Information Loss in a Nutshell

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Abstract

This note is a nutshell of our previous note which aims to support the arguments of information loss by Unruh and Wald (`arXiv:1703.02140`).

1 Introduction

First we mention two extreme answers to the information paradox of the black-hole evaporation. Hawking [1] claimed the recovery of the information. Unruh and Wald [2] argued the loss of the information.

Why were they led to different conclusions? We ascribe it to the difference of the type of the algebra. Our discussion[1] proceeds in the framework of the algebraic quantum theory [3, 4]. The algebra of observables is the von Neumann algebra.

Hawking's claim is based on the unitarity of the evolution of the state vector. This unitary evolution is only allowed for the type-I von Neumann algebra. The Hilbert space employed is finite dimensional. The non-relativistic quantum mechanics is described by the type-I von Neumann algebra [5, 6].

On the other hand, the relativistic quantum field theory is described by the type-III von Neumann algebra [5, 6]. The Hilbert space employed is infinite dimensional. The evolution of the state is non-unitary so that the information loss occurs without a blackhole.

Unruh and Wald pointed out that any system which we can observe is an open system[2]. Such an open system [3] is described by the type-III von Neumann algebra in the relativistic quantum field theory.

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[1] This note is a nutshell of our previous note, http://hdl.handle.net/2324/1955688.

[2] Unruh and Wald's example of an open system, a living room, also has nothing to do with a blackhole.

In the following we concentrate on the property of the vacuum in the relativistic quantum field theory. The vacuum shows apparently contradictory properties, entanglement and mixing. The mixing leads to the loss of information.

2 Entanglement

2.1 Local net

The algebraic quantum theory starts with the algebra $\mathcal{A}(\mathcal{O})$ of observables at the space-time region $\mathcal{O}$. The net $\mathcal{A}$ of local algebras $\mathcal{A}(\mathcal{O})$ constitutes the intrinsic mathematical description [3].

2.2 GNS construction

In the GNS construction[3] the vacuum vector Ω can be identified with the identity operator $\mathbb{I}$ of the net $\mathcal{A}$. We can think both the vacuum and the identity as an object which contains every possibility.

2.3 Reeh-Schlieder property

The Reeh-Schlieder property, the denseness of $\overline{\mathcal{A}(\mathcal{O})\Omega}$ in $\overline{\mathcal{A}\Omega}$, is a direct consequence of the weak additivity [4]. Loosely speaking, an event which occurs in a space-time region $\mathcal{O}$ can occur in any other space-time region. This kind of vector Ω is called cyclic. If Ω is cyclic, it is shown to be separating [4]. Thus the vacuum vector Ω is cyclic and separating.

Although the Reeh-Schlieder property is for the vector in the Hilbert space with infinite degrees of freedom, we can catch its image in the following toy space[4] with finite degrees of freedom. Let us consider the space-time with only two domains, L and R. The dimension of the Hilbert space d is the same for L and R. Via the Schmidt decomposition the vacuum vector Ω of the total system is expressed as

$$\Omega = \sum_{i=1}^{d} \sqrt{\lambda_i} \left|\phi_i^{\mathrm{L}}\right\rangle \left|\phi_i^{\mathrm{R}}\right\rangle. \tag{1}$$

The Reeh-Schlieder property is embodied if all λ_i's are positive. In this case Ω is entangled. Thus the Reeh-Schlieder property leads to the entanglement.

[3] In the GNS construction everything is described by operators. Thus the Reeh-Schlieder property can be understood within the algebra.

[4] See, for example, the section 2.2 in arXiv:1811.05052v1.

If all λ_i's are the same, Ω is maximally entangled. In this case the reduced density matrix ρ^{L}

$$\rho^{\mathrm{L}} = \sum_{i=1}^{d} \lambda_i \left|\phi_i^{\mathrm{L}}\right\rangle \left\langle \phi_i^{\mathrm{L}}\right|, \tag{2}$$

is proportional to the identity operator. This is consistent with the choice of the vacuum in the GNS construction.

2.4 Unruh effect

Two space-time domains also appear in the discussion of the Unruh effect. They are the Rindler wedges. The Unruh effect results from the entanglement in the Lorentz vacuum. The reduced density[5] matrix becomes thermal. The degrees of freedom unseen from the observer's wedge plays the role of the reservoir.

3 Mixing

3.1 Causality

The (Einstein) causality requires that the observations at $\mathcal{O}_1$ and $\mathcal{O}_2$ are independent, if $\mathcal{O}_1$ and $\mathcal{O}_2$ are space-like.

Apparently such a requirement seems to conflict with the entanglement of the vacuum. However, in the following we will guarantee the causality.

3.2 Statistical independence

According to the abstract of [5], one of von Neumann's motivations for developing the theory of operator algebras and the classification of factors was the question of possible decompositions of quantum systems into independent parts.

The entanglement prevents us from decomposing the state vector. However, even in the presence of the entanglement we can accomplish an independence of the observations at $\mathcal{O}_1$ and $\mathcal{O}_2$ in space-like separation. The independence is expressed in the expectation value [5, 6] as

$$\omega(A_1 A_2) = \omega(A_1) \cdot \omega(A_2), \tag{3}$$

[5]The density matrix is a convenient tool in the quantum mechanics with finite degrees of freedom. In the algebraic quantum theory with infinite degrees of freedom the Unruh effect is explained via the Bisognano-Wichmann theorem [3].

where ω is the map to give the expectation value and A_i is an observable in the space-time region $\mathcal{O}_i$ with $i = 1, 2$.

3.3 Split property

The statistical independence in the expectation value described in the previous subsection is the consequence of the mixing property explained in the following. The mixing property of the expectation value is led from the split property [5, 6] of the algebra.

The local algebra has the isotony property: $\mathcal{A}(\mathcal{O}_1) \subset \mathcal{A}(\tilde{\mathcal{O}}_1)$ if $\mathcal{O}_1 \subset \tilde{\mathcal{O}}_1$. When the difference between $\mathcal{O}_1$ and $\tilde{\mathcal{O}}_1$ consists of finite number of space-time points, we can say

$$\omega(A_1) = \tilde{\omega}(A_1), \tag{4}$$

since $\mathcal{O}_1$ consists of infinite number of space-time points. Here ω is the expectation value for $\mathcal{O}_1$ and $\tilde{\omega}$ for $\tilde{\mathcal{O}}_1$. Thanks to the split property $\tilde{\omega}$ is expressed in terms of a density matrix of a mixed state. The expectation values of these mixed states show the statistical independence. At the same time the information is lost.

3.4 Takesaki property

The vacuum expectation value of the local observables, A_1 and B_1, shows the KMS property

$$\langle\Omega|A_1(t)B_1|\Omega\rangle = \langle\Omega|B_1A_1(t-i)|\Omega\rangle. \tag{5}$$

This vacuum fluctuation [7, 8] clearly shows the mixing property.

4 Summary

- The vacuum of the relativistic quantum field theory is entangled.
- The vacuum expectation value shows the mixing property.
- Due to the mixing the information is lost.

References

[1] Hawking: arXiv:1509.01147.

[2] Unruh and Wald: arXiv:1703.02140.

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[7] Fredenhagen: “On the Modular Structure of Local Algebras of Observables” in *Quantum Field Theory*, eds. Jaffe, Lehmann and Mack (Springer-Verlag, 1985).

[8] Fredenhagen: *The vacuum is not empty* (2017) Slides on Web.