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Fast analysis method of time-periodic nonlinear fields

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Abstract. A fast analysis method is proposed to obtain time-periodic nonlinear fields in the presence of extremely slow decay fields. First, the analysis variables are time-averaged to reduce effects of the harmonic wave components, and next, second order time-derivatives of the time-averaged values are used to correct the variables toward the time-periodic steady-state field. The time width in time-averaging operation can be set much shorter than one half period, and so the variables can be corrected in early stage of time evolution. The presented method was validated in two-variable simultaneous equations as a simple problem and a magnetic field simulation by the finite element method as a multivariable problem. Furthermore, harmonic TDC and a serial usage of (harmonic) TDC and TP-EEC are proposed for the case that higher order time-harmonic waves are included in the corrected objectives. In addition, the conventional simplified three-phase AC TP-EEC method is expanded to a general form for the three-phase AC system.

Keywords. time-periodic, steady-state, correction, transient field

1. Introduction

A highly accurate time-periodic solution of nonlinear time-differential equation can be derived in several ways. As a standard technique, the shooting method [1, 2] is more commonly used in a case of a small-scale system like an electric circuit. The time-periodic finite element method [3, 4] can be powerfully used in the two-dimensional finite element analysis of electromagnetic field. The harmonic balance finite element method [5] uses nonlinear analysis in frequency domain where a large-scale matrix equation must be analyzed.

A step-by-step solution in the transient analysis is corrected toward a steady-state one-half periodic solution every one half period in TP-EEC (Time periodic explicit error correction) method [6, 7], while every one sixth of period in three-phase AC TP-EEC method [8, 9] which is generally proposed by Tokumasu as the polyphase AC TP-EEC method. The TP-EEC and three-phase AC TP-EEC methods are very powerful techniques to secure periodic solutions in several corrections. In other words, however, the TP-EEC and three-phase AC TP-EEC methods require one half and one sixth period of calculations for one correction, respectively. In this paper, a new correction method named as TDC (Time Differential Correction) is presented with requiring only transient calculation much shorter than one half period with no or a little of higher time-harmonic waves. Furthermore, the simplified three-phase AC TP-EEC is expanded to a general form in this paper.

2. Principle and formulations of TDC

The principle of TDC is clearly described here. The variables have harmonic wave components and a decay term as well as a basic wave component. The variable $x(\theta)$ can be written as follows:

$$x(\theta) = a_0 e^{-\gamma\theta} + \sum_{k=1}^{p} x_{n_k}(\theta) + \sum_{l\neq k} x_{n_l}(\theta). \tag{1}$$

In the right hand side of the above equation, the first term indicates a decay term, the second term is composed of main harmonic wave components including basic one, and the last term has the other additional harmonic wave components. The variable θ is an electric angle working as a time variable. To reduce the harmful effect of the additional harmonic wave components, the variable $x(\theta)$ is averaged over the angle width 2ϕ to become $y(\theta) = \langle x(\theta)\rangle$. The averaged variable $y(\theta)$ is approximated by $\hat{y}_p(\theta)$ defined as

$$\hat{y}_p(\theta) := a_0' e^{-\gamma\theta} + \sum_{k=1}^{p} y_{n_k}(\theta), \tag{2}$$

where

$$y_{n_k}(\theta) = \langle x_{n_k}(\theta)\rangle = \left(\frac{\sin n_k\phi}{n_k\phi}\right) x_{n_k}(\theta-\phi). \tag{3}$$

The $2m$-th order derivative of Eq. (2) is approximated by $\hat{y}_p^{(2m)}(\theta)$ defined by

$$\hat{y}_p^{(2m)}(\theta) := \sum_{k=1}^{p} (-1)^m n_k^{2m} y_{n_k}(\theta). \tag{4}$$

Using the derivatives of the order of even-number up to $2p$, we obtain the harmonic wave components $y_{n_k}(\theta)$ ($k = 1, 2, \ldots, p$). Equations (3) and (4) generate the following equation to correct $x(\theta)$ toward a one-half periodic steady state field,

$$x_{\text{new}}(\theta - \phi) = \sum_{k=1}^{p} g_{n_k} y_{n_k}(\theta), \qquad g_{n_k} = \frac{n_k \phi}{\sin n_k \phi}. \tag{5}$$

A $2p$-th order derivative can be obtained by $2p + 1$ steps calculation, and therefore the variable $x(\theta)$ is more accurately corrected at the time point with the electric angle $\theta - \phi - p\Delta\theta$ where $\Delta\theta$ is the electric angle width corresponding to the time step width in the step-by-step calculation. The higher order derivative is used, the more the accuracy of calculation is reduced. Therefore, a large value of p is not of practical use for the TDC method, and so it is useful that $p = 1, 2, 3,$ or 4.

In this paper, the name "TDC" is used in the case that $p = 1$, i.e., for only use of basic wave component, while the name "n-harmonic TDC" is used in the case that $p = 1 + n$ ($n = 1, 2,$ or 3).

When the corrected variables have no time-harmonics, TDC is used. On the other hand, n-harmonic TDC is used when n is the number of main time-harmonic wave components included in the corrected variables.

