power, one half of the input power remains in the
center waveguide and the other half is equally shared by the two outermost waveguides. For further increase of input power, the NLDC behaves again like an inefficient coupler and the rate of power transfer is rapidly decreased. The minimum propagation length for the maximum power transfer is estimated as $z = 0.428\,[\text{mm}]$, which is very near the coupling length of the identical linear three-waveguide coupler for the center-waveguide excitation. It is noted that the maximum of the power fraction in waveguide $b$ is always slightly smaller than unity. This is because a small amount of residual powers remains in other waveguides due to an intrinsic cross-talk of coupled optical waveguides.

Figure 4.9 shows the normalized output power $\eta_i(z) = \rho_i(z)/P_{b,\text{in}}$, ($i = a, b, c$) at $z = 0.428\,[\text{mm}]$ as a function of the input power $P_{b,\text{in}}$. It is seen that for the input power around $P_{b,\text{in}} = 9.3\,[\text{W/m}]$, the output of the center waveguide is strongly suppressed and the input power is equally divided into the output ends of two outermost waveguides. This input/output characteristic is useful for constructing a power-dependent optical power divider.

We consider next the case of excitation of two outermost waveguides. Figure 4.10 shows the normalized output power $\rho_i/(P_{a,\text{in}} + P_{c,\text{in}})$, ($i = a, b, c$) at $z = 0.428\,[\text{mm}]$ as functions of the input power $P_{a,\text{in}} = P_{c,\text{in}}$, when waveguides $a$ and $c$ are both excited by the respective linear TE$_0$ modes with equal intensity and same phase. We can see that for the input power around $P_{a,\text{in}} = P_{c,\text{in}} = 5.4\,[\text{W/m}]$, the outputs into two outermost waveguides are strongly suppressed and the two input powers are combined at the output end of the center waveguide $b$. This input/output characteristic is useful for constructing a power-dependent optical power combiner. Figure 4.11 shows the input/output characteristics at $z = 0.428\,[\text{mm}]$ when waveguides $a$ and $c$ are both excited by the respective linear TE$_0$ modes with equal intensity but opposite phase. In this case the NLDC behaves like
the isolated three waveguides for the broad range of input powers. Two input powers almost exit from the initially launched waveguides and the output powers from the center waveguide $b$ is nearly zero. This is because two incident fields with the opposite phase cancel each other in the overlap area and do not cause the power-induced change of refractive index in the center waveguide. The behaviors of the power transfer depicted in Figures 4.10 and 4.11 suggests a possibility of the nonidentical three-waveguide NLDC as an optical logic element.
Figure 4.7: The side view of a three-waveguide nonlinear directional coupler with two identical outer linear waveguide and a center nonlinear Kerr-like waveguide. The linear waveguide a is identical to the linear waveguide c. Only the center waveguide has a nonlinear core with the narrower width compared with the linear waveguides.
Figure 4.8: Power fraction in the center waveguide $b$ as functions of the propagating length $z$ for several input powers when waveguide $b$ is initially excited.
Figure 4.9: Normalized output powers of three waveguides at $z = 0.428\text{[mm]}$ as functions of input powers when the center waveguide $b$ is initially excited.
Figure 4.10: Normalized output powers of three waveguides at $z = 0.428\,[mm]$ as functions of input powers when waveguides $a$ and $c$ are both excited by the incident fields with equal intensity and same phase.
Figure 4.11: Normalized output powers of three waveguides at $z = 0.428 \text{[mm]}$ as functions of input powers when waveguides $a$ and $c$ are both excited by the incident fields with equal intensity but opposite phase.
4.5 Conclusions

