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Kinari, Yusuke
Faculty of Economics, Kyushu University

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Yusuke Kinari

Abstract

Theories of asset allocation predict that higher forecasts of returns on risky assets lead to a higher share of risky assets to wealth. This study tests this prediction on household asset allocation by examining the relationship between the share of risky assets held by households and their forecasts of returns on risky assets by using a household survey that asks respondents their highest and lowest forecasts for the Nikkei 225 index. Constructing the median forecasts from these two forecasts, the share of risky assets to wealth is regressed on the three forecast variables: the highest, median and lowest. It was found that the share of risky assets to wealth is positively but weakly correlated with the forecast variables as the theories predict. Moreover, the lowest forecast has a greater effect on the share than the other two forecasts. These results indicate that households determine the amount they hold in risky assets based on the lowest forecast rather than the highest or median forecasts of returns on risky assets.

JEL Classification: D84; G11

Keywords: Stock Price Forecast; Share of Risky Assets; Household Survey

1. Introduction

Classical theories of asset allocation, such as those of Merton (1969) and Samuelson (1969), assign importance to forecasts of returns on risky assets. These theories reveal how household shares of risky assets to wealth depend on their forecasts of returns on risky assets. The classical prediction confirms expected household behavior: households with higher forecasts of returns on risky assets hold more risky assets than those with lower forecasts of returns on risky assets. Therefore, the share of risky assets to wealth is higher for households with higher forecasts than for those with lower forecasts. This prediction is also supported by the extended asset allocation model of Friend and Blume (1975), which incorporates the tax rate and human capital into the classical asset allocation models.

Logically, higher forecasts of returns on risky assets would lead to higher shares of risky assets to wealth. However, there is no empirical research on the relationship between the forecasts of returns on risky assets and the share of risky assets to wealth. Therefore, it is not clear whether the forecasts are positively correlated with household shares of risky assets to wealth or how the forecasts affect the shares of risky assets to wealth.

This study analyzes the prediction by using a household survey asking respondents their highest and lowest forecasts for the Nikkei 225 index. Constructing the median forecast variable from the highest and lowest forecasts, the shares of risky assets to wealth are regressed on the highest, median, and lowest forecasts for the Nikkei 225 index by applying the bivariate sample selection model. If the classical theory holds, then the coefficients of these forecast variables are positive because higher forecasts lead to higher shares of risky assets to wealth. In addition, this study examines how the share and ownership of risky assets are affected by the highest, median, and lowest forecasts.

The results indicate that there is a positive but weak correlation between the share of risky assets to wealth and forecast variables as predicted by the classical theories. Moreover, the lowest forecast has a greater effect on the share of risky assets to wealth than the highest and median forecasts. These results indicate that households determine the amount they hold in risky assets based on the lowest forecast rather than the highest or median forecasts of returns on risky assets.

The rest of the paper is organized as follows. Section 2 briefly introduces the classical theories of asset allocation and their predictions. Section 3 explains the data and method of analysis. Section 4 reports the estimation results. Section 5 discusses why households determine their shares of risky assets to wealth based on the lowest forecast. Finally, section 6 summarizes the findings of this study.

2. Classical Theory and Its Prediction

According to classical theories of asset allocation, such as those of Merton (1969) and Samuelson (1969), assuming that the market is perfect, that there is no labor income, and that households have a constant relative risk aversion utility function, then the share of risky assets to wealth is determined by relative risk tolerance, forecasts of returns on risky assets, the variance of returns on risky assets, and returns on safe assets as follows:

$$W_j^* = \theta_j \times \frac{E_J[R_r] - R_f}{\sigma_{r,j}^2}. \tag{1}$$

In equation(1), θ_j is the relative risk tolerance of household j, $E_j[R_r]$ is the subjective expected value of returns on risky assets of household j, R_f is the returns on safe assets, and $\sigma_{r,j}^2$ is the subjective variance of the returns on risky assets of household j. W_j^* is the theoretically predicted value of the share of risky assets.

This theory's prediction confirms the expected household behavior: households with a higher forecast of returns on risky assets hold more risky assets than those with a lower forecast of returns on risky assets. Therefore, the share of risky assets to wealth is higher for households with a higher forecast than for those with a lower forecast. This prediction is also supported by the extended asset allocation model of Friend and Blume (1975), which incorporates the tax rate and human capital into the classical asset allocation model.