2.1. Formulation of TDC

Using one basic wave component, we will derive the TDC formula. In this case, Eq. (2) becomes

$$\hat{y}_1(\theta) = a_0' e^{-\gamma\theta} + y_1(\theta). \tag{6}$$

The second derivative of Eq. (6) is approximately written as

$$\hat{y}_1^{(2)}(\theta) = -y_1(\theta). \tag{7}$$

Then, we obtain the following correction formula,

$$x_{\text{new}}(\theta - \phi) = g_1 y_1(\theta). \tag{8}$$

2.2. Formulation of 1-harmonic TDC

Using one harmonic wave component added to one basic wave one, we will derive the 1-harmonic TDC formula. In this case, Eq. (2) becomes

$$\hat{y}_2(\theta) = a_0' e^{-\gamma\theta} + y_1(\theta) + y_n(\theta). \tag{9}$$

The second and forth derivatives of Eq. (9) are approximately written as

$$\hat{y}_2^{(2)}(\theta) = -y_1(\theta) - n^2 y_n(\theta), \tag{10}$$

$$\hat{y}_2^{(4)}(\theta) = y_1(\theta) + n^4 y_n(\theta). \tag{11}$$

Then, we have

$$y_1(\theta) = -\frac{n^2 \hat{y}_2^{(2)}(\theta) + \hat{y}_2^{(4)}(\theta)}{n^2 - 1}, \quad \text{and} \tag{12}$$

$$y_n(\theta) = \frac{\hat{y}_2^{(2)}(\theta) + \hat{y}_2^{(4)}(\theta)}{n^2(n^2 - 1)}. \tag{13}$$

Thus, we obtain the following correction formula,

$$x_{\text{new}}(\theta - \phi) = g_1 y_1(\theta) + g_n y_n(\theta). \tag{14}$$

2.3. Formulation of 2-harmonic TDC

Using two harmonic wave components added to one basic wave one, we will derive the 2-harmonic TDC formula. In this case, Eq. (2) becomes

$$\hat{y}_3(\theta) = a_0' e^{-\gamma\theta} + y_1(\theta) + y_n(\theta) + y_m(\theta). \tag{15}$$

The second, forth and sixth order derivatives of Eq. (15) are approximately written as

$$\hat{y}_3^{(2)}(\theta) = -y_1(\theta) - n^2 y_n(\theta) - m^2 y_m(\theta), \tag{16}$$

$$\hat{y}_3^{(4)}(\theta) = y_1(\theta) + n^4 y_n(\theta) + m^4 y_m(\theta), \tag{17}$$

$$\hat{y}_3^{(6)}(\theta) = -y_1(\theta) - n^6 y_n(\theta) - m^6 y_m(\theta), \tag{18}$$

respectively. Then, we have

$$y_1(\theta) = -\frac{n^2 m^2 \hat{y}_3^{(2)}(\theta) + (n^2 + m^2)\hat{y}_3^{(4)}(\theta) + \hat{y}_3^{(6)}(\theta)}{(n^2 - 1)(m^2 - 1)}, \tag{19}$$

$$y_n(\theta) = -\frac{m^2 \hat{y}_3^{(2)}(\theta) + (m^2 + 1)\hat{y}_3^{(4)}(\theta) + \hat{y}_3^{(6)}(\theta)}{n^2(n^2 - 1)(n^2 - m^2)}, \tag{20}$$

$$y_m(\theta) = -\frac{n^2 \hat{y}_3^{(2)}(\theta) + (n^2 + 1)\hat{y}_3^{(4)}(\theta) + \hat{y}_3^{(6)}(\theta)}{m^2(m^2 - 1)(m^2 - n^2)}. \tag{21}$$

Thus, we obtain the following correction formula,

$$x_{\text{new}}(\theta - \phi) = g_1 y_1(\theta) + g_n y_n(\theta) + g_m y_m(\theta). \tag{22}$$

2.4. Formulation of 3-harmonic TDC

Using three harmonic wave components added to one basic wave one, we will derive the 3-harmonic TDC formula. In this case, Eq. (2) becomes

$$\hat{y}_4(\theta) = a_0' e^{-\gamma\theta} + y_1(\theta) + y_n(\theta) + y_m(\theta) + y_k(\theta). \tag{23}$$

The second, forth, sixth and eighth order derivatives of Eq. (23) are approximately written as

$$\hat{y}_4^{(2)}(\theta) = -y_1(\theta) - n^2 y_n(\theta) - m^2 y_m(\theta) - k^2 y_k(\theta), \tag{24}$$

$$\hat{y}_4^{(4)}(\theta) = y_1(\theta) + n^4 y_n(\theta) + m^4 y_m(\theta) + k^4 y_k(\theta), \tag{25}$$

$$\hat{y}_4^{(6)}(\theta) = -y_1(\theta) - n^6 y_n(\theta) - m^6 y_m(\theta) - k^6 y_k(\theta), \tag{26}$$

$$\hat{y}_4^{(8)}(\theta) = y_1(\theta) + n^8 y_n(\theta) + m^8 y_m(\theta) + k^8 y_k(\theta). \tag{27}$$

Then, we have

$$\begin{aligned}y_1(\theta) = &-[n^2m^2k^2\hat{y}_4^{(2)}(\theta) + (n^2m^2 + m^2k^2 + k^2n^2)\hat{y}_4^{(4)}(\theta)\\ &+ (n^2 + m^2 + k^2)\hat{y}_4^{(6)}(\theta) + \hat{y}_4^{(8)}(\theta)]\\ &/(n^2-1)(m^2-1)(k^2-1),\end{aligned} \tag{28}$$

$$\begin{aligned}y_n(\theta) = &[m^2k^2\hat{y}_4^{(2)}(\theta) + (m^2k^2 + m^2 + k^2)\hat{y}_4^{(4)}(\theta)\\ &+ (m^2 + k^2 + 1)\hat{y}_4^{(6)}(\theta) + \hat{y}_4^{(8)}(\theta)]\\ &/n^2(n^2-1)(n^2-m^2)(n^2-k^2),\end{aligned} \tag{29}$$

$$\begin{aligned}y_m(\theta) = &[k^2n^2\hat{y}_4^{(2)}(\theta) + (k^2n^2 + k^2 + n^2)\hat{y}_4^{(4)}(\theta)\\ &+ (k^2 + n^2 + 1)\hat{y}_4^{(6)}(\theta) + \hat{y}_4^{(8)}(\theta)]\\ &/m^2(m^2-1)(m^2-k^2)(m^2-n^2),\end{aligned} \tag{30}$$

$$\begin{aligned}y_k(\theta) = &[n^2m^2\hat{y}_4^{(2)}(\theta) + (n^2m^2 + n^2 + m^2)\hat{y}_4^{(4)}(\theta)\\ &+ (n^2 + m^2 + 1)\hat{y}_4^{(6)}(\theta) + \hat{y}_4^{(8)}(\theta)]\\ &/k^2(k^2-1)(k^2-n^2)(k^2-m^2).\end{aligned} \tag{31}$$