In this chapter, we investigated two kinds of planar three-waveguide NLDCs. Firstly, the identical three-waveguide NLDC showed sharper switching curve around the critical power than the two-waveguide NLDC. This numerical result agreed very well in all the three waveguides with that of coupled-mode analysis based on the singular perturbation method, which asserted the validity of both analysis techniques each other. This characteristic is preferable as a power-dependent optical filter. From another point of view, this result can be applied also for a high-pass or a low-pass filter of the optical power level by selecting the input and output waveguide appropriately. We found the novel phenomena that the standing point of this power filtering curve can be controlled by illuminating the center nonlinear waveguide with an additional control light of fractional power level. We can raise the standing point by using the opposite phase control light with the signal light, whereas we can reduce it by using the same phase control light. The idea of phase-controlled switching has been already demonstrated by Wabnitz et al.[9] for a two-waveguide NLDC. However the three-waveguide NLDC has an advantage such that the output signal channels can be separated from the channel of weak control beam.

Secondly, the nonidentical three-waveguide NLDC was analyzed. This NLDC provided the characteristics of phase-dependent optical signal combiner and divider when the outermost two linear waveguides are illuminated simultaneously by the same intensity. The two optical signals with same phase and same intensity are combined into one signal with the double intensity in the center nonlinear waveguide, while the opposite-phase signals passes through the NLDC without any coupling. This result demonstrate that the NLDC with three nonidentical waveguide cores is useful for the phase dependent optical signal switch.
Chapter 5

Concluding Remarks

The electromagnetic waves propagating in various nonlinear directional couplers (NLDC) are demonstrated by making use of the finite-difference beam propagation method (FD-BPM). The nonlinear refractive index term is carefully evaluated by the second order iteration procedure in FD-BPM. The results obtained in this thesis are briefly concluded as follows:

1. Two-waveguide symmetric NLDC

   For the symmetric NLDC composed of two identical linear waveguide-cores and a common nonlinear gap layer, the numerical results such as critical power level agreed well with those of the improved coupled-mode theory in the wide range of gap layer width. The intensity of the propagating electric field was also illustrated and the interesting behaviour of the field depending on the input power level was observed. The analysis of the other symmetric NLDC, composed of two identical nonlinear waveguide-cores situated in linear claddings, provided the very close numerical results of critical power level compared to the coupled-mode theory using singular perturbation method. These facts suggest that the FD-BPM and these
coupled-mode analysis techniques assert the validity and reliability on the study of NLDC each other.

2. Two-waveguide asymmetric NLDC

A novel asymmetric NLDC structure was proposed that consists of a nonlinear waveguide-core and a linear waveguide-core situated in linear claddings. The numerical demonstration indicated that the proposed asymmetric NLDC structure is useful for constructing a power band-pass or a power band-reject filter of light wave, by setting the waveguide parameters properly. Additionally, we simulated the case that the asymmetric NLDC is excited initially by a signal light and a control light with a fractional power level. These results suggest that we can control the power coupling between two waveguides using an additional light beam. It is possible to apply the asymmetric NLDC as an optically controlled switch.

3. Three-waveguide NLDC with identical waveguide cores

The identical three-waveguide NLDC showed sharper switching curve around the critical power than the two-waveguide NLDC. This numerical result agreed very well in all the three waveguides with that of coupled-mode analysis based on the singular perturbation method, which asserted the validity of both analysis techniques each other. This characteristic is preferable as a power-dependent optical filter. From another point of view, this result can be applied also for a high-pass or a low-pass filter of the optical power level by selecting the input and output waveguide appropriately. We found a novel phenomena that the standing point of this power filtering curve can be controlled by illuminating the center nonlinear waveguide with an additional control light beam of fractional power level. We can raise the standing point by using the same phase control light with the signal light, whereas we can reduce it by using the opposite phase control light. The three-waveguide NLDC
has an advantage such that the output signal channels can be separated from the channel of weak control beam, while the two-waveguide NLDC provide the signal and the control light in the same waveguide.

