A viscoelastic model for time-dependent simulating analysis of the Wenchuan earthquake fault

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A viscoelastic model for time-dependent simulating analysis of the Wenchuan earthquake fault

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Abstract. The sudden big earthquake which happened in the Wenchuan county of China in 2008 was an extraordinary type of earthquake. Its exact mechanism is still unknown. The paper presents a simplified computational approach to develop a model to simulate the long-term evolution of the Wenchuan earthquake fault by means of a viscoelastic finite-element method, in order to investigate the dynamic mechanism of the 2008 Wenchuan earthquake. The relevant characteristics of crustal stress fields and displacement fields around the fault are analyzed. It is suggested that the accumulated earthquake energy was mainly due to deep crust motion rather than to surface motion. The study helps show that viscoelastic modeling is a powerful tool for simulating natural phenomena such as crustal movement and its implication for earthquakes.

Keywords. viscoelastic, finite-element method, Wenchuan earthquake, crustal fault

1. INTRODUCTION

The 12 May 2008 Wenchuan earthquake (M = 8.0) was the strongest earthquake in China in over 50 years. The magnitude 8.0 earthquake occurred on the well-recognized Longmen Shan (Chinese name meaning dragon's gate mountains) fault zone, which is on the eastern side of the Tibet plateau and rises 6,000 m above the Sichuan basin (Fig. 1). Modern geological surveys combined with Global Positioning System (GPS) measurements indicate a very low rate of ground surface motion (east-west shortening, $< 3\,\mathrm{mm/yr}$) across the Longmen Shan. It confirms the 1–2 millimetres per year suggested by Burchfiel [1]. The very low displacement rate had been considered as evidence in support of the Longmen Shan fault's historical seismicity that shows no earthquake with magnitude over 7 occurred for at least the last 2000 years [2]. From a geological viewpoint, that rate seems to be relatively moderate, which implies that major quakes would not have been happening in this region [1, 3, 4]. Faults store up potential earthquake energy in proportion to regional crust motion. So how had the energy accumulated to trigger the great Wenchuan earthquake when the surface motion of crust was so very low? The energy accumulation mechanism represents a problem worth researching.

In this paper, on the bases of regional tectonic characteristics of the Longmen Shan area, a computational model for time-dependent numerical simulation is constructed by means of a viscoelastic finite-element method. The time-evolution simulation is performed for 2500 years, considering a long-term accumulation for the Wenchuan earthquake energy. Differences in both the crustal stress and displacement fields around the fault between surface material and deep material of crust are examined. The simulation results suggest that the accumulated energy was mainly contributed by deep crust motion rather than by surface motion. Based on these results, the dynamic mechanism of the Wenchuan earthquake has been discussed. This study also shows that viscoelastic modeling is a powerful tool for simulating natural phenomena such as crustal movement and its implication for earthquakes.

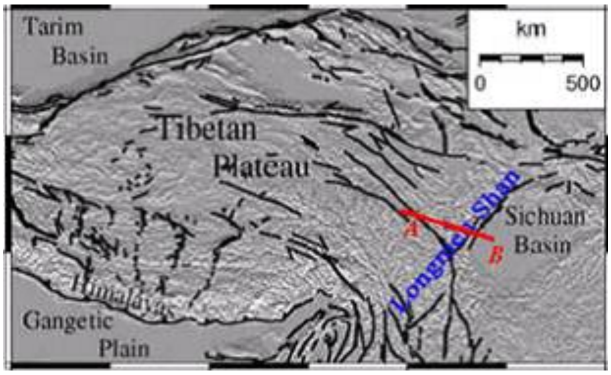

Figure 1: Topographic map of the Longmen Shan and surrounding areas, modified from Godard et al., 2009 [8]

2. MODELING DESCRIPTION

We consider a simplified and practical approach by focusing the investigation on the section along the red line AB

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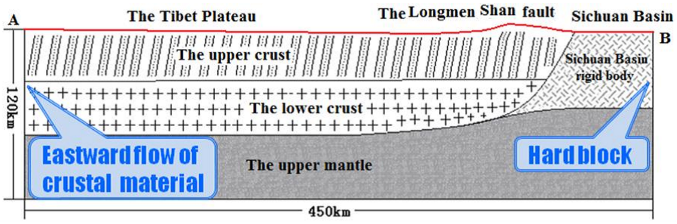