Thus, we obtain the following correction formula,

$$x_{\text{new}}(\theta-\phi) = g_1y_1(\theta) + g_ny_n(\theta) + g_my_m(\theta) + g_ky_k(\theta). \tag{32}$$

When the initial field is far from the final steady-state one, the field may fluctuate drastically at the initial stage of calculation; e.g. eddy current field in electromagnetic field. The similar fluctuation may occur immediately after the TDC or harmonic TDC correction. In these cases, the above operation of TDC or harmonic TDC may as well execute after the fluctuation cools down in order to get a larger correction effect.

3. Generalized simplified three-phase AC TP-EEC

In a third order symmetric system driven by a three-phase alternating current, the simplified three-phase AC TP-EEC is applied as a fast steady-state analysis method. The simplified three-phase AC TP-EEC proposed in Refs. [8, 9] is derived by the theory of EEC [10, 11]. In this section, the simplified three-phase AC TP-EEC is expanded to a general form.

We introduce the three-phase variables U_1, V_1, and W_1 which become U_2, V_2, and W_2, respectively, after one-sixth period. Then, we define dU, dV, and dW where $dU = U_2 - U_1$, $dV = V_2 - V_1$, $dW = W_2 - W_1$. In a phaser diagram drawing the behavior of the three-phase variables, we directly obtain the following correction formula,

$$U_{\text{new}} = -dW, \quad V_{\text{new}} = -dU, \quad W_{\text{new}} = -dV, \tag{33}$$

for Case A with $(6n+1)$-th order harmonic waves in normal phase and $(6n-1)$-th order harmonic waves in reversed phase,

$$U_{\text{new}} = -dV, \quad V_{\text{new}} = -dW, \quad W_{\text{new}} = -dU, \tag{34}$$

for Case B with $(6n-1)$-th order harmonic waves in normal phase and $(6n+1)$-th order harmonic waves in reversed phase, and

$$U_{\text{new}} = dU/2, \quad V_{\text{new}} = dV/2, \quad W_{\text{new}} = dW/2, \tag{35}$$

for Case C with $(6n+3)$-th order harmonic waves in normal or reversed phase. Using a condition equation satisfied in a steady state:

$$dU + dV + dW = 0, \tag{36}$$

the correction formula (33)–(35) are generalized to

$$\begin{aligned}U_{\text{new}} &= \alpha_1(dU + dV + dW) - dW,\\ V_{\text{new}} &= \alpha_2(dU + dV + dW) - dU,\\ W_{\text{new}} &= \alpha_3(dU + dV + dW) - dV,\end{aligned} \tag{37}$$

for Case A,

$$\begin{aligned}U_{\text{new}} &= \beta_1(dU + dV + dW) - dV,\\ V_{\text{new}} &= \beta_2(dU + dV + dW) - dW,\\ W_{\text{new}} &= \beta_3(dU + dV + dW) - dU,\end{aligned} \tag{38}$$

for Case B, and

$$\begin{aligned}U_{\text{new}} &= \gamma_1(dU + dV + dW) + dU/2,\\ V_{\text{new}} &= \gamma_2(dU + dV + dW) + dV/2,\\ W_{\text{new}} &= \gamma_3(dU + dV + dW) + dW/2,\end{aligned} \tag{39}$$

for Case C. A condition that $U + V + W = 0$, satisfied in the steady state, can not be introduced into the correction formula, because of including a slow decay term contrary to the condition that $dU + dV + dW = 0$. Case A is the most important to which the basic wave in normal phase belongs. For Case A, we set $\alpha_1 = \alpha_2 = \alpha_3 = 1/2$ to get the correction formula

$$\begin{aligned}U_{\text{new}} &= (dU + dV - dW)/2,\\ V_{\text{new}} &= (dV + dW - dU)/2,\\ W_{\text{new}} &= (dW + dU - dV)/2,\end{aligned} \tag{40}$$

which can be changed to the equations proposed in Refs. [7, 8] using the variable $Z = -W$.

We have no correction formula effective for all Cases A, B, and C. When using Eq. (37) for Case A, the harmonics in Cases B and C are not effectively corrected to the time-harmonic steady-state fields, and therefore the correction of harmonics in Case A is disturbed by the harmonics in Cases B and C. The zero phase harmonics are also not effectively corrected by Eq. (37) and slightly disturb the correction of the harmonics in Case A.

4. Numerical simulation of 2-variable models

The correction effect of TDC/harmonic TDC can be validated in simple 2-variable models written in the following simultaneous equations,

$$\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \tau\begin{pmatrix} dx/d\theta \\ dy/d\theta \end{pmatrix} = \begin{pmatrix} 0 \\ \sum_{\text{odd } k}\alpha_k \sin k\theta \end{pmatrix}. \tag{41}$$

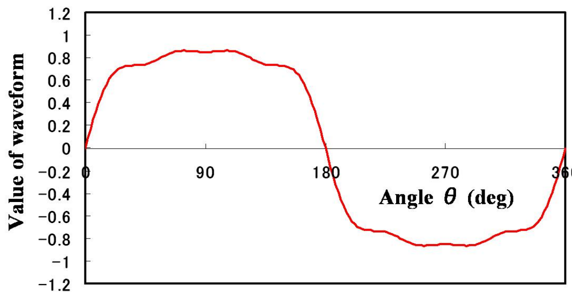


Figure 1: Waveform of source term in Case II.