4. Three-waveguide NLDC with nonidentical waveguide cores

The power transfer of the nonidentical three-waveguide NLDC was analyzed. This NLDC provided the characteristics of phase-dependent optical signal combiner and divider when the outermost two linear waveguides are illuminated simultaneously by the same intensity. The two optical signals with same phase and same intensity are combined into one signal with the double intensity in the center nonlinear waveguide, while the opposite-phase signals passes through the NLDC without any coupling. This result is useful for the phase dependent optical signal switch.

Finally, let us mention some future subjects to the present study:

1. As we can expect easily from the power band-pass filtering characteristics shown in chapter 3, the asymmetric NLDC structure have a function of pulse shaping. However, the continuous input wave has been applied and the static state has been analyzed in this study. We need to clarify the transient phenomena in NLDC considering the response of the nonlinear polarization.

2. We have analyzed two-waveguide and three-waveguide directional couplers in this thesis. The characteristics of multi-waveguide (more than four waveguides) couplers have not been known yet. In the analysis of such multi-waveguide couplers, we should take care of the conditions for numerical analysis such as the width of computation window and the distance of discrete points in the limit of the computational speed and the memory space. The reasonable modeling is required in the
computation.

3. The increase of the nonlinear refractive index of the material at the peak intensity of the electric field was less than several percent of the linear part throughout the thesis. In this range, we can use the paraxial condition and Eq. (2.11). In the analysis of the NLDC with larger nonlinearity, we must pay attention to the quantity of the nonlinear refractive index term and the difference from the linear part.
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Bibliography


Glossary of Symbols Used

\[ n(x, y, z) \] : Refractive index profile
\[ \mu_0 \] : Permeability
\[ \varepsilon \] : Electric vector field
\[ \mathcal{H} \] : Magnetic vector field
\[ \nabla \] : Nabla operator
\[ i_x, i_y, i_z \] : Unit vectors parallel to the each coordinate axis
\[ \partial/\partial x, \partial/\partial y, \partial/\partial z \] : Partial differential operators
\[ k_0 \] : Wave number in vacuum
\[ j \] : Unit imaginary number
\[ \omega \] : Angular frequency of light
\[ t \] : Time
\[ E \] : Scalar electric complex field amplitude along \( y \) axis
\[ n_1, n_2 \] : Linear part of refractive index
\[ n_{NL} \] : Nonlinear coefficient of Kerr material
\[ \beta \] : Reference wave number
\[ \Delta x, \Delta z \] : Small discretizing step size
\[ E_m \] : Discretized electric field in FD-BPM
\[ x_m \] : Discretization point in FD-BPM
\[ M \] : Total number of \( x_m \)
\[ p_m, q_m \] : Coefficients of tridiagonal equation in FD-BPM
\[ a_m, b_m \] : Coefficients of \( p_m \) and \( q_m \)
\[ E_m^{(0)} \] : Discretized electric field of zero order iteration
\[ E_m^{(1)} \] : Discretized electric field of first order iteration
\[ E_m^{(2)} \] : Discretized electric field of second order iteration
\( P \) : Tridiagonal coefficient matrix
\( x \) : Column vector of unknown electric field \( E_m(z + \Delta z) \)
\( Q \) : Column vector calculated from known electric field
\( t_m, s_m \) : Temporary variables in Gaussian elimination procedure
\( \eta_a(z), \eta_b(z), \eta_c(z) \) : Normalized output power in each labeled waveguide
\( \rho_a(z), \rho_b(z), \rho_c(z) \) : Output power in each labeled waveguide
calculated from the discretized electric field
by trapezoid formula
\( P_{a,\text{in}}, P_{b,\text{in}}, P_{c,\text{in}} \) : Input power launched into each labeled waveguide
\( x_{ab}, x_{bc} \) : Terminal grid of the numerical integration
\( E_{i,\text{max}} \) : Maximum electric field amplitude of input light
at the center of the input waveguide
\( P_c \) : Critical power
\( L_c \) : Coupling length
\( P_0 \) : Input power that provide the maximum power transfer
in an identical three-waveguide NLDC
\( \theta \) : Phase difference between signal and control light