Logically, higher forecasts of returns on risky assets would lead to higher shares of risky assets to wealth. However, there is no empirical research on the relationship between the forecasts of returns on risky assets and the share of risky assets to wealth. Therefore, it is not clear whether the forecasts are positively correlated with household shares of risky assets to wealth or how the forecasts affect the share of risky assets to wealth. This study examines the above mentioned issues and the effect of the three forecasts on the share and ownership of the risky assets.

3. Data and Method

This study implements the results of the "Preference and Life Satisfaction Survey" conducted in Japan by Osaka University in February 2006. In this survey, the following questions about respondents' forecasts of the Nikkei 225 index were asked.

- At present, the stock price (Nikkei 225 index) is about 16000 yen. On April 1, 2006, between what yen range do you think the stock price will be? Write in a range of which you are confident.
  — The Lowest price ______ to the Highest price ______

This study denotes the answers as *Lowest* and *Highest*, respectively, and defines *Median* as the average of the two forecast variables.

The survey also asked about respondents' shares of risky asset to financial wealth.

- What percentage of your financial assets of your entire household are in the investment trusts, stocks, futures/options, corporate bonds, foreign currency deposits, and government bonds of foreign countries?

Using the answer to this question, which is denoted by α, the share of risky assets to wealth is defined as

$$Share \equiv \frac{\alpha \times financial\ \ wealth}{financial\ \ wealth + real\ \ wealth} \tag{2}$$

This study regards investment trusts, stocks, futures/options, corporate bonds, foreign currency deposits, and government bonds of foreign countries as the risky assets, and defines the share of risky assets as the ratio of the total amount of the risky assets to the total amount of financial and real wealth.

By regressing *Share* on the forecast variables, this paper tests whether the *Share* is positively correlated with the forecast variables. If the classical theory holds, the coefficients of these forecast variables are positive because the positive signs indicate that higher forecasts lead to higher shares of risky assets to wealth. Moreover, it investigates how Japanese household shares of risky assets to wealth and the ownership of the risky assets respond to *Lowest, Median,* and *Highest.*

In addition to the forecast variables, this model includes *Risk Tolerance, Variance, Optimist, Pessimist, Wealth, Age* and *Liability* in the regression. *Risk Tolerance* is calculated from the survey responses on gambles over lifetime income by applying a method by Barsky et al. (1997). Appendix briefly explains their estimation method. *Variance* is defined as the logarithm of the difference between *Highest* and *Lowest*; it captures the spread of the forecasts. *Optimist* and *Pessimist* are the dummy variables that take unity if the lowest (highest) forecast is over (under) the present price 16000 yen, and zero otherwise. *Wealth* is defined as financial wealth plus the real wealth of the respondent. *Age* is the respondent's age, and *Liability* is the dummy variable that takes unity if the respondent has a liability, including housing loans, and zero otherwise.

To deal with sample selection biases, this paper applies the bivariate sample selection model that models the decisions about ownership and the share of risky assets separately and allows unobserved determinants of the two decisions to be correlated[1]. The model requires some variables that may affect the ownership decision but not the decision about the share of risky assets. Following Sekita (2007), this model includes dummy variables *region* and *city-size* when making decisions about the ownership of risky assets.

4. Results

4.1 Distribution of Forecasts

Figures 1, 2, and 3 show the frequency ratios of *Median, Lowest,* and *Highest*, respectively. Horizontal axes represent the forecasts classified into categories (e.g., 15000 to 16000 yen). Vertical axes represent the ratios of the number of households in each category to the total number of households. All figures reveal that forecasts of majority of the households are distributed around the current stock price index, 16000 yen. About 70% of households expect the *Median, Lowest,* and *Highest* forecasts ranging from 15000 to 17000 yen.