The theoretical solution is written as follows

$$x_p = \sum_k \alpha_k x_k, \qquad y_p = \sum_k \alpha_k y_k, \tag{42}$$

where

$$\begin{aligned} x_k &= a_{1k} \sin k\theta + b_{1k} \cos k\theta, \\ y_k &= a_{2k} \sin k\theta + b_{2k} \cos k\theta, \\ a_{1k} &= \frac{3 - g_k^2}{D_k}, \qquad b_{1k} = -\frac{4g_k}{D_k}, \\ a_{2k} &= \frac{2(3 + g_k^2)}{D_k}, \qquad b_{2k} = -\frac{g_k(5 + g_k^2)}{D_k}, \\ D_k &= (1 + g_k^2)(9 + g_k^2), \qquad g_k = k\tau. \end{aligned}$$

As the test model, we set that $\tau = 10$ with the initial condition that $x = y = 1$, and two cases will be shown in the following. One case (Case I) includes no harmonic wave; $\alpha_1 = 1$, $\alpha_k = 0$ $(k > 1)$, while the other case (Case II) includes harmonic waves; $\alpha_1 = 1$, $\alpha_3 = 0.1$, $\alpha_5 = -0.02$, $\alpha_7 = 0.01$, $\alpha_k = 0$ $(k > 7)$ as shown in Fig. 1.

In Case I, the time-averaging operation is not required with only basic wave component, and so 3-step calculation gives 2nd time differential coefficients for TDC. In Case II, time-averaging operation should be executed beside 3-harmonic TDC. The main order of harmonic waves is 3 for TDC, 5 for 1-harmonic TDC, and 7 for 2-harmonic TDC. The number of time-divisions in one period is 96 for Case I, and 360 for Case II to accurately calculate the seventh order harmonic wave. The number of time steps for time-averaging operation are 120 (= 360/3), 72 (= 360/5) and 51 (= 360/7) for TDC, 1-harmonic TDC, and 2-harmonic TDC, respectively, because the orders of main harmful harmonic wave are 3, 5, and 7 for TDC, 1-harmonic TDC, and 2-harmonic TDC, respectively. Time-averaging operation is not required for 3-harmonic TDC.

The results of calculation are shown in Figs. 2–5 with Figs. 2 and 3 for Case I, and with Figs. 4 and 5 for Case II. In Figs. 3–5, the error δ indicates the difference from the theoretical solution as mentioned above, which is defined as

$$\delta = \sqrt{(x - x_p)^2 + (y - y_p)^2} \tag{43}$$

The number of time-steps indicates a time itself in Fig. 2, while the number of steps indicate the total steps including

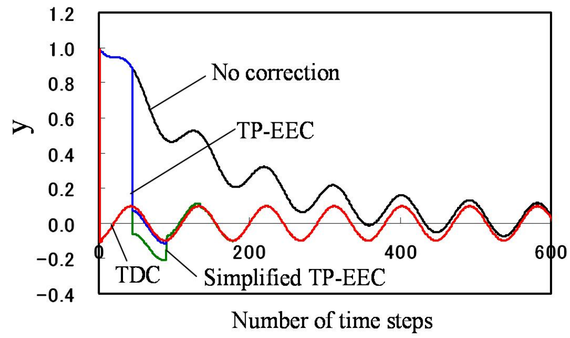


Figure 2: Waveform of y on the 2-variable model (Case I).

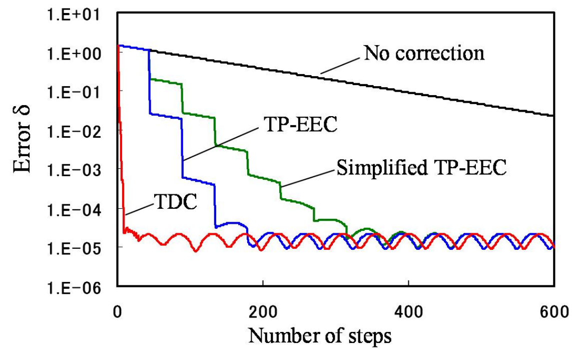


Figure 3: Comparizon of convergence among various correction methods on the 2-variable model. (Case I, Number of divisions per one period: 90)

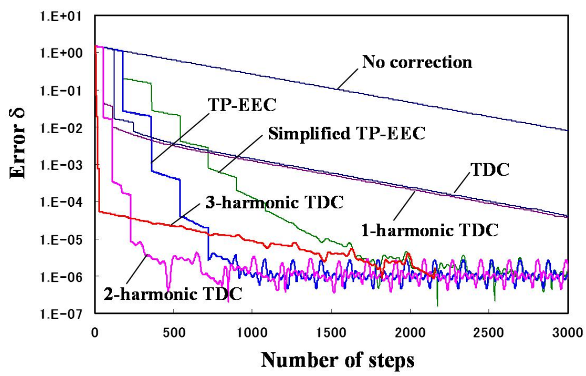


Figure 4: Comparison of convergence among various correction methods on the 2-variable model. (Case II, Number of divisions per one period: 360)

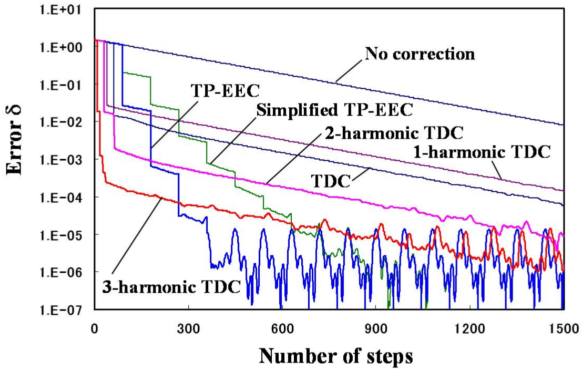


Figure 5: Comparison among various correction methods on the 2-variable model. (Case II, Number of divisions per one period: 180)