Figure 2: Geological cross-section A-B

as shown in Fig. 1. It traverses the very long distance from the eastern part of the Tibet plateau, across the Longmen Shan region, to the western part of Sichuan basin. The line AB also crosses over the exact location of the Wenchuan earthquake. The current approach was motivated by earlier work using finite-element methods to simulate the dynamic evolution of fault [5, 6]. This simulation represents a reasonable and appropriate way to approach the problem of modeling crustal motion. Figure 2 shows the line AB's geological cross-section, which highlights the significant tectonic differences of the Longmen Shan and its fault zone [1, 7].

Based on these tectonic details, along with geometrical and mechanical characteristics of the section, a 3-D dynamic computational model was constructed, as shown in Fig. 3, to simulate the long-term evolution of the Longmen Shan fault. The dynamic modeling and numerical computation were assumed to follow the principles of the continuum mechanics and established by using viscoelastic finite-elements.

The simulation of the Longmen Shan fault was performed using the commercial finite-element code ANSYS. The upper crust in eastern Tibet was modeled using elastic elements, while the lower crust in eastern Tibet and the upper mantle were modeled using viscoelastic elements. The total numbers of elements and nodes were 4824 and 7160, respectively. Fine meshing was assigned for the Longmen Shan region, because of its complicated tectonics. The interfaces shown as red and yellow lines in Fig. 3 were modeled using contact elements, which are defined as a contact pair. There are no direct or indirect available friction coefficients for the study area. So it was assumed that a low friction coefficient held for the interfaces between the upper and the lower crust, and between the lower crust and the upper mantle. The friction coefficient for the Longmen Shan fault zone was assumed to be high.

The computational conditions include displacement boundary conditions: the eastern boundary (the right side in Fig. 3) of the model is fixed in the horizontal direction and free in the vertical direction; the bottom of the model is free-slip in the horizontal direction and fixed in the vertical direction; the top surface of the model is free.

The displacement velocity condition, as an applied load, is prescribed on the western boundary (the left side in Fig. 3) of the model. Taking into account displacement rates of about 20 mm/yr for the eastward movement of upper crust away from the central Tibet plateau [7], it is assumed that the horizontal velocity of the west boundary is $V = 20\,\mathrm{mm/yr}$. In addition, an acceleration of $9.8\,\mathrm{m/s^2}$, due to gravity, is applied vertically in the calculation.

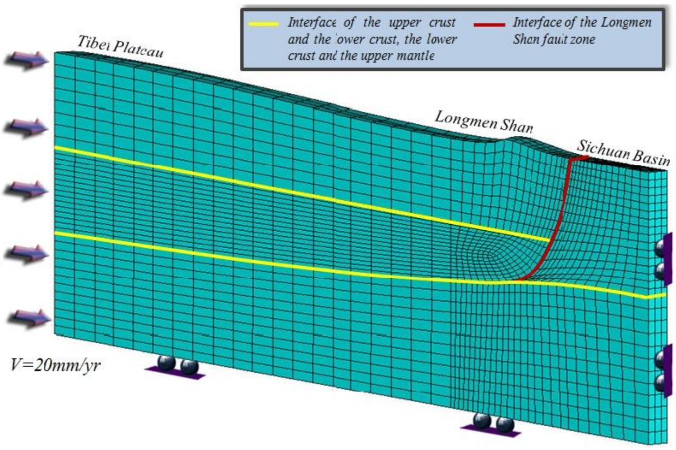

Figure 3: dynamic model and analysis conditions

In order to account for the very long-term accumulation of the Wenchuan earthquake energy which comes from the eastern margin of the Tibet plateau, the simulation for the Longmen Shan fault is run for 2500 years. This is because, before the 2008 Wenchuan earthquake, the Longmen Shan fault lacked a big earthquake for more than 2000 years [2]. The Wenchuan earthquake may have a millenary recurrence cycle.

