Finite Element Analysis of Corrugated Board under Bending Stress

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INTRODUCTION

Physical distribution has become more mechanized, modernized, and industrialized, and as a result, it has become necessary to understand box compression strength in packaging, which is the first stage of physical distribution. Further, the diverse quality requirements for corrugated boards need to be satisfied.

A corrugated board is a sandwich–type engineering material. The mechanical behavior of a corrugated board can be affected by the quality of the material used for constructing the board and the structural union of the liner and the corrugating medium that compose a corrugated board. In order to maintain the strength of a corrugated board, high–quality paperboard or functional additives can be used. However, in order to enhance the value of the corrugated board, the strength optimization of a board combination should be accomplished.

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Corrugated board is an engineering structure, whose strength characteristics depend on the cross–section geometry and the material properties of liner and corrugated medium. Therefore, the board combination (combination of material for preparing corrugated board) of corrugated boards should be optimized with considering the quality characteristics of corrugated board demanded to achieve the optimum packaging design.

The board combinations of corrugated boards can be determined by employing the configuration method for the paperboard stiffness and the basis weight. The results of board combinations can greatly affect the buckling resistance and compression strength of the box. In order to analyze the effect of the configuration method for the board stiffness and the basis weight on the flexural stiffness of corrugated board, we performed experiments based on cost– and time–consuming procedures. However, by using a simulation method such as finite element analysis (FEA) has the advantages such as saving of the time and cost in experiments and sample production as well as detailed analysis by adjusting several parameters. In the case of the simulation method, the estimation of reliability and reproducibility of the results should be strongly considered.

In this study, we observed the strength–optimal design conditions for the corrugated board combinations by analyzing the flexural stiffness index (FSI). The FEA method was used in the study using the four–point loading conditions. We found that the difference between the FEA result and the test results were approximately 18–31%. However, it was estimated that the variations in FSI estimated by using both methods are similar. Therefore, we expect that the flexural balance for different board combinations would be successfully analyzed on the basis of the qualitative comparison of FSI values corresponding to the combinations when the stiffness for each of the liner and the corrugating medium are assumed to be with 2–3 levels.

Key words: board combination, corrugated board, finite element analysis, flexural stiffness, four–point bending test, orthotropic material
the effect of the configuration method for the paper-board stiffness and the basis weight on the flexural stiffness of corrugated board, we performed experiments based on cost- and time-consuming procedures. However, by using a simulation method such as finite element analysis (FEA) has the advantages such as saving of the time and cost in experiments and sample production as well as detailed analysis by adjusting several parameters. In the case of the simulation method, the estimation of reliability and reproducibility of the results should be strongly considered.

The compressive strength, crush strength, bending deflection, flexural stiffness, and creep and recoverability of corrugated boards were investigated (Hahn et al., 1992; Lee and Park, 2004; Urbanik, 2001; Guo et al., 2008). By using some finite element models and commercial finite element codes ABAQUS or ANSYS, the mechanical behaviors of the corrugated board such as buckling, transverse shear, elasticity, stability, collapse, and ultimate failure were studied (Gilchrist et al., 1990; Nordstrand and Allansson, 2003; Aboura et al., 2004; Rami et al., 2009; Talbi et al., 2009). Within the finite element (FE) models (e.g. thin shell element, simplified homogenization model), both geometric nonlinearity (large deformation) and material nonlinearity (anisotropy or orthotropy) effects were considered. Sek and Kirkpatrick, 1997 analyzed the cushioning property and its predictive model, the vibration transmissibility and frequency response of single-wall corrugated board. Guo et al., 2010 measured the dynamic packaging properties such as dynamic cushioning curves and vibration transmissibility for corrugated board pads.

The purpose of this study is to determine the optimal design conditions of flexural strength for board combination; these conditions are determined by analyzing the flexural stiffness characteristics of flute types of corrugated boards by employing the FEA method under the four-point loading condition.