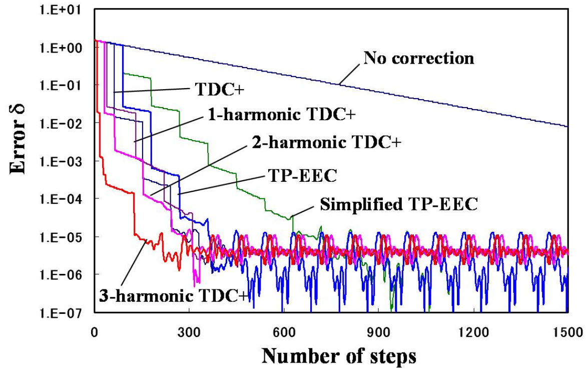


Figure 6: Convergence in the serial use of (harmonic) TDC and TP-EEC. (Number of divisions per one period: 180)

the number of stepping back corresponding to the electric angle width $\phi + p\Delta\theta$ in TDC and harmonic TDC in Figs. 3–5.

In Case I, the number of corrections for all the correction methods is 10. Figures 2 and 3 indicate that TDC has the best performance of correction in the case of no harmonic wave.

On the other hand, in Case II, 2-harmonic and 3-harmonic TDC largely surpass TDC as shown in Fig. 4, because harmonic waves depress the performance of TDC. The TDC and 1-harmonic TDC have weak power of corrections, and negative effects after the second correction so that the number of corrections is 2 for TDC and 1-harmonic TDC. The TP-EEC and simplified TP-EEC allow successive correction in the presence or absence of harmonic waves, and so surpass TDC and 1-harmonic TDC after about 400 steps.

Since the harmonic TDC uses higher order time derivatives, the performance of correction depend on the number of time-divisions. Figure 5 shows the result with the number of time-divisions in one period 180. The number of time-divisions in one period is about 26 (= 180/7) for 7th order harmonic wave which can be almost exactly described in the simulation.

The number of corrections is 8 for TP-EEC and simplified TP-EEC, while 1 for TDC and 1-harmonic TDC, 2 for 2-harmonic TDC, and 4 for 3-harmonic TDC. In TDC and harmonic TDC, the corrections over the above values have negative effects.

As shown in Fig. 5, 3-harmonic TDC has a maximum performance of correction. However, TP-EEC and simplified TP-EEC surpass 3-harmonic TDC under the error level of 1×10^{-4} since the 3-harmonic TDC does not allow successive corrections.

The initial performances of TDC and harmonic TDC are superior to the TP-EEC and simplified TP-EEC. However, it is a problem that TDC and harmonic TDC do not allow a large number of corrections due to harmful effects by harmonic wave components.

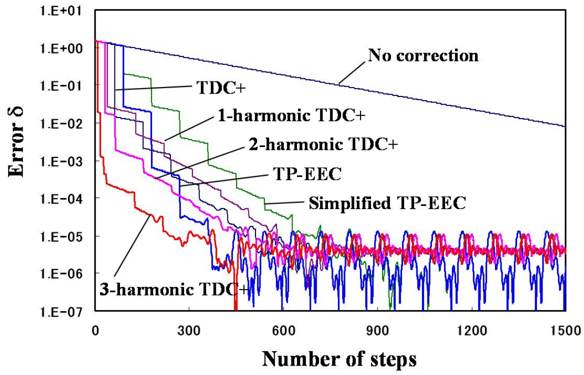
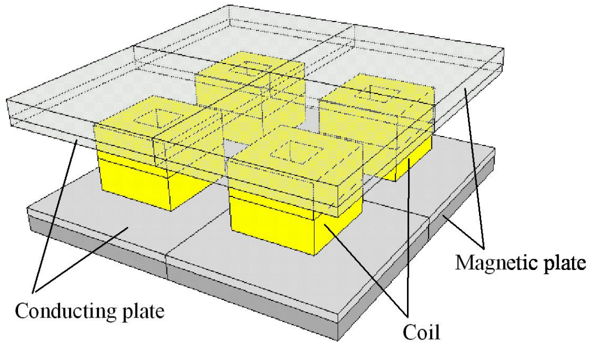


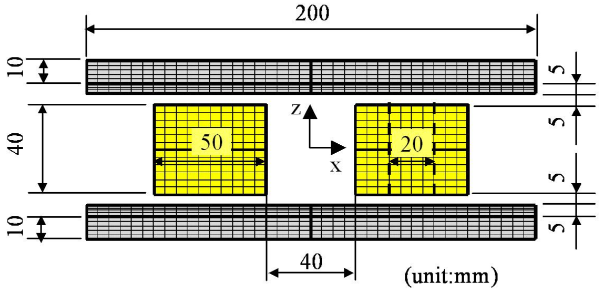
Figure 7: Convergence in the serial use of (harmonic) TDC and simplified TP-EEC. (Number of divisions per one period: 180)

5. Serial usage of TDC and TP-EEC

When neither TDC nor harmonic TDC can provide a highly precise steady-state solution due to time-harmonics, the following serial use of TDC and TP-EEC or simplified TP-EEC is very useful. When TDC or harmonic TDC is used for initial one or two corrections with subsequent use of TP-EEC or simplified TP-EEC, we can get the best performance of both the correction methods. Figure 6 shows the calculation result of the serial usage of TDC (harmonic TDC) and TP-EEC, while Fig. 7 shows one of the serial usage of TDC (harmonic TDC) and simplified TP-EEC. Here TDC+ and n-harmonic TDC+ ($n = 1, 2, \text{and } 3$) indicate the serial usage with subsequent use of TP-EEC or simplified TP-EEC. The calculation results shown in Figs. 6 and 7 indicate that the serial usage of TDC (harmonic TDC) and (simplified) TP-EEC is very powerful compared with the single usage of each correction method.



