Stable systolic category of the product of spheres

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（有理ホモロジー球面の積空間における安定シストリックカテゴリーに関する研究）

論　文　内　容　の　要　旨

In this paper, a manifold is assumed to be closed, connected, orientable and smooth. The systole of a manifold M is the least length of non-contractible closed loops in M. One can generalize this concept to the least volume of k–dimensional nonzero homology classes, so called as the homology systole. Now we can imagine such systoles have some kind of relations with the entire volume of M, and it is natural to ask what kind of relationship exists.

As an answer, Gromov proved a theorem that says that the existence of non-trivial cup product implies the existence of the stable isosystolic inequality as follows.

Gromov's Theorem
Let M be an n–manifold. If there exist some reduced real cohomology classes $\alpha_{1*}, \cdots, \alpha_{k*}$ with α_{i*} in $H^{d_i}(M;R)$ and a nonzero cup product $\alpha_{1*} \cup \cdots \cup \alpha_{k*}$ in $H^n(M;R)$, then there exists C>0 satisfying

$$\prod_{i=1}^{k} \mathrm{stsys}_{d_i}(M,\mathcal{G}) \leq C \cdot \mathrm{mass}([M],\mathcal{G})$$

for all Riemannian metric on M where stsys_{d_i} is the stable d_i–systole and [M] is the fundamental class of M with coefficients in Z/2Z.

The greatest k satisfying the stable isosystolic inequality is called as the stable systolic category of M. The stable systolic category is introduced by Katz and Rudyak, and it is known as a homotopy invariant also by Katz and Rudyak. We will show the stable systolic category of 0-universal manifold is also invariant under the rational equivalences.

For an orientable manifold M, Gromov's Theorem implies that the stable systolic category is not smaller than the real cup-length. So, is there some manifold M such that the stable systolic category is greater than the real cup-length? If such M exists, then the inversion of Gromov's Theorem will fail for M, while this interesting question is not answered yet. Instead of the answer, it is known the equality of them for some manifolds. In this paper, we also show equality for the product space of rational spheres.