**FEA SIMULATION AND THEORETICAL CONSIDERATION**

**Finite element modeling**

The corrugated board was modeled on the basis of flute types of boards by using a finite element method. In modeling, it was used the cell element to qualitatively analyze the flexural behavior that changes with the thickness and mechanical properties of the liner and the corrugating medium. The cell element has six–degree of freedom, and three types of displacements corresponding to the x, y, and z–axes and three types of rotations corresponding to the three axes are possible (ANSIS Inc).

As shown in Fig. 1, the geometrical shape of the flute was modeled as a cosine function on the basis of the Korean Standard (KS T 1034). For the CD, double–wall (DW) corrugated boards named AA/F, AB/F, and BC/F were modeled on the basis of the largest width of the flute and SW corrugated boards named A/F, B/F, and C/F were modeled on basis of the corresponding width with the single wave of each flute. Also, in case of the MD, both the DW and SW corrugated boards were modeled with a width of 10 mm, i.e., 20% of 50 mm (Markstrom, 1988; Urbanik, 2001).

Fig. 2 shows the solid models of the CD and MD for one kind of SW and DW corrugated boards, respectively.

**Fig. 1.** Geometry parameters for the flute of the corrugated board investigated.

![Fig. 1. Geometry parameters for the flute of the corrugated board investigated.](image)

**Fig. 2.** Examples of FE models for simulation.

![Fig. 2. Examples of FE models for simulation.](image)

**Fig. 3.** Loading and boundary conditions under four-point bending.

![Fig. 3. Loading and boundary conditions under four-point bending.](image)

**FEA procedure and material properties**

ANSYS software was used to simulate the corrugated board under the four–points bending condition as a post–processor, and nonlinear–large deflection conditions were considered in the analysis.

A load equivalent to FE model was divided by width-wise node number and applied to each node. Figure 3 shows that the equivalent loads were applied to inner liner and outer liner as compression strength and shear strength, respectively, with considering the buckling
which can be a critical factor of the load–bearing ability of box.

Because a displacement toward y–axis at the contact position with supporting anvil was not occurred at all when the specimen was tested under the four–points bending condition, node equivalents of the contacts were also restrained. The span between supporting points was 200 mm and between loading points was 400 mm, respectively (Markstrom, 1988; Urbanik, 2001).

Table 1 shows the material properties used in the FEA. In case of DW corrugated board, totally 144 kinds of combination which composed of the liner and corrugating medium were used in simulations. Each case can be distinguished by stiffness conditions of liner and corrugating medium. In case of SW corrugated board, 18 combinations were applied in simulation cases which were also classified by the stiffness conditions of liner and corrugating mediums.

The material properties of paperboards shown in Table 1 include the diverse materials of paperboard in corrugated board (Biancolini and Brutti, 2003; Park, 2003) and satisfy the characteristic equation (1) of orthotropic material (Biancolini and Brutti, 2003; Park, 2003; Pilkey, 1994). Material properties in Table 1 should be applied with coinciding both principal material direction of corrugated board and composed paperboard direction.

\[ E_{MD}/E_{CD} = 2, \mu_{MD}/\mu_{CD} = 2 \]

Here, \( E_{MD} \) and \( E_{CD} \) are the Young’s modulus of paperboard in MD and CD, respectively, and \( \mu_{MD} \) and \( \mu_{CD} \) are the Poisson’s ratio of paperboard in MD and CD, respectively.

**Bending test and related equations**

A flexural stiffness of corrugated board was defined as the curvature–to–bending moment ratio in the elastic range and calculated as the multiplication of Young’s modulus and moment of inertia (KSMISO, 1924; Lee and Park, 2004; Pilkey, 1994).

The schematic designs of two–point, three–point, and four–point flexural stiffness tests of corrugated board according to the load application points (supporting & loading point) are shown in Fig. 4. A significant error was observed in the case of the two–point and three–point bending tests, and this error was attributed to the shearing force. In particular, in the case of the three–point bending test, the specimen length was expected to be sufficiently long to prevent the unacceptable error, which can be significantly affected by the specimen length (Maltenfort, 1989; Markstrom, 1988).