(a) Bird's eye view



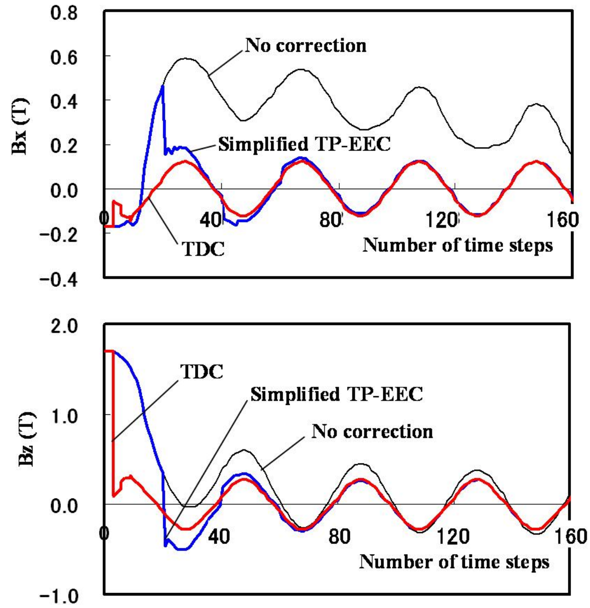
(b) Side view and model size

Figure 8: Static apparatus model. (Number of elements: 41,650, Number of nodes: 45,360; The mesh of air region is not visualized.)

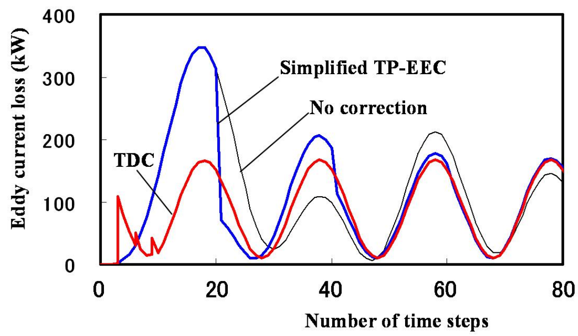
6. Numerical simulation of eddy current field of static apparatus

The model in Eq. (41) is a very simple model of 2 variables. Next, the performance of TDC and harmonic TDC should be validated in the system of a very large number of variables. We have applied the correction methods to a nonlinear magnetic field analysis by the finite element method. This section presents a numerical simulation of static apparatus, and Section 7 presents a numerical simulation of synchronous motor coupled with an electric circuit energized by an external voltage.

An apparatus model is shown in Fig. 8. Two plates stacked with a conducting plate and a magnetic material one are arranged at the top and bottom apart from four rectangular parallelepiped coils with rectangular holes. Each part of the four lateral sides is all in the same size. The conductivity of the conducting plate is 3.6×10^7 S/m. The basic wave current is $100 \cos(2\pi f t)$ kAT with the frequency f of 200 Hz with no higher time-harmonics, and the all coils have the same currents in the same direction. The magnetic plate has the same initial magnetization curve as the magnetic steel sheet 35A300. Only one-eighth of the entire analysis region was analyzed for the symmetry of the analyzed model. The number of elements is 41,650 where all the elements are hexahedrons for highly accurate calculation.



(a) Magnetic flux density



(b) Eddy current loss

Figure 9: Comparison among TDC and simplified TP-EEC methods in eddy current analysis of the static apparatus model with no higher time-harmonics in coil currents.

The number of time-divisions over one period of the basic wave is 40 to calculate the basic wave in high accuracy (time width: $125\,\mu$s).

The correction results by the simplified TP-EEC and TDC are displayed in Fig. 9 with respect of the x and y directional components of magnetic field at the point of $x = y = 42$ mm and $z = 35$ mm and the eddy current loss in the conducting plates. The number of corrections is 3 both for TDC and simplified TP-EEC. The time-averaging process is not required because of no higher time-harmonics.

As shown in Fig. 9, a steady state field is obtained at about a 80th time-step for simplified TP-EEC, while 14th time-step for TDC in consideration of stepping back in time by the second time derivative operation.

Next, the case including higher time-harmonics in the coil currents will be shown here. The wave form of coil current is the same as shown in Fig. 1. The number of

Table 1: Parameters of the synchronous motor.

Diameter of rotor	54.8 mm
Inner diameter of stator	56.0 mm
Outer diameter of stator	103 mm
Minimum air gap length	0.6 mm
Motor length	55 mm
Remanent magnetization of neodymium magnet	1.315 T

time-divisions over one period of the basic wave is 360 to calculate the harmonic waves up to the seventh-order in high accuracy (time width: 13.89 μs). The correction results by the simplified TP-EEC, TDC and harmonic TDC are displayed in Fig. 10 with respect of the x and y directional components of magnetic field at the same point as Fig. 9 and the eddy current loss in the conducting plates. The number of corrections is 4 only for the simplified TP-EEC and 2 for the others. The number of time-averaging steps is 120 for TDC, 72 for 1-harmonic TDC, and 51 for 2-harmonic TDC. The correction by 3-harmonic TDC has failed due to a large error for the 8th order time derivative.

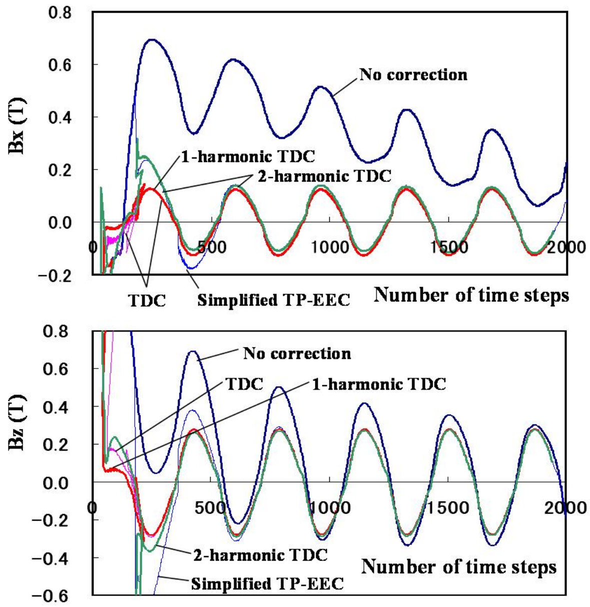
As shown in Fig. 10, a steady state field is obtained at about a 200th time step for TDC and 1-harmonic TDC. The number of time steps required for time average process of 1-harmonic TDC is smaller than one of TDC by 45 steps. The 2-harmonic TDC is relatively less precise presumably because the 6th order time derivative can not be obtained precisely.