3. FUNDAMENTAL EQUATIONS

The computational model simulates crustal motion as deformation in continuum mechanical terms by solving the dynamic equilibrium equation of force balance:

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial \sigma_{ij}}{\partial x_j} + \rho g_i \ , \tag{1}$$

where u_i is the displacement vector $(i = 1, 2, 3)$ related to the crustal motion, σ_{ij} is the ij-component of Cauchy's stress tensor $(i, j = 1, 2, 3)$ corresponding to the crustal stress, ρ is the density, and g_i is the gravitational acceleration.

The strain tensor ε_{ij} $(i, j = 1, 2, 3)$ derived from the displacement field u_i is a geometrical measure of the crust motion, which takes the form:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) . \tag{2}$$

The above relation is known as the kinematical equation, the relation between the strain and displacement.

The crust motion is related to crustal stress through the constitutive equation, which is based on the theory of continuum mechanics and is given by:

$$\sigma_{ij} = \frac{Ev}{(1+v)(1-2v)}\varepsilon_{kk}\delta_{ij} + \frac{E}{1+v}\varepsilon_{ij} \ , \tag{3}$$

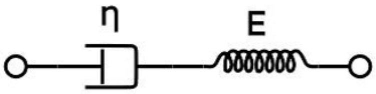

Figure 4: A linear combination of spring and dashpot

where the parameters E and v are Young's modulus and Poisson's ratio, respectively; δ_{ij} is the Kronecker delta. The above equation is known as the generalized Hooke's law, linearly correlating the stress and the strain. In this model, the strain is composed of elastic and viscous components.

The model calculates, for each time step, the incremental displacement u_i as well as the incremental strain ε_{ij}, which includes elastic and viscous components: *i.e.* each time-evolution increment includes elastic and viscous components:

$$\begin{cases} u_i = u_i^{elastic} + u_i^{viscous} \\ \varepsilon_{ij} = \varepsilon_{ij}^{elastic} + \varepsilon_{ij}^{viscous} \end{cases} \tag{4}$$

where superscripts *elastic* and *viscous* denote the elastic and viscous part, respectively.

The equivalent stress σ is computed. It is a scalar stress that is computed from the stress tensor:

$$\sigma = \left\{ \frac{1}{2} \left[\begin{matrix} (\sigma_{11}-\sigma_{22})^2 + (\sigma_{22}-\sigma_{33})^2 + \\ (\sigma_{11}-\sigma_{33})^2 + 6(\sigma_{12}^2+\sigma_{23}^2+\sigma_{13}^2) \end{matrix} \right] \right\}^{1/2} . \tag{5}$$

The equivalent stress smooths out the contribution of the directional dependence of the stress tensor and is, therefore, a measure of stress strength. The corresponding equivalent strain ε is given by:

$$\varepsilon = \left\{ \frac{2}{9} \left[\begin{matrix} (\varepsilon_{11}-\varepsilon_{22})^2 + (\varepsilon_{22}-\varepsilon_{33})^2 + \\ (\varepsilon_{11}-\varepsilon_{33})^2 + 6(\varepsilon_{12}^2+\varepsilon_{23}^2+\varepsilon_{13}^2) \end{matrix} \right] \right\}^{1/2} . \tag{6}$$

In our model,the viscoelastic behavior of the crustal motion is assumed to be Maxwellian, which consists of an elastic element and a viscous element connected in series as shown in the Fig. 4 [9]. The elastic element describes a spring relationship between the stress σ and the strain ε with modulus E, while the viscous element equation describes a dashpot relationship between time-dependent stress σ and strain rate $\dot{\varepsilon}$ with viscosity η. Combining the elastic and viscous elements for the Maxwell model yields the time-evolution relationship [10]

$$\dot{\varepsilon}(t) = \frac{1}{E}\dot{\sigma}(t) + \frac{1}{\eta}\sigma(t) . \tag{7}$$

The equations (1)–(7) were used in our model to simulate the time evolution of the crustal stress and displacement fields in the Wenchuan earthquake fault zone caused by the eastward movement from Tibet plateau.