In the four–point bending test, the corrugated board was in a state of pure bending [BMD in Fig. 4(c)] and was bent to form a circular arc (pure moment beam) because the shearing force was not exerted between the supporting points [SFD in Fig. 4(c)]. Accordingly, the four–point bending test has been regarded as the most reliable method to estimate the relationship between pure bending moment and deflection.

The equations for flexural stiffness in the different bending tests (Fig. 4) were derived as equations (2)–(4) by using the differential equation of an elastic line for a neutral axis and the load–deflection relation (Lee and Park, 2004; Markstrom, 1988; Park, 2003; Urbanik, 2001).

<table>
<thead>
<tr>
<th>Items</th>
<th>Liner</th>
<th>Corrugating medium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus (GPa)</td>
<td>A: 2.01 (MD), 3.02 (MD), 4.02 (MD), 2.01 (MD), 3.02 (MD), 1.01 (CD)</td>
<td>E_{MD}/E_{CD} = 2, (MD, CD) a:b:c = 1:1.5:2</td>
</tr>
<tr>
<td></td>
<td>B: 1.01 (CD)</td>
<td>E_{MD}/E_{CD} = 2, (MD, CD) (1:2) = 1:1.5</td>
</tr>
<tr>
<td></td>
<td>C: 1.51 (CD)</td>
<td></td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.36 (MD), 0.18 (CD), ( \mu_{MD}/\mu_{CD} = 2 )</td>
<td>0.36 (MD), 0.18 (CD), ( \mu_{MD}/\mu_{CD} = 2 )</td>
</tr>
</tbody>
</table>

Note : (1) examples of board combination : (DW) OL/ML/IL/OC/IC = b/a/c/(1)/(2), (SW) OL/L/C = b/a/(2) (OL = outer liner, ML = middle liner, IL = inner liner, OC = outer corrugating medium (B/F in AB/F and BC/F–DW), IC = inner corrugating medium (A/F in AB/F–DW, C/F in BC/F–DW)

**Fig. 4.** Kinds and characteristics of bending test for corrugated board.
four–point bending:

\[ S_f = \frac{EI}{\omega} = \frac{1}{16} \left( \frac{F}{\delta} \right) \left( \frac{L}{\omega} \right) \left( \frac{a}{L} \right) \quad (4) \]

Here, \( S_f \) is the flexural stiffness per unit width (N–m), \( E \) is Young’s modulus (Pa), \( I \) is the moment of inertia (m\(^4\)), \( F \) is the load applied (N), \( \delta \) is the deflection of the sample (measuring at the center for three and four–point bending tests and at the end for the two–point bending test), \( \omega \) is the width of the specimen (m), \( L \) is the distance between the supporting points (m), and \( a \) is the distance between the supporting point and loading point (m).

**SIMULATION RESULTS AND CONSIDERATIONS**

**Flexural stiffness by flute types**

Corrugated boards used in FEA were AA/F, AB/F, and BC/F for DW and A/F, B/F, and C/F for SW. The corrugated boards were represented to the FE model for MD and CD, respectively and the material properties listed in Table 1 were applied to the models.

Fig. 5 and 6 show the examples of FEA results with MD and CD for DW and SW corrugated boards.

The flexural stiffness per unit width was estimated by equation (4) with the FEA results of the applied load–deflection relation at the mid–point of the specimen as shown in Fig. 5 and 6. The flexural stiffness of c/a/b/(1)/(1) and a/a/(2) for the MD and CD according to flute type are shown in Fig. 7. The c/a/b/(1)/(1) and a/a/(2) denote typical board combinations for both the DW and SW corrugated boards, respectively.

In the case of the DW corrugated board, the flexural stiffness along the MD was approximately 1.7 times more than that along the CD. In the SW corrugated board, the MD was more than approximately 1.5 times more than that along the CD. This indicates that flexural stiffness is significantly affected by the principal material direction of the corrugated board.