7. Numerical simulation of synchronous motor coupled with an electric circuit

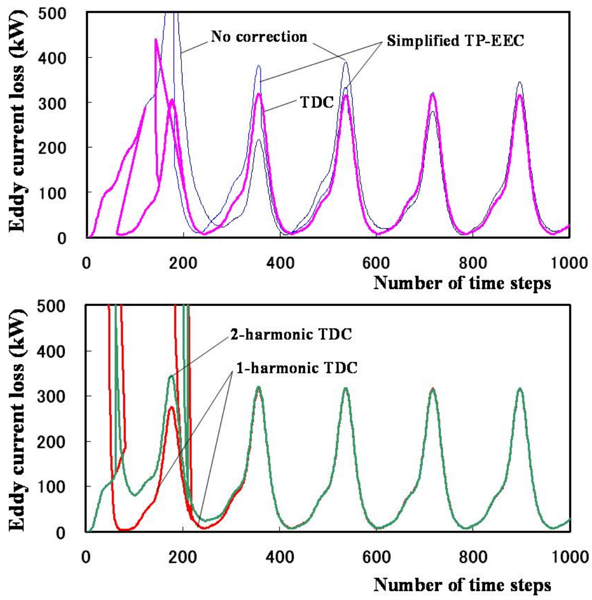
The TDC and harmonic TDC are applied to a numerical simulation of magnetic field of a permanent magnet synchronous motor coupled with an electric circuit to verify the performances by comparing with simplified TP-EEC. The discretized model of the synchronous motor is shown in Fig. 11, where the number of elements is 8,682. The parameters of the motor are listed in Table 1. The motor with 4 poles and 6 slots has a 2-electric-period structure, and therefore a half model was analyzed in one electric period. The slide surface between the rotor and the stator is divided by 180 in the circumferential direction. In the rotor moving simulation, the rotor is stepwisely rotated with the step width the minimum mesh size, and so the number of time-divisions in one electric period is 180.

The stator coils are connected to a three-phase star connection through electric resistances of 0.2 ohm including coil resistances.

The motor model in a three-phase AC system has a third order symmetric configuration so that the simplified three-phase AC TP-EEC can be applied in order to obtain a fast



(a) Magnetic flux density



(b) Eddy current loss

Figure 10: Comparison among TDC and simplified TP-EEC methods in eddy current analysis of the static apparatus model with higher time-harmonics in coil currents.

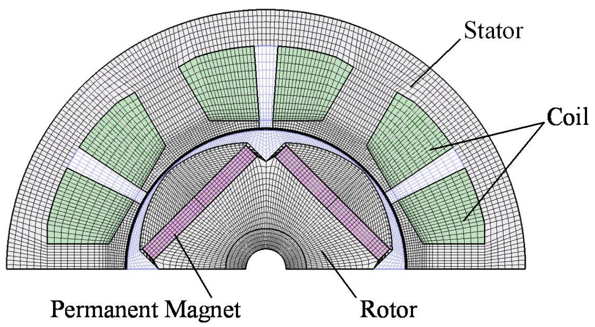


Figure 11: 2D finite element model of a synchronous motor. (Number of elements: 8682)

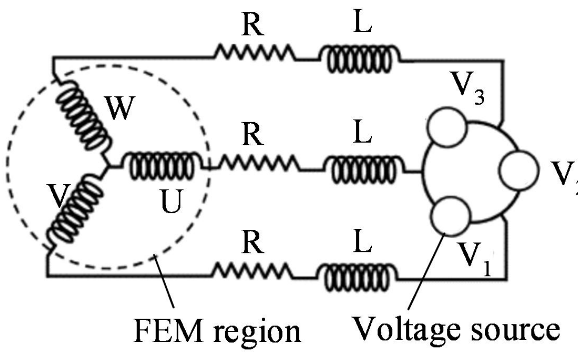


Figure 12: External electric circuit connected with coils embedded in the FEM region.

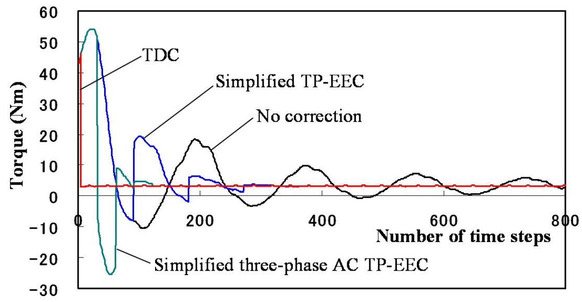


Figure 13: Comparison among TDC, simplified TP-EEC and simplified three-phase TP-EEC methods in torque analysis of the synchronous motor with no higher time-harmonics of external applied voltage.

converged steady state solution.

Now, the three-phase external applied voltage has only a basic wave component with no higher time-harmonics. The RMS value of external applied voltage is 200 V. The torque analysis results by TDC, simplified TP-EEC and simplified three-phase TP-EEC are shown in Fig. 13. In the case, time averaging operation is not required for TDC because of very few harmonics in the magnetic field. The number of corrections is 2 for TDC, while 4 both for simplified TP-EEC and simplified three-phase TP-EEC. For obtaining the steady state, simplified TP-EEC and simplified three-phase AC TP-EEC require about 380 time-steps and 130 time-steps, respectively, while TDC requires only about 15 time-steps in consideration of stepping back in time by the second time derivative operation. In the standard motor analysis, higher time-harmonics are not included in the external applied voltage with the exception of inverter driven motors, and so TDC is a very powerful method obtaining the steady-state performance of synchronous motors.