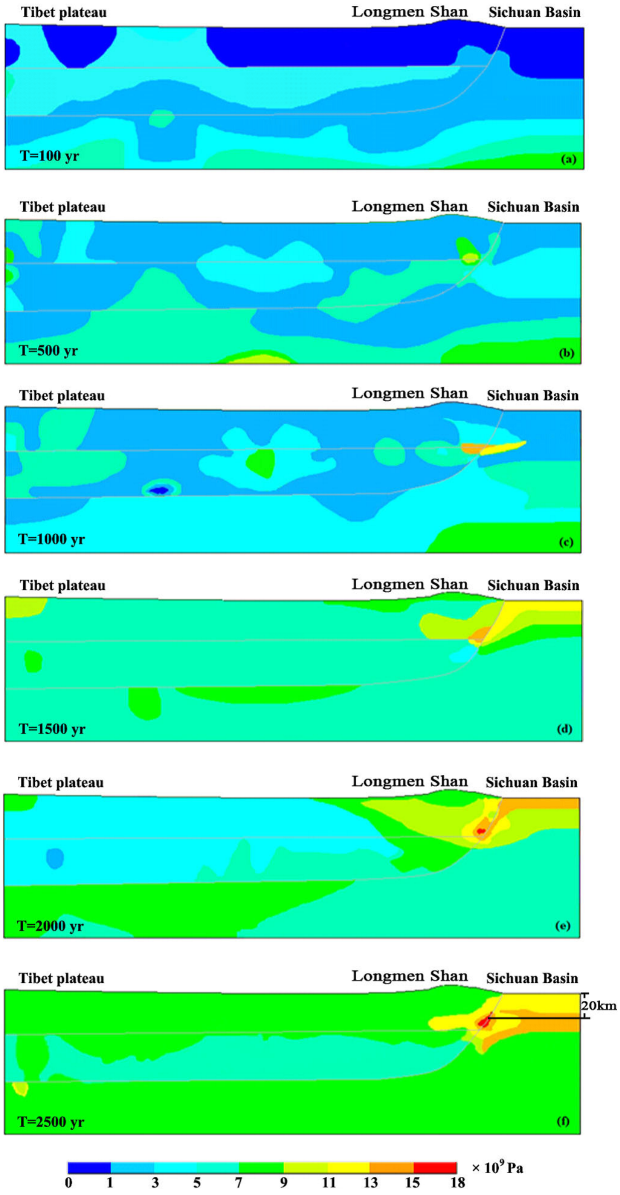

Figure 5: Time evolution of equivalent stresses

4. SIMULATION RESULTS

4.1. STRESS EVOLUTION

After the simulation of a period of 2500 years in process of the eastern crustal flow of the Tibetan plateau, stress evolution results show the equivalent stress continuously increases with time evolution and that there is concentration under the Longmen Shan fault. Figure 5 illustrates the equivalent stress evolution with time of 2500 years. With such a long time evolution, the stress concentration area gets closer and closer to the Longmen Shan fault, and specially after 2000 years elapsed time (see Fig. 5 (e)–(f)), the stress concentration increases significantly.

The equivalent stress evolution results also indicate that the peak stress values at the level of 15–18 × 10^9 Pa located under the Longmen Shan (see Fig. 5 (e)–(f)), may have already gone beyond the theoretical strength of crustal materials. From a viewpoint of stress criterion, it could be expected that stress of the concentration area will exceed the stress intensity limit and a following earthquake may occur in the Longmen Shan fault nearby. Furthermore, the largest stress concentration is found at a depth of about 20 km, as shown in Fig. 5 (f), which is near the focal depth of 19 km determined by the China Earthquake Network Center.

4.2. DISPLACEMENT EVOLUTION

Figure 6 shows the simulation results of displacement evolution in the same period. Results show that, displacements continuously increase with time evolution and deep crustal displacements are larger than the shallow parts in general, especially the deep crustal displacements below the Longmen Shan fault zone increase quickly after 1000 years of elapsed time (see Fig. 6 (c)–(f)).

With crustal stress data (as shown in Fig. 5) and crustal displacement data (as shown in Fig. 6), we can calculate crustal strain-energy density $w = 0.5\sigma_{ij}\varepsilon_{ij} = 0.5\sigma_{ij}\,\partial u_i/\partial x_j$. The energy w has the physical significance of the potential energy stored within crustal structures. Thus, crustal faults store up potential earthquake energy in proportion to regional crustal strain and applied stress. Then it could be expected that the accumulated earthquake energy of the Longmen Shan surrounding region may also exceed its strength and failure limit finally and a following earthquake may occur nearby. This is reasonable from a viewpoint of energy criterion.

