

# Efficient Implementation of $\eta$ T Pairing on Supersingular Elliptic Curves in Characteristic 3

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(標数 3 の超特異楕円曲線上の  $\eta_T$  ペアリングの高速実装)

区 分： 甲

## 論 文 内 容 の 要 旨

Pairing-based cryptosystems can provide cryptographic schemes which have novel and useful properties, such as Identity-based encryption schemes, and they have been attracted in cryptography. These schemes are constructed by using pairings, such as the Tate and Weil pairings, hash functions, and group computations. Miller proposed the first polynomial-time algorithm for computing the Weil pairing on algebraic curves, and various pairings are indicated until now.  $\eta_T$  pairing over  $F_{3^m}$  is one of the fastest pairings now.

We propose efficient algorithms of addition and subtraction in  $F_3$  and MapToPoint, which is a hash function to compute a point on elliptic curves, for efficient implementation of the  $\eta_T$  pairing over  $F_{3^m}$ . Firstly, we construct instruction sequences of addition and subtraction in  $F_3$  with the minimum number of logical instructions, since all functions of the  $\eta_T$  pairing is based on them. Every  $F_3$ -element is assigned to two bits, and the  $F_3$ -addition and subtraction is considered as a map  $(F_2)^2 \times (F_2)^2 \rightarrow (F_2)^2$ . We perform an exhaustive search for finding the instruction sequences of the  $F_3$ -addition that use seven or fewer logical instructions. Indeed, we find many implementations of the  $F_3$ -addition and subtraction with only six logical instructions, and no sequence that can compute them with less than six logical instructions for any assignment of elements or logical instructions. In other words, we have proven that the minimum number of logical instructions required for the  $F_3$ -addition and subtraction is six.

MapToPoint algorithm is used to compute a point of an elliptic curve from identity. There exists two conventional algorithms for supersingular elliptic curves over  $F_{3^m}$ : one is computed by using a square root computation in  $F_{3^m}$ , which needs  $O(\log m)$  multiplications and  $O(m)$  cubings in  $F_{3^m}$ ; another is computed by using an  $(m-1) \times (m-1)$  matrix over  $F_3$ , which needs the off-line memory to store it. We construct an efficient MapToPoint algorithm on the supersingular elliptic curves by using  $1/3$ -trace over  $F_{3^m}$ . The  $1/3$ -trace can compute a solution  $x$  of  $x^3 - x = c$  by using no multiplication in  $F_{3^m}$ . The proposed algorithm is computed by  $O(1)$  multiplications and  $O(m)$  cubings in  $F_{3^m}$ , and it requires less than  $m$   $F_3$ -elements to be stored in the off-line memory to efficiently compute  $trace$  over  $F_{3^m}$ .

Finally, the implementation of the  $\eta_T$  pairing, arithmetic over  $E(F_{3^m})$  and  $F_{3^m}$  are described. We implement them in C and Java as programming languages, and measure the running time of these functions on an Intel Core i7-950 processor.