The figures also show that some respondents report extremely high or low forecasts. Figure 2 shows that about 3% of households expect that the stock price index to be less than 8000 yen in the worst-case scenario, while Figure 3 shows that about 2% of households expect that the stock price index to be more than 24000 yen in the best-case scenario. The stock price index is not likely to increase (or decrease) by more than 50% in one month; therefore, these forecasts seem unrealistic and may lead to biases. In order

1 For details, see Cameron and Trivedi (2005).

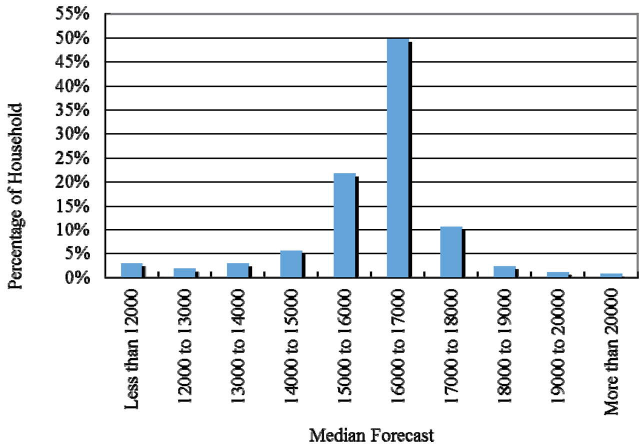

Figure 1: Distribution of Median Forecasts

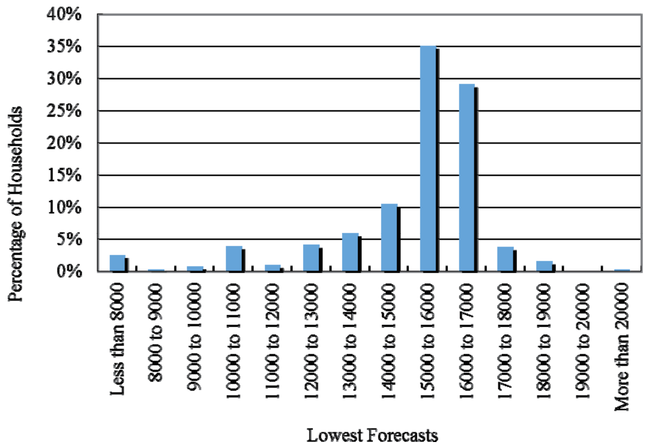

Figure 2: Distribution of Lowest Forecasts

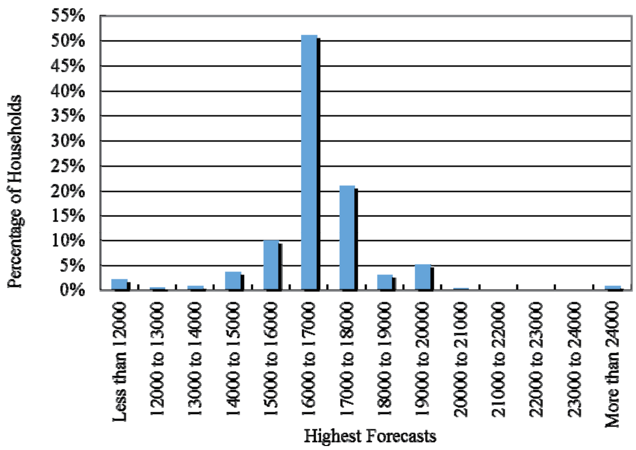

Figure 3: Distribution of Highest Forecasts

Figure 4: The Nikkei 225 Index from February to April 2006

to eliminate the biases, this paper excludes the samples whose *Highest* is more than 24000 yen and *Lowest* is less than 8000 yen from the regression. The number of observations thus excluded is 30 and 52, respectively[2].

4.2 Estimation Results

Table 1 presents the summary statistics of the variables used for the regression. It shows that the means of *Lowest, Median,* and *Highest* are approximately 15000, 16000, and 17000 yen, respectively, thus indicating that households on an average expect the stock price index on April 01, 2006 to range from 15000 to 17000 yen. Figure 4 shows the trends of the Nikkei 225 stock price index from February to April 2006. On April 03, 2006, the earliest trading day of the month, the Nikkei 225 is about 17333 yen, which exceeds the mean of *Highest*. This indicates that the stock price increased beyond the respondents' expectations.