Next, the case including higher time-harmonics in the external applied voltage will be shown here. The three-phase external applied voltage has 7th, 9th, and 11th order harmonics, the relative amplitudes of which are 0.1, 0.04, -0.05 with a unit amplitude of basic wave. All the waves have the same phase at the initial time. The amplitudes of harmonics are intentionally large in order to clearly validate harmonic TDC and the serial usage of (harmonic) TDC and simplified TP-EEC.

The results by TDC, 1-harmonic TDC, and 3-harmonic TDC are shown in Fig. 14 as poorly effective correction methods with the number of corrections are all 3 and with the number of time-steps for averaging 26 (nearly equals to 180/7) for TDC depressing the 7th order harmonic and 16 (nearly equals to 180/11) for the others depressing the 11th order harmonic. The steady state solution can not be obtained by these corrections until 800 time steps.

Next, the results by 2-harmonic TDC, simplified TP-EEC, and simplified three-phase AC TP-EEC are shown in Fig. 15 in a close-up scale as relatively effective correction methods in this case with the number of corrections are

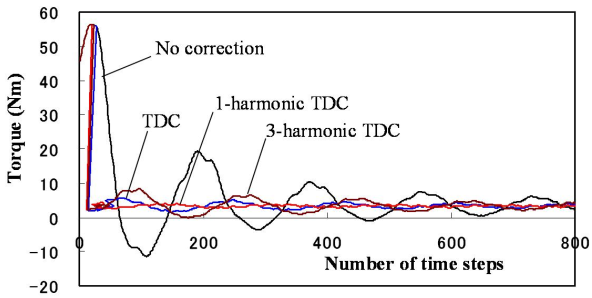


Figure 14: Comparison among TDC and 1, 3-harmonic TDC methods in torque analysis of the synchronous motor.

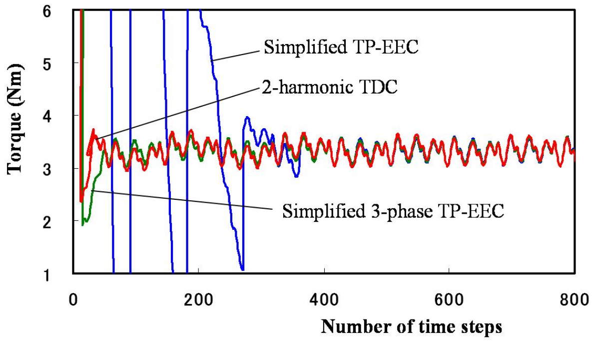


Figure 15: Comparison among 2-harmonic TDC, simplified TP-EEC, and simplified three-phase TP-EEC in torque analysis of the synchronous motor.

3, 6, and 4, respectively. The simplified three-phase AC TP-EEC gave the same results for $\alpha = 0$, $1/2$, and 1 in Eq. (37) with $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$. Figure 15 indicates that the steady state can be obtained after 400th time-step in all the cases using the three correction methods.

The 2-harmonic TDC and simplified three-phase AC TP-EEC work effectively as methods for fast obtaining an approximate steady state solution. The successive simplified TP-EEC can provide a precious steady state solution, while the simplified three-phase AC TP-EEC can not provide precious one due to time-harmonic waves in Cases B and C described in Section 3.

Finally, the results by the serial use of (harmonic) TDC and simplified TP-EEC are shown in Fig. 16 where the serial use is indicated as "TDC+" or "(n-harmonic) TDC +". The serial use can fast provide a precious steady state solution. In the serial use, the simplified TP-EEC is executed 2 times after one time of (harmonic) TDC. The four torque curves obtained by TDC and 1, 2, and 3-harmonic TDCs are in good agreement with each other after 200th time step showing a precious steady state periodic curve with a 180-time-step period. These curves are in good agreement with the steady state curve obtained by simplified TP-EEC after 400th time-step shown in Fig. 16. The result indicates that the serial usage of (harmonic) TDC and simplified TP-

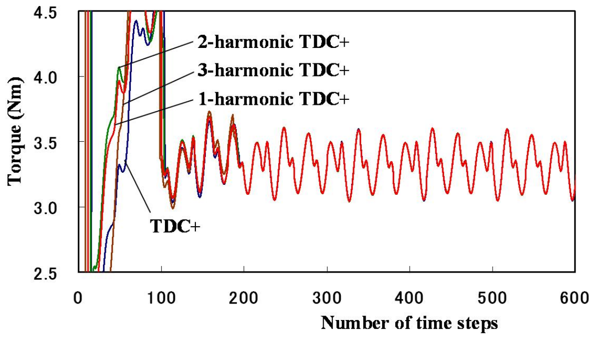


Figure 16: Comparison among serial uses of (harmonic) TDC and simplified TP-EEC methods in torque analysis of the synchronous motor.

EEC can provide a steady solution faster than simplified TP-EEC by a factor of 2 in this case.

8. Conclusions

As a fast analysis method to obtain a steady state solution of nonlinear magnetic field, we have proposed TDC and harmonic TDC methods using the second and higher order time-derivatives of the time-averaged variables having a time-derivative term in the transient analysis. The computational cost of TDC is very low like simplified TP-EEC. In the case of no higher order time-harmonic waves in the driving source term, the correction effect of TDC is very large surpassing the conventional methods. In the case that time-harmonic waves are included in the driving source term, the harmonic TDC and the serial usage of (harmonic) TDC and (simplified) TP-EEC have a profound effect to provide a steady-state field. In addition, the simplified three-phase AC TP-EEC has been expanded to a general form for the three-phase AC system where the three-phase AC field is corrected every one sixth period toward the steady state field.

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