Relaxations of Hard Graph Problems Using Finite Groups, and Characterizations of Graphs for Efficient Algorithms

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論 文 名　：Relaxations of Hard Graph Problems Using Finite Groups, and Characterizations of Graphs for Efficient Algorithms
（有限群を用いた計算困難なグラフ問題の緩和と，効率的アルゴリズムのためのグラフの特徴づけ）

区　　分　：甲

論 文 内 容 の 要 旨

It was 1971 when the notion of NP-completeness was introduced by Cook. Karp proposed 21 NP-complete problems, and more than half of them are graph problems, including Hamiltonian cycle problem, vertex cover, and chromatic number. These hard problems, however, are solved efficiently when input graphs have specific structures. Understanding graph structures is an important and prospective approach for designing efficient graph algorithms.

Relaxation of a problem is a standard strategy to attack hard problems; consider a relaxed variant of a hard problem, find good characterizations for it, and apply the idea of the characterizations to the original hard problem. Motivated by the development of new technologies for relaxation, this paper focuses on relaxations using finite groups.

The target of relaxation is not unique; relaxation of the constraints of a problem, relaxation of "for any input" condition (restricting the class of inputs), relaxation of objective function (approximation), etc. To begin with, this thesis considers a variant of the Hamiltonian cycle problem by relaxing the constraint. We give rise to a new problem, parity Hamiltonian cycle problem, which is a relaxed variant of the Hamiltonian cycle problem. A parity Hamiltonian cycle is a closed walk (possibly using each edge more than once) which visits every vertex an odd number of times. We are concerned with the problem in both undirected and directed graphs. We give some good characterizations of graphs which have parity Hamiltonian cycles. Based on the characterizations, we present efficient algorithms for both problems. Particularly, the existence of a parity Hamiltonian cycle in a directed graph is characterized by a linear system over GF(2), thus the problem is solved by solving the linear system. Next we consider the Hamiltonian cycle problem for covering graphs, which are defined by finite groups. Batagelj et al. (1982) showed a very simple characterization of the Hamiltonicity of the Cartesian product of a tree and a cycle which is represented as a covering graph. We show the same characterization as Batagelj et al.'s is applicable for two larger graph classes.

Finding a spanning tree of minimum weight with bounded diameter is known to be NP-hard. We give rise to a variant of the problem, the odd depth tree problem. An odd depth tree is a rooted spanning tree such that the distance of each leaf and the root is odd. We show the NP-hardness for non-bipartite graphs, while we present a complete characterization for bipartite graphs. We also consider the problem for directed graphs, and show similar characterizations and complexity results to the undirected case.