Table 2 lists the estimation results from the bivariate sample selection model. Model 1, 2, and 3 include *Median, Lowest,* and *Highest* as the forecast variables, respectively, and Model 4 includes both *Lowest* and *Highest.*

In Model 1, the coefficient on *Median* is positively significant at the 10% level in the ownership equation and insignificant but positive in the share equation, indicating that the median forecast weakly affects the ownership of risky assets but does not affect the share of risky assets. In Model 2, the coefficients on *Lowest* are positively significant at the 1% level in the share equation and at the 10% level in the ownership equation. In Model 3, the coefficients on *Highest* are insignificant in both ownership and share equations, indicating that the highest forecast has no effect on either the ownership of the risky assets or the share

2 The results of this study do not change very much when these samples are included in the regression.

of risky assets. In Model 4, *Lowest* has a significantly positive coefficient in the share equation, and *Highest* has insignificant coefficients in both equations.

These results indicate that the share of risky assets to wealth is positively but weakly correlated with the forecasts of returns on risky assets as predicted by the classical theories because many of the coefficients on the forecast variables are positive and some of them are significant (Model 2 and 4). Moreover, the lowest forecast has a greater effect on the share of risky assets to wealth than the highest

Table 1: Summary Statistics

	Observations	Mean	S.D.	Minimum	Maximum
Share	1752	4.704	11.111	0	95
Lowest	2314	14905.660	1784.524	8000	20000
Median	2314	15975.620	1222.299	9000	20500
Highest	2314	17045.580	1329.802	10000	24000
Pessimist	2314	0.074	0.262	0	1
Optimist	2314	0.158	0.365	0	1
Risk Tolerance	2001	0.356	0.632	0.068	2.862
Variance	2314	7.265	0.970	0.000	9.547
Wealth	1903	38.521	47.420	1.301	339.768
Age	2314	49.302	12.917	20	73
Liability	2257	0.490	0.500	0	1
Region 1	2314	0.040	0.195	0	1
Region 2	2314	0.057	0.231	0	1
Region 3	2314	0.344	0.475	0	1
Region 4	2314	0.039	0.194	0	1
Region 5	2314	0.037	0.188	0	1
Region 6	2314	0.118	0.323	0	1
Region 7	2314	0.168	0.374	0	1
Region 8	2314	0.056	0.229	0	1
Region 9	2314	0.028	0.165	0	1
Region 10	2314	0.113	0.317	0	1
City-Size 1	2314	0.257	0.437	0	1
City-Size 2	2314	0.442	0.497	0	1
City-Size 3	2314	0.223	0.416	0	1
City-Size 4	2314	0.078	0.269	0	1

Notes: This table shows number of observations, mean, standard deviation, minimum, and maximum of variables. The value of *Wealth* is represented by yen in millions.

Table 2: Estimation Results from Bivariate Sample Selection Model

	Model 1				Model 2				Model 3				Model 4			
	Share		Ownership		Share		Ownership		Share		Ownership		Share		Ownership	
	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.
Constant	−7.64	6.59	−0.57	1.23	−10.58	6.49	−0.51	1.22	−0.44	5.56	0.24	1.02	−9.25	6.64	−0.62	1.25
Lowest $\times 10^{-3}$					0.53 **	0.24	0.07 *	0.04					0.65 **	0.27	0.06	0.05
Median $\times 10^{-3}$	0.50	0.31	0.10 *	0.05												
Highest $\times 10^{-3}$									0.07	0.31	0.07	0.05	−0.33	0.35	0.04	0.06
Pessimist	1.79	1.27	0.07	0.22	1.80	1.16	−0.01	0.19	0.74	1.30	0.02	0.23	1.23	1.31	0.06	0.23
Optimist	0.08	0.90	0.10	0.18	0.11	0.86	0.14	0.17	0.56	0.91	0.11	0.18	0.39	0.91	0.11	0.18
Risk Tolerance	0.07	0.43	0.01	0.08	0.04	0.43	0.01	0.08	0.13	0.43	0.02	0.08	0.04	0.43	0.01	0.08
Variance	−0.49	0.30	−0.09	0.06	−0.04	0.37	−0.02	0.07	−0.59	0.38	−0.17 **	0.07	0.32	0.53	−0.07	0.11
Wealth	0.04 ***	0.01	0.01 **	0.00	0.04 ***	0.01	0.01 **	0.00	0.04 ***	0.01	0.01 **	0.00	0.04 ***	0.01	0.01 **	0.00
Wealth^2	−0.02	0.05	−0.01	0.01	−0.02	0.05	−0.01	0.01	−0.02	0.05	−0.01	0.01	−0.02	0.05	−0.01	0.01
Age	0.32 **	0.16	0.05 *	0.03	0.31 *	0.16	0.05 *	0.03	0.33 **	0.16	0.05 *	0.03	0.31 *	0.16	0.05 *	0.03
Age^2	−2.73	1.71	−0.63 **	0.29	−2.66	1.71	−0.63 **	0.29	−2.85 *	1.71	−0.65 **	0.29	−2.65	1.71	−0.63 **	0.29
Liability	−4.93 ***	0.58	0.17	0.11	−4.94 ***	0.58	0.17	0.11	−4.92 ***	0.58	0.17	0.11	−4.94 ***	0.58	0.17	0.11
Region			yes				yes				yes				yes	
City-Size			yes				yes				yes				yes	
ρ	−0.06	0.12			−0.06	0.12			−0.06	0.11			−0.06	0.12		
σ	10.52 ***	0.19			10.51 ***	0.19			10.53 ***	0.19			10.51 ***	0.19		
λ	−0.60	1.23			−0.58	1.26			−0.60	1.20			−0.58	1.25		
Log likelihood		−6120.71				−6119.56				−6122.71				−6118.96		

