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Mathematical Relationships between Basic Growth Function and Bondi K–Factor

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This study was designed to investigate mathematical relationships between basic growth function and Bondi K–factor from the viewpoint of 4–dimensional phenomena. The results obtained were as follows. Basic growth function was mathematically related to Bondi K–factor. The product of relative growth rate and time ($r{\cdot}t$) where $W{\cdot}c^2$ replaced W showed a closer mathematical relationship to natural logarithm of Bondi K–factor. If based on these mathematical relationships, then basic growth phenomena look like 4–dimensional phenomena. Applying basic growth function to 3–dimensional body frame size was also discussed. In addition, appendix was added to give corrected versions to problems that were found in previous reports.

Key words: basic growth function, Bondi K–factor, exponential function, 4–dimension

INTRODUCTION

Basic growth analysis of a plant (Blackman, 1919) or an animal (Brody, 1945) is conducted using exponential function to analyze the weight increase with the passage of time. It is well known that exponential function is equal to Bondi K–factor. Bondi K–factor is one of the useful tools to investigate and explain 4–dimensional phenomena in physics (Bondi, 1964). Shimojo (2011a, 2011c) also investigated relationships between them, but in a report (Shimojo, 2011c) there are problems found that require further investigation.

The present study was designed to investigate mathematical relationships between basic growth function and Bondi K–factor from the viewpoint of 4–dimensional phenomena.

BASIC GROWTH FUNCTION AND BONDI K–FACTOR

Relative growth rate and basic growth function

The following are differential equation for relative growth rate (1) and its solution (2),

$$\frac{1}{W} \cdot \frac{dW}{dt} = r, \tag{1}$$

$$W = W_0 \exp(r{\cdot}t), \tag{2}$$

where W = weight, t = time, W_0 = weight at t = 0, r = relative growth rate.
In the present study, W is regarded as the weight of matter instead of a plant or an animal.

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Bondi K–factor and exponential function

Hyperbolic functions relate Lorentz transformation with exponential function, leading to expressions (3) and (4),

$$\exp(\theta) = \sqrt{\frac{1 + v/c}{1 - v/c}}, \tag{3}$$

$$\theta = \ln\left(\sqrt{\frac{1 + v/c}{1 - v/c}}\right), \tag{4}$$

where v = velocity of an object, c = speed of light, $0 \leq v < c$, $0 \leq \theta < \infty$, $1 \leq \exp(\theta) < \infty$, ln = natural logarithm. Bondi K–factor is exponential function. Instead of Bondi K–factor, exponential function may be used because the value of v is determined from θ. Boldly presuming from the mathematical viewpoint, exponential increases look like 4–dimensional phenomena.

Basic growth function and Bondi K–factor

A mathematical application of expression (3) to basic growth function (2) leads to

$$\exp(r{\cdot}t) = \sqrt{\frac{1 + v/c}{1 - v/c}}. \tag{5}$$

Since r is actually the mean value over the interval t_1 to t_2, expression (6) is given,

$$\frac{\ln W_2 - \ln W_1}{t_2 - t_1} \cdot (t_2 - t_1)$$
$$= \frac{\ln(W_2 \cdot c^2) - \ln(W_1 \cdot c^2)}{t_2 - t_1} \cdot (t_2 - t_1)$$
$$= \ln\left(\sqrt{\frac{1 + v/c}{1 - v/c}}\right). \tag{6}$$

Expression (5) shows that basic growth function is mathematically related to Bondi K–factor. Expression (6) shows that the product of relative growth rate and time ($r \cdot t$) where $W \cdot c^2$ replaces W has a closer mathematical relationship to natural logarithm of Bondi K–factor, a mathematical relationship between $r \cdot t$ and v/c. If based on these mathematical relationships, then basic growth phenomena look like 4–dimensional phenomena. Basic growth function may also be applied to the analysis of 3–dimensional body frame size (*X, Y, Z*) of matter, for example, a report by Shimojo *et al.* (2009) for a plant or an animal. Whether or not the growth of matter (*W, X, Y, Z*) looks like 4–dimensional phenomenon requires further investigation. In addition, expressions (5) and (6) are corrected versions to problems that were found in a previous report (Shimojo, 2011c) where only r was related to the natural logarithm of Bondi K–factor.

Properties of Bondi K–factor including Lorentz factor

Bondi K–factor (B) includes Lorentz factor (L) as follows,

$$\sqrt{\frac{1+v/c}{1-v/c}} = \frac{1}{\sqrt{1-(v/c)^2}}\left(1+\frac{v}{c}\right). \qquad (7)$$

If Bondi K–factor and Lorentz factor are hypothetically regarded as functions of v, then

$$\frac{(dB(v)/dt)^2}{B(v)(d^2B(v)/dt^2)} = \frac{c}{c+2v}, \qquad (8)$$

$$\frac{(dL(v)/dt)^2}{L(v)(d^2L(v)/dt^2)} = \frac{v^2}{c^2+2v^2}. \qquad (9)$$

Since $0 \leq v < c$, the following inequality is given,

$$0 \leq \frac{v^2}{c^2+2v^2} < \frac{1}{3} < \frac{c}{c+2v} \leq 1. \qquad (10)$$

This is compared with the case of exponential function ($e(\theta)$) that is given by expression (11),

$$\frac{(de(\theta)/dt)^2}{e(\theta)(d^2e(\theta)/dt^2)} = 1. \qquad (11)$$

What the inequality (10) shows remains to be investigated.

Appendix: the form of expression (11) and complex wave function

Shimojo (2011b) showed that expression (11) was given by complex wave function (12) as well as basic growth function (2),

$$\phi = A \cdot \exp((\boldsymbol{i} \cdot F) \cdot t), \quad (12) \qquad W = W_0 \exp(r \cdot t), \quad (2)$$

where ϕ = wave function, $\boldsymbol{i}$ = imaginary unit, A = amplitude, F = term related to frequency. Shimojo (2011b) compared functions (12) and (2) to investigate hypothetic wave–matter relationships, but there was a problem in the hypothetic interpretation of F as f/A, the number of frequencies (f) divided by amplitude (A). The corrected version is F = frequency, and hypothetic wave–matter relationships are: (high A, high F) – (high W_0, high r), (high A, low F) – (high W_0, low r), (low A, high F) – (low W_0, high r), (low A, low F)–(low W_0, low r).

In addition, complex wave function (12) is extended as follows,

$$\phi(x, t) = A \cdot \exp((\boldsymbol{i}/(h/2\pi)) \cdot (p \cdot x - E \cdot t)), \qquad (13\text{–}1)$$

$$\phi^*(x, t) = A \cdot \exp((-\boldsymbol{i}/(h/2\pi)) \cdot (p \cdot x - E \cdot t)), \qquad (13\text{–}2)$$

$$\phi^*(x, t) \cdot \phi(x \cdot t) = |\phi(x, t)|^2, \qquad (13\text{–}3)$$

where h = Planck's constant, π: = circular constant, p = momentum, x = position, E = energy. The expression (13–3) is a corrected version to $|\phi(x, t)|^2 = 1$ in a previous report (Shimojo, 2011c).

Conclusions

It is suggested that basic growth phenomena look like 4–dimensional phenomena when the product of relative growth rate and time is mathematically related to natural logarithm of Bondi K–factor.

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