In terms of flexural stiffness based on flute types, AB/F and BC/F corrugated board were estimated as approximately 60% and 44%, respectively, for the thickest flute of AA/F in DW. The SW corrugated board B/F and C/F were estimated to be approximately 32.2% and 61%, respectively, for the biggest flute of A/F.

The results suggest that the difference in flexural stiffness can be greatly affected by the cross–sectional geometry of the flute or the combined board caliper of the corrugated board. The flexural stiffness along the MD was higher than that along the CD because the liner stiffness along the MD, which is a result of the coincidence of the MD of the corrugated board and the MD of the liner, has a greater influence than the increase in the moment of inertia which is because of the difference between the cross–sections along the MD and that along the CD.

In order to analyze the effect of reinforcing the liner...
and corrugating medium into the DW and SW corrugated boards, the a/a/a(1) combination for the DW and the a/a(1) combination for the SW were chosen. Only one element of each board was reinforced and the variation in the flexural stiffness was estimated.

The flexural stiffness along the MD and CD in AA/F–DW (Fig. 8) was increased by 21% and 15%, respectively when the inner and outer liner was reinforced with a/a/b(1)/1 and b/a/a(1)/1(1) from the a/a/a(1) combination. And the flexural stiffness of MD and CD were also increased by 7.7–13.1% and 0.4–0.8%, respectively, when the outer and inner corrugating medium were reinforced with the a/a/a(2)/1(1) and a/a/a(1)/2(1) from the a/a/a(1)/1(1) combination.

The flexural stiffness in AB/F–DW (Fig. 9) was increased by 24.8%, 16.4% along the MD and by 18.3%, 12.1% along the CD when the inner and outer liner were reinforced with a/a/b(1)/1(1) and b/a/a(1)/1(1) from the a/a/a(1) combination, respectively. The flexural stiffness was also increased by 0.7% along the MD and 6.7–8.5% along the CD when the outer and inner corrugating medium were reinforced with a/a/a(2)/1(1) and a/a/a(1)/2(1) from the a/a/a(1)/1(1), respectively. However, no variation in the flexural stiffness was found in both AA/F and AB/F when the middle liner was reinforced.

The flexural stiffness of the SW corrugated board (Fig. 10) was increased by 19.7% in the case of the MD and 16.7% in the case of the CD when the inner and outer liner was reinforced with b/a(1) and b/a(1) from the a/a/1, respectively. Further, the flexural stiffness of the SW corrugated board was also increased by 1.2% in the case of the MD and 11.1% in the case of the CD when the corrugating medium was reinforced with a/a(2) from a/a(1), respectively. Accordingly, it was estimated that the effect of the reinforcement of the liner to flexural stiffness was more significant than that of the corrugating medium.

This method facilitates limited quantitative comparison of the flexural stiffness according to the reinforcement of the liner and the corrugating medium, and moreover, shows interaction effects of the flexural stiffness of both directions on the box compression strength (Maltenfort, 1989; McKee et al., 1963; Park, 2003). To compare the qualitative buckling-resistant characteristics, Park (2003) had used a flexural stiffness index (FSI) defined as a product of the flexural stiffness of the MD and the flexural stiffness of the CD, which considered both directions of the corrugated board.

\[
\text{FSI} = [S_{f}]_{\text{MD}} \times [S_{f}]_{\text{CD}} \tag{5}
\]

Here, \([S_{f}]_{\text{MD}}\) and \([S_{f}]_{\text{CD}}\) are flexural stiffness per unit width in the MD and CD, respectively.

From Fig. 7, the FSI values for the DW corrugated boards, were evaluated as 1352, 491, and 264 for AA/F, AB/F, and BC/F, respectively, and for AB/F and BC/F, the FSI values were found to be 36.3% and 19.5% of FSI.
value of the AA/F, respectively. The FSI values for SW corrugated boards, were 31.39, 3.14, and 11.61 for A/F, B/F, and C/F, respectively, and for B/F and C/F, the FSI values were found to be 10.1% and 37.2% of the FSI value of A/F, respectively.