Notes: Out of 1633 observations, 115 observations are censored. ***, **, and * indicate that the values are statistically significant at the 1%, 5%, and 10% levels, respectively.

Table 3: Estimation Results from Bivariate Sample Selection Model (Restricted Sample)

	Model 1				Model 2				Model 3				Model 4			
	Share		Ownership		Share		Ownership		Share		Ownership		Share		Ownership	
	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.
Constant	−4.165	8.014	−1.045	1.535	−9.400	7.826	−1.069	1.460	4.385	6.274	0.379	1.197	−6.690	8.101	−1.189	1.530
Lowest $\times 10^{-3}$					0.516 *	0.280	0.092 *	0.048					0.647 **	0.298	0.087 *	0.052
Median $\times 10^{-3}$	0.371	0.385	0.121 *	0.068												
Highest $\times 10^{-3}$									−0.207	0.363	0.065	0.067	−0.496	0.385	0.020	0.069
Risk Tolerance	−0.034	0.494	0.092	0.103	−0.075	0.494	0.093	0.104	0.013	0.493	0.099	0.103	−0.072	0.493	0.092	0.104
Variance	−0.617 *	0.337	−0.029	0.065	−0.094	0.461	0.059	0.088	−0.538	0.426	−0.117	0.081	0.430	0.614	0.038	0.117
Wealth	0.037 **	0.016	0.005	0.003	0.037 **	0.016	0.005 *	0.003	0.037 **	0.016	0.005	0.003	0.037 **	0.016	0.005 *	0.003
Wealth^2	−0.026	0.059	−0.012	0.012	−0.026	0.059	−0.012	0.012	−0.025	0.059	−0.011	0.012	−0.026	0.058	−0.012	0.012
Age	0.313 *	0.179	0.036	0.032	0.301 *	0.179	0.035	0.032	0.327 *	0.179	0.038	0.032	0.302 *	0.179	0.035	0.032
Age^2	−2.700	1.867	−0.477	0.328	−2.602	1.865	−0.464	0.328	−2.842	1.866	−0.496	0.327	−2.632	1.864	−0.466	0.328
Liability	−4.637 ***	0.634	0.126	0.118	−4.656 ***	0.633	0.126	0.118	−4.635 ***	0.634	0.127	0.118	−4.671 ***	0.633	0.126	0.118
Region			yes				yes				yes				yes	
City-Size			yes				yes				yes				yes	
ρ	−0.078	0.108			−0.078	0.110			−0.076	0.108			−0.079	0.109		
σ	10.213 ***	0.212			10.202 ***	0.212			10.216 ***	0.212			10.195 ***	0.212		
λ	−0.799	1.105			−0.797	1.122			−0.781	1.105			−0.804	1.110		
Log likelihood		−4703.725				−4702.250				−4705.202				−4701.393		

Notes: Out of 1265 observations, 92 observations are censored. ***, **, and * indicate that the values are statistically significant at the 1%, 5%, and 10% levels, respectively.

and median forecasts. This suggests that households determine how amount they hold in risky assets based on the forecast in the worst-case scenario rather than the best-case scenarios.