5. DISCUSSION AND CONCLUSIONS

We deal simply with the concept of crustal strength and failure that should be related to the mechanism of earthquake. As shown above in the simulation results, stress or energy parameter has been optionally used as selection criterion of strength and failure in determining what had been accumulated to trigger the great Wenchuan earthquake. Our analysis is least successful in explaining that higher levels of stress or potential energy at depths may be useful in estimating occurrence of large earthquakes, and may be the important factors resulting in the Wenchuan earthquake.

Because of uncertainties about the complexity of various stress criteria for crustal materials, we hereby choose to apply the energy criterion. Therefore, our analysis means that the accumulated earthquake energy to trigger the Wenchuan earthquake is mainly contributed by deep crust motion rather than by surface motion. This may help explain why such a large earthquake was unexpected in Wenchuan because the surface motion of crust had been very low for a very long time before the 2008 Wenchuan earthquake.

The model results have shown that viscoelastic modeling is a powerful tool for simulating natural phenomena such as

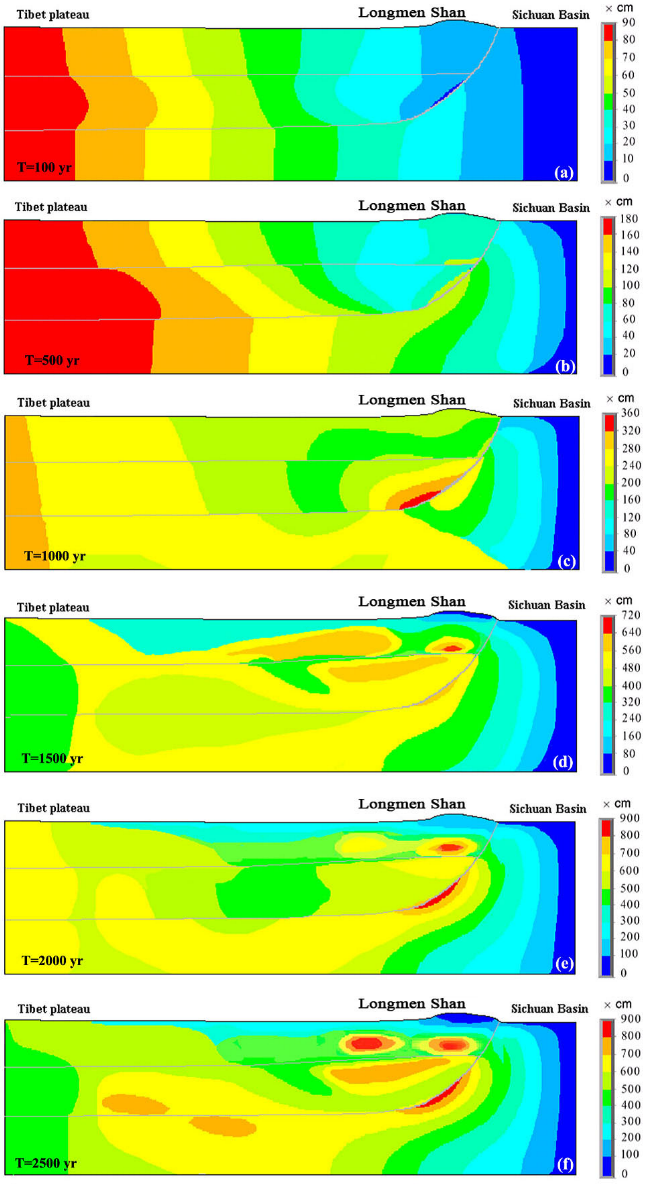

Figure 6: Time evolution of displacements

crustal movement and its implication for earthquakes. As a real crustal structure has more complicated characteristics, the modeling data in this paper have been much simplified, and it should be refined. We are convinced that using a more accurate model will enable us to do further research in the future.

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