To compare the FSI with the qualitative data the FSI per unit basis weight for the flute types of corrugated boards, the basis weights for each of the liner and the corrugated medium were considered (listed in Table 2). The FSI per unit basis weights of the DW corrugated boards were evaluated as 1.33, 0.5, and 0.27 for AA/F, AB/F, and BC/F, respectively, and for AB/F and BC/F, it is 37.4% and 20.4% of the FSI per unit basis weight of the AA/F, respectively. In the case of the SW corrugated board, FSI per unit basis weights were 0.05, 0.006, and 0.02 for A/F, B/F, and C/F, respectively, and for B/F and C/F, FSI per unit basis weights were estimated to be 10.5% and 37.5% of the FSI per unit basis weight of the A/F, respectively.

Flexural balance
A corrugated board is an engineering structure whose strength characteristics depend on the cross-section geometry and the material properties of liner and corrugated medium. Therefore, the quality characteristics of the corrugated board and the material properties should be observed under distressed conditions to achieve optimum packaging design. It was analyzed the relationship between the board combination and the flexural balance of the corrugated board based on the FSI values, which were the significant factors in the box compression strength.

Fig. 11 and 12 show that the variation of the FSI according to the board combination of the corrugated board. In Fig. 11, the same flute constitution such as AA/F is shown, and in Fig. 12, a different flute constitution such as AB/F and BC/F is displayed.

In the AA/F–DW corrugated board, reinforcement in order of outer, inner, and middle liner has shown the greatest stiffness for all kinds of board combinations of the corrugating medium, and the effect of the reinforcement in the middle liner on the FSI is the lowest. These results show a slight difference in view of quantitative estimation, but have similar aspects in view of qualitative comparison in the AB/F–DW corrugated board.

Accordingly, it was shown the strength merits when the stiffness of liner was ranked in order of inner, outer, and middle liner, for the same condition in the DW corrugated board. It was also found that the variation of the FSI was more affected by the stiffness reinforcement and configuration of the liner than that of the corrugating medium. However, in the SW corrugated board as shown in Fig. 13, the variation of the FSI was significantly

Table 2. FSI per unit basis weight for various flute combinations

<table>
<thead>
<tr>
<th>Items</th>
<th>DW (c/a/b)(1)(1)</th>
<th>SW (a/a)(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AA/F</td>
<td>AB/F</td>
</tr>
<tr>
<td>Total basis weight (g/m²)</td>
<td>1020</td>
<td>990</td>
</tr>
<tr>
<td>FSI per unit basis weight</td>
<td>1.33</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Note: (1) assumed basis weight of liner and corrugating medium:
‘a’–150 g/m², ‘b’–180 g/m², ‘c’–210 g/m², ‘1’–150 g/m², ‘2’–180 g/m²
(2) take-up factor applied: A/F=1.6, B/F=1.4, C/F=1.5
affected by the stiffness reinforcement of the liner and corrugating medium, rather than by the stiffness of the liner configuration.

LABORATORY TESTING

In order to experimentally verify the FEA results, FEA simulation and the four–point bending test were executed simultaneously for each of the two kinds of DW and SW corrugated board as shown in Table 3.

In the four–point bending test, specimen size and loading rate were determined by TAPPI T (Urbanik, 2001) as 500×50 mm, 25.4 mm/min for both MD and CD, respectively. Also, specimens were stored in a chamber with constant temperature and humidity 48 h equilibrate with a standard atmospheric condition (23±1˚C, rh 50±2%).

A Young’s modulus of paperboards used in FEA was measured by using the universal tensile machine (AG–1, JAPAN, SHIMADZU Co.) as shown in Fig. 15 and tests were performed with Korean Industrial Standard Test Method (KS M ISO 1924–1).

As shown in Table 3, the difference between the FEA results and test results was approximately 18–31%. In the case of the SW corrugated board, the difference between the results of the two methods was more than that in case of the DW corrugated board. The $F/\delta$ value, which is described in equation (4) was calculated as a deflection for uniform loads in FEA and the slope of the line for the load–deflection curve measured by the four–point bending test in test method. We expect that this difference method for $F/\delta$ may result in the difference

<table>
<thead>
<tr>
<th>Board combination (OL/OC/ML/IC/IL)</th>
<th>Flute types</th>
<th>FSI Experimental</th>
<th>FEA</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>KLB175/K180/KLB175/K180/KLB175</td>
<td>AA/F</td>
<td>1,970</td>
<td>1,616</td>
<td>18</td>
</tr>
<tr>
<td>KLB175/K180/KLB175/K180/KLB175</td>
<td>AB/F</td>
<td>891</td>
<td>722</td>
<td>19</td>
</tr>
<tr>
<td>KLB175/K180/KLB175</td>
<td>A/F</td>
<td>125</td>
<td>89</td>
<td>29</td>
</tr>
<tr>
<td>KLB175/K180/KLB175</td>
<td>B/F</td>
<td>16</td>
<td>11</td>
<td>31</td>
</tr>
</tbody>
</table>

Note: (1) material properties applied:
KLB175: (Thickness) 0.22 mm, (Ring Crush) 25.0 kgf, (ECD) 4.24 MPa, (ECD) 0.98 MPa
K180: (Thickness) 0.24 mm, (Ring Crush) 20.2 kgf, (ECD) 3.83 MPa, (ECD) 0.78 MPa
(2) KLB: outer liner contained UKP (40%) + AOC (30%) + KOC (30%)
K: KOC (100%) (UKP = unbleached kraft pulp, AOC = American old corrugated container, KOC = Korean old corrugated container)
error.

In the study, it was estimated that the variations of FSI by both methods (bending test and FEA) have the similar results. Therefore, we expect that the flexural balance for different board combinations would be successfully analyzed by using the qualitative comparison of FSI according to the board combination with supposing the stiffness of the liner and corrugating medium, respectively, with 2–3 levels.

SUMMARY AND CONCLUSIONS

Corrugated boards have been used in various industries as a packaging material because they are environment–friendly, affordable, and have diverse qualities required for various applications. Specifically, edgewise compression strength and flexural stiffness of corrugated board should be considered as important factors when using a corrugated board as a structural member for transport packaging or as a raw material for preparing a packaging box.

In this study, we analyzed the FSI to determine the strength–optimal design conditions for the board combinations of a corrugated board. The FEA method was performed under the four–point loading conditions.

The following are conclusions and summaries.

(1) In the case of the DW and SW corrugated boards, the flexural stiffness along the MD was approximately 1.5–1.7 times more than that along the CD. This indicates that flexural stiffness is significantly affected by the principal material direction of the corrugated board.

(2) The flexural stiffness of the DW corrugated board was greatly increased when the inner or outer liner was reinforced than corrugating medium was reinforced. However, no variation in the case of SW corrugated board was found when the middle liner was reinforced. Also, in the case of SW corrugated board, it was estimated that the effect of the reinforcement of the liner to flexural stiffness was more significant than that of the corrugating medium.

(3) In the case of DW corrugated board, reinforcement in order of outer, inner, and middle liner has shown the greatest stiffness for all kinds of board combinations of the corrugating medium, and the effect of the reinforcement in the middle liner on the FSI is the lowest. Accordingly, it was shown the strength merits when the stiffness of liner was ranked in order of inner, outer, and middle liner, for the same condition. However, in the SW corrugated board, the variation of the FSI was significantly affected by the stiffness reinforcement of the liner and corrugating medium, rather than by the stiffness of the liner configuration.

(4) The difference between the FEA result and the test results were approximately 18–31%. Therefore, we expect that the flexural balance for different board combinations would be successfully analyzed on the basis of the qualitative comparison of FSI values corresponding to the combinations when the stiffness for each of the liner and the corrugating medium are assumed to be with 2–3 levels.

POSTSCRIPT

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