These results are constant when the optimistic and pessimistic samples are excluded. Table 3 presents the estimation results from the bivariate sample selection model that excludes *Optimist* and *Pessimist* from the sample; it reveals that the share of risky assets to wealth is positively but weakly correlated with the forecasts of returns on risky assets. *Lowest* also had a greater effect on *Share* than *Highest* and *Median.* The results are also unchanged when dummy variables of the borrowing constraint, self-employment, unemployment, respondent's job, respondent's health condition, respondent's educational level, housing, and respondent's income are involved in the regression as the independent variables. Thus, the estimation results of this study are fairly robust.

5. Discussion

The estimation results indicate that households determine the amount they hold in risky assets based on the forecast in the worst-case scenario. This section discusses the reason to do so.

There are two possible explanations. The first explanation concerns subjective probability. In Model 4, both *Highest* and *Lowest* are included as independent variables. The estimated coefficients on these variables can be interpreted as estimated subjective probabilities when household expectations of stock price index performance are represented as follows:

$$E[P_t] = \pi^L Lowest + \pi^H Highest, \tag{3}$$

where π^L is the probability of realizing the worst-case scenario and π^H is the probability of realizing the best-case scenario. In this case, the estimation results imply that households expect a higher probability of realizing the worst-case scenario than the best-case scenario. Therefore, they determine their shares of risky assets to wealth based on the forecast in the worst-case scenario.

The second explanation relates to loss aversion as proposed by Kahneman and Tversky (1979). Loss aversion refers to the tendency of individuals to be more sensitive to decrease in their levels of well being than to increase. Empirical estimates of loss aversion are typically in the neighborhood of 2, indicating that the disutility of giving something up is twice as great as the utility of acquiring it[3]. If households are loss averse, they may decide to allocate a lower share of their wealth to risky assets to avoid losses compared with the level predicted by their expectations of stock price performance. In this case, the share of risky assets may respond to the lowest forecasts rather than the highest or median forecasts. Probably the estimation results capture the effect of loss aversion on the share of risky assets.

3　For details, see Benartzi and Thaler (1995).

6. Summary

Classical theories of asset allocation predict that if households expect that future returns on risky assets are high (low), then they will be willing to hold more (less) risky assets. Consequently, their share of risky assets to wealth becomes higher (lower). Logically, higher forecasts of returns on risky assets lead to higher shares of risky assets to wealth. However, there is no empirical research on the relationship between the forecasts of returns on risky assets and the share of risky assets to wealth. Therefore, it is not clear that the forecast really affects household shares of risky assets to wealth or how the forecast affects the share of risky assets to wealth.

The study examined the classical prediction on asset allocation and revealed how Japanese household shares of risky assets to wealth respond to their forecasts of returns on risky assets by using the household survey, asking respondents' highest and lowest forecasts for the Nikkei 225 index. Constructing the median forecast variable from the highest and lowest forecasts, this study regressed the shares of risky assets to wealth on the highest, median, and lowest forecasts for the Nikkei 225 index using the bivariate sample selection model.

The results found that there is a positive but weak correlation between the share of risky assets to wealth and the forecast variables as predicted by classical theories. Moreover, the lowest forecast has a greater effect on the share of risky assets to wealth than the highest and median forecasts. These results indicate that households determine the amount they hold in risky assets based on the forecast of the worst-case scenario.

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Appendix: Estimating Relative Risk Tolerance

This appendix briefly explains Barsky et al. (1997)'s method of estimating relative risk tolerance in the survey responses on gambles over lifetime income[4]. Respondents are required to answer the following questions:

・Considering the following two ways of receiving your monthly income, which is preferable to you? Assume the job assignment is the same under these situations. If you are a dependent (e.g., student, housewife, etc.), answer this question taking your living expense as your monthly income.

1. Your monthly income has a 50% chance of doubling, but also has a 50% chance of decreasing by 30%.

2. Your monthly income is guaranteed to increase by 5%.

If the answer to this question is 1, the respondent is required to answer the next question with a higher downside risk.

