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A computer-assisted proof for the pattern formation on reaction-diffusion systems

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Abstract

In this paper we give a method by computer-assistance to prove a pattern formation. As a typical model we consider two dimensional time-dependent reaction-diffusion equations with Neumann boundary conditions. For suitable system parameters we solve (approximately) the parabolic problem, hoping for some convergence to some pattern formation stationary (approximate) solution, and improve the approximation to a stationary solution by Newton's method, then enclose the stationary solution by our numerical verification method. Next we prove that the operator linearized at the exact stationary solution is a sectorial operator and compute a bound for the resolvent of the linearized operator which is needed for semigroup estimates. By using the semigroup estimate we analytically compute a domain of attraction for the stationary solution, i.e. some (norm-)neighborhood of the stationary solution such that, for initial data within this neighborhood, the parabolic solution converges to the stationary solution. For suitable initial conditions, if we enclose the solution of the parabolic problem until, for some time T, the enclosing set is a subset of the domain of attraction, then we can conclude that from time T on, convergence to the stationary solution takes place. This gives a complete convergence result, proving a pattern formation, for the initial conditions used for the parabolic problem.