1. Your monthly income has a 50% chance of doubling, but also has a 50% chance of the monthly income being cut in half.

2. Your monthly income is guaranteed to increase by 5%.

4 For details, see Barsky et al. (1997).

If the answer to the first question is 2, the respondent is required to answer the following question with a lower downside risk.

1. Your monthly income has a 50% chance of doubling, but also has a 50% chance of the monthly income decreasing by 10%.
2. Your monthly income is guaranteed to increase by 5%.

Depending on the responses to the two questions, the respondents are classified into four distinct risk preference categories from the most risk-tolerant category (both answers are 1) to the most risk-averse category (both answers are 2).

Assuming that respondents maximize their expected utility and they choose the 50-50 gamble to double lifetime income as opposed to having it fall by the fraction $1-\pi$ if

$$0.5U(2W)+0.5U((1-\pi)W)\geq U(1.05W). \tag{4}$$

W is original lifetime income, and $U(\cdot)$ is the constant relative risk aversion utility function,

$$U(W)=\frac{W^{1-1/\theta}}{1-1/\theta} \tag{5}$$

where θ is the coefficient of relative risk tolerance. Under these assumptions, the upper and lower bounds of relative risk tolerance of the four categories can be calculated. For example, consider the group that accepts the gamble when π is 0.3 and rejects when π is 0.5. In this case, the lower bound $\underline{\theta}$ is the value for satisfying equation (6) and the upper bound $\overline{\theta}$ is the value for satisfying equation (7).

$$0.5\frac{2W^{1-1/\underline{\theta}}}{1-1/\underline{\theta}}+0.5\frac{0.7W^{1-1/\underline{\theta}}}{1-1/\underline{\theta}}=\frac{1.05W^{1-1/\underline{\theta}}}{1-1/\underline{\theta}} \tag{6}$$

$$0.5\frac{2W^{1-1/\overline{\theta}}}{1-1/\overline{\theta}}+0.5\frac{0.5W^{1-1/\overline{\theta}}}{1-1/\overline{\theta}}=\frac{1.05W^{1-1/\overline{\theta}}}{1-1/\overline{\theta}} \tag{7}$$

Furthermore, assuming that true risk tolerance follows a log-normal distribution,

$$\log\theta\equiv x\sim N(\mu,\ \sigma_x^2). \tag{8}$$

This assumption allows to express the probability of being in category j as equation (9),

$$P(c=j)=P(\log\underline{\theta}_j<x<\log\overline{\theta}_j)$$
$$=\Phi\left(\frac{\overline{\theta}_j-\mu}{\sigma_x}\right)-\Phi\left(\frac{\underline{\theta}_j-\mu}{\sigma_x}\right) \tag{9}$$

where $\Phi(\cdot)$ is the cumulative normal distribution function. Summing across individuals provides the log-likelihood

$$\mathcal{L}(\mu,\ \sigma_x|c)=\Sigma_{i\in N}\Sigma_{j\in J}1[c_i=J]\log P(c_i=j) \tag{10}$$

where $1[\cdot]$ is an indicator function, N is the set of respondents, and J is the set of possible responses to the survey. By using the grouping data and maximizing the log-likelihood, μ and σ_x in the population are obtained. Finally, using the moment generating function and the estimated population parameters, risk tolerance conditional on a survey response in category j is calculated as

$$E(\theta|c=j)=E(\theta|\log\underline{\theta}_j<x<\log\overline{\theta}_j)$$
$$=exp(\mu+\sigma_x^2/2)\frac{\int_{\log\underline{\theta}_j}^{\log\overline{\theta}_j}\frac{1}{\sqrt{2\pi}\,\sigma_x}exp\left(\frac{-(y-\mu-\sigma_x^2)^2}{2\sigma_x^2}\right)dy}{\int_{\log\underline{\theta}_j}^{\log\overline{\theta}_j}\frac{1}{\sqrt{2\pi}\,\sigma_x}exp\left(\frac{-(y-\mu)^2}{2\sigma_x^2}\right)dy}. \tag{11}$$

[Assistant Professor, Faculty of Economics